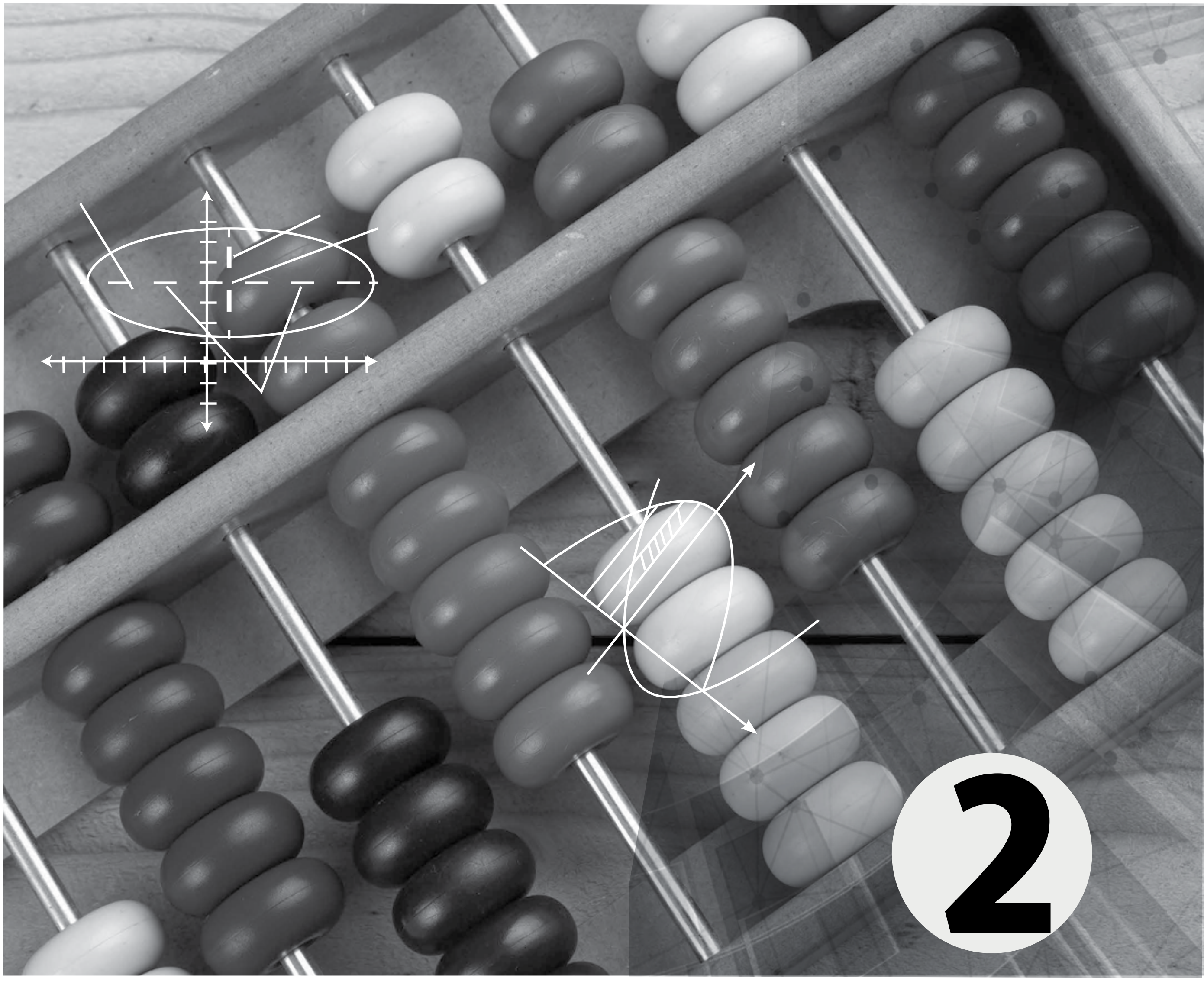
7th EDITION

NEW SYLLABUS MATHEMATICS
TEACHER'S RESOURCE BOOK



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OXFORD
UNIVERSITY PRESS

CONTENTS

Syllabus Matching Grid

Cambridge O Level Mathematics (Syllabus D) 4024/4029. Syllabus for examination in 2018, 2019 and 2020.

Theme or Topic	Subject Content	Reference
1. Number	Identify and use: • Natural numbers • Integers (positive, negative and zero) • Prime numbers • Square numbers • Cube numbers • Common factors and common multiples • Rational and irrational numbers (e.g. π, $\sqrt{2}$) • Real numbers	Book 1: Chapter 1 Chapter 2
2. Set language and notation	• Use set language, set notation and Venn diagrams to describe sets and represent relationships between sets • Definition of sets: e.g. $A = \{x : x \text{ is a natural number}\}$, $B = \{(x, y): y = mx + c\}$, $C = \{x : a \leqslant x \leqslant b\}$, $D = \{a, b, c, \ldots\}$	Book 2: Chapter 14 Book 4: Chapter 2
2. Squares, square roots, cubes and cube roots	Calculate • Squares • Square roots • Cubes and cube roots of numbers	Book 1: Chapter 1 Chapter 2
4. Directed numbers	• Use directed numbers in practical situations	Book 1: Chapter 2
5. Vulgar and decimal fractions and percentages	• Use the language and notation of simple vulgar and decimal fractions and percentages in appropriate contexts • Recognise equivalence and convert between these forms	Book 1: Chapter 2
6. Ordering	• Order quantities by magnitude and demonstrate familiarity with the symbols $=, \neq, <, >, \leqslant, \geqslant$.	Book 1: Chapter 2 Chapter 5
7. Standard form	• Use the standard form $A \times 10^n$, where n is a positive or negative integer, and $1 \leqslant A < 10$.	Book 3: Chapter 4
8. The four operations	Use the four operations for calculations with: • Whole numbers • Decimals • Vulgar (and mixed) fractions including correct ordering of operations and use of brackets.	Book 1: Chapter 2
9. Estimation	• Make estimates of numbers, quantities and lengths • Give approximations to specified numbers of significant figures and decimal places • Round off answers to reasonable accuracy in the context of a given problem	Book 1: Chapter 3
10 Limits of accuracy	• Give appropriate upper and lower bounds for data given to a specified accuracy • Obtain appropriate upper and lower bounds to solutions of simple problems given to a specified accuracy	Book 3: Chapter 3

11. Ratio, proportion, rate	• Demonstrate an understanding of ratio and proportion • Increase and decrease a quantity by a given ratio • Use common measures of rate • Solve problems involving average speed	Book 1: Chapter 9 Book 2: Chapter 1
12. Percentages	• Calculate a given percentage of a quantity • Express one quantity as a percentage of another • Calculate percentage increase or decrease • Carry out calculations involving reverse percentages	Book 1: Chapter 8 Book 3: Chapter 5
13. Use of an electronic calculator	• Use an electronic calculator efficiently • Apply appropriate checks of accuracy • Enter a range of measures including 'time' • Interpret the calculator display appropriately	Book 1: Chapter 2 Chapter 4 Book 2: Chapter 11 Book 3: Chapter 10 Book 4: Chapter 4
14. Time	• Calculate times in terms of the 24-hour and 12-hour clock • Read clocks, dials and timetables	Book 1: Chapter 9
15. Money	• Solve problems involving money and convert from one currency to another	Book 3: Chapter 5
16. Personal and small business finance	• Use given data to solve problems on personal and small business finance involving earnings, simple interest and compound interest • Extract data from tables and charts	Book 3: Chapter 5
17. Algebraic representation and formulae	• Use letters to express generalised numbers and express arithmetic processes algebraically • Substitute numbers for words and letters in formulae • Construct and transform formulae and equations	Book 1: Chapter 4 Chapter 5 Book 2: Chapter 2 Book 3: Chapter 1
18. Algebraic manipulation	• Manipulate directed numbers • Use brackets and extract common factors • Expand product of algebraic expressions • Factorise where possible expressions of the form: $ax + bx + kay + kby$ $a^2x^2 - b^2y^2$ $a^2 + 2ab + b^2$ $ax^2 + bx + c$ • Manipulate algebraic fractions • Factorise and simplify rational expressions	Book 1: Chapter 4 Book 2: Chapter 3 Chapter 4 Chapter 6
19. Indices	• Understand and use the rules of indices • Use and interpret positive, negative, fractional and zero indices	Book 3: Chapter 4

20. Solutions of equations and inequalities	• Solve simple linear equations in one unknown • Solve fractional equations with numerical and linear algebraic denominators • Solve simultaneous linear equations in two unknowns • Solve quadratic equations by factorisation, completing the square or by use of the formula • Solve simple linear inequalities	Book 1: Chapter 5 Book 2: Chapter 2 Chapter 5 Book 3: Chapter 1 Chapter 3
21. Graphical representation of inequalities	• Represent linear inequalities graphically	Book 4: Chapter 1
22. Sequences	• Continue a given number sequence • Recognise patterns in sequences and relationships between different sequences • Generalise sequences as simple algebraic statements	Book 1: Chapter 7
23. Variation	• Express direct and inverse variation in algebraic terms and use this form of expression to find unknown quantities	Book 2: Chapter 1
24. Graphs in practical situations	• Interpret and use graphs in practical situations including travel graphs and conversion graphs • Draw graphs from given data • Apply the idea of rate of change to easy kinematics involving distance-time and speed-time graphs, acceleration and deceleration • Calculate distance travelled as area under a linear speed-time graph	Book 1: Chapter 6 Book 2: Chapter 2 Book 3: Chapter 7
25. Graphs in practical situations	• Construct tables of values and draw graphs for functions of the form ax^n where a is a rational constant, $n = -2, -1, 0, 1, 2, 3$, and simple sums of not more than three of these and for functions of the form ka^x where a is a positive integer • Interpret graphs of linear, quadratic, cubic, reciprocal and exponential functions • Solve associated equations approximately by graphical methods • Estimate gradients of curve by drawing tangents	Book 1: Chapter 6 Book 2: Chapter 1 Chapter 2 Chapter 5 Book 3: Chapter 1 Chapter 7
26. Function notation	• Use function notation, e.g. $f(x) = 3x - 5$; $f : x \mapsto 3x - 5$, to describe simple functions • Find inverse functions $f^{-1}(x)$	Book 2: Chapter 7 Book 3: Chapter 2
27. Coordinate geometry	• Demonstrate familiarity with Cartesian coordinates in two dimensions • Find the gradient of a straight line • Calculate the gradient of a straight line from the coordinates of two points on it • Calculate the length and the coordinates of the midpoint of a line segment from the coordinates of its end points • Interpret and obtain the equation of a straight line graph in the form $y = mx + c$ • Determine the equation of a straight line parallel to a given line • Find the gradient of parallel and perpendicular lines	Book 1: Chapter 6 Book 2: Chapter 2 Book 3: Chapter 6

28. Geometrical terms	• Use and interpret the geometrical terms: point; line; plane; parallel; perpendicular; bearing; right angle, acute, obtuse and reflex angles; interior and exterior angles; similarity and congruence • Use and interpret vocabulary of triangles, special quadrilaterals, circles, polygons and simple solid figures • Understand and use the terms: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment	Book 1: Chapter 10 Chapter 11 Book 2: Chapter 8 Book 3: Chapter 9 to Chapter 13
29. Geometrical constructions	• Measure lines and angles • Construct a triangle, given the three sides, using a ruler and a pair of compasses only • Construct other simple geometrical figures from given data, using a ruler and protractor as necessary • Construct angle bisectors and perpendicular bisectors using a pair of compasses as necessary • Read and make scale drawings • Use and interpret nets	Book 1: Chapter 12 Chapter 14 Book 2: Chapter 8 Book 4: Chapter 8
30. Similarity and congruence	• Solve problems and give simple explanations involving similarity and congruence • Calculate lengths of similar figures • Use the relationships between areas of similar triangles, with corresponding results for similar figures, and extension to volumes and surface areas of similar solids	Book 2: Chapter 8 Book 3: Chapter 11 Chapter 12
31. Symmetry	• Recognise rotational and line symmetry (including order of rotational symmetry) in two dimensions • Recognise symmetry properties of the prism (including cylinder) and the pyramid (including cone) • Use the following symmetry properties of circles: (a) equal chords are equidistant from the centre (b) the perpendicular bisector of a chord passes through the centre (c) tangents from an external point are equal in length	Book 2: Chapter 13 Book 3: Chapter 13
32. Angles	• Calculate unknown angles and give simple explanations using the following geometrical properties: (a) angles at a point (b) angles at a point on a straight line and intersecting straight lines (c) angles formed within parallel lines (d) angle properties of triangles and quadrilaterals (e) angle properties of regular and irregular polygons (f) angle in a semi-circle (g) angle between tangent and radius of a circle (h) angle at the centre of a circle is twice the angle at the circumference (i) angles in the same segment are equal (j) angles in opposite segments are supplementary	Book 1: Chapter 10 Chapter 11 Book 3: Chapter 13
33. Loci	• Use the following loci and the method of intersecting loci for sets of points in two dimensions which are: (a) at a given distance from a given point (b) at a given distance from a given straight line (c) equidistant from two given points (d) equidistant from two given intersecting straight line	Book 4: Chapter 8
34. Measures	• Use current units of mass, length, area, volume and capacity in practical situations and express quantities in terms of larger or smaller units	Book 1: Chapter 13 Chapter 14

35. Mensuration	• Solve problems involving: (a) the perimeter and area of a rectangle and triangle (b) the perimeter and area of a parallelogram and a trapezium (c) the circumference and area of a circle (d) arc length and sector area as fractions of the circumference and area of a circle (e) the surface area and volume of a cuboid, cylinder, prism, sphere, pyramid and cone (f) the areas and volumes of compound shapes	Book 1: Chapter 13 Chapter 14 Book 2: Chapter 12 Book 3: Chapter 10
36. Trigonometry	• Interpret and use three-figure bearings • Apply Pythagoras' theorem and the sine, cosine and tangent ratios for acute angles to the calculation of a side or an angle of a right-angled triangles • Solve trigonometrical problems in two dimensions involving angles of elevation and depression • Extend sine and cosine functions to angles between 90° and 180° • Solve problems using the sine and cosine rules for any triangle and the formula area of triangle = $\frac{1}{2}ab \sin C$ • Solve simple trigonometrical problems in three dimensions	Book 2: Chapter 10 Chapter 11 Book 3: Chapter 8 Chapter 9
37. Vectors in two dimensions	• Describe a translation by using a vector represented by $\begin{pmatrix} x \\ y \end{pmatrix}$, $\overrightarrow{AB}$ or **a** • Add and subtract vectors • Multiple a vector by a scalar • Calculate the magnitude of a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ as $\sqrt{x^2 + y^2}$ • Represent vectors by directed line segments • Use the sum and difference of two vectors to express given vectors in terms of two coplanar vectors • Use position vectors	Book 4: Chapter 7
38. Matrices	• Display information in the form of a matrix of any order • Solve problems involving the calculation of the sum and product (where appropriate) of two matrices, and interpret the results • Calculate the product of a matrix and a scalar quantity • Use the algebra of 2 × 2 matrices including the zero and identity 2 × 2 matrices • Calculate the determinant $\|\mathbf{A}\|$ and inverse $\mathbf{A}^{-1}$ of a non-singular matrix **A**	Book 4: Chapter 5
39. Transformations	• Use the following transformations of the plane: reflection (M), rotation (R), translation (T), enlargement (E) and their combinations • Identify and give precise descriptions of transformations connecting given figures • Describe transformations using coordinates and matrices	Book 2: Chapter 9 Book 4: Chapter 6
40. Probability	• Calculate the probability of a single event as either a fraction or a decimal • Understand that the probability of an event occurring = 1 – the probability of the event not occurring • Understand relative frequency as an estimate of probability • Calculate the probability of simple combined events using possibility diagrams and tree diagrams where appropriate	Book 2: Chapter 15 Book 4: Chapter 3

41. Categorical, numerical and grouped data	• Collect, classify and tabulate statistical data • Read, interpret and draw simple inferences from tables and statistical diagrams • Calculate the mean, median, mode and range for individual and discrete data and distinguish between the purposes for which they are used • Calculate an estimate of the mean for grouped and continuous data • Identify the modal class from a grouped frequency distribution	Book 1: Chapter 15 Book 2: Chapter 17 Book 4: Chapter 4
42. Statistical diagrams	• Construct and interpret bar charts, pie charts, pictograms, simple frequency distributions, frequency polygons, histograms with equal and unequal intervals and scatter diagrams • Construct and use cumulative frequency diagrams • Estimate and interpret the median, percentiles, quartiles and interquartile range for cumulative frequency diagrams • Calculate with frequency density • Understand what is meant by positive, negative and zero correlation with reference to a scatter diagram • Draw a straight line of best fit by eye	Book 1: Chapter 15 Book 2: Chapter 16 Book 4: Chapter 4

Secondary 2 Mathematics Scheme of Work

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
1	**1 Direct and Inverse Proportions**	**1.1 Direct Proportion** (pp. 3 – 5)	• Explain the concept of direct proportion • Solve problems involving direct proportion	Demonstrate an understanding of ratio and proportion	Investigation – Direct Proportion (p. 3) Class Discussion – Real-Life Examples of Quantities in Direct Proportion (p. 4)			Investigation – Direct Proportion (p. 3) Class Discussion – Real-Life Examples of Quantities in Direct Proportion (p. 4)
1		**1.2 Algebraic and Graphical Representation of Direct Proportion** (pp. 6 – 12)	• Explain the concept of direct proportion using tables, equations and graphs • Solve problems involving direct proportion	Express direct variation in algebraic terms and use this form of expression to find unknown quantities Construct tables of values and draw graphs for functions of the form ax^n where a is a rational constant, n = –2, –1, 0, 1, 2, 3, and simple sums of not more than three of these and for functions of the form ka^x where a is a positive integer	Thinking Time (p. 6) Investigation – Graphical Representation of Direct Proportion (pp. 6 – 7) Thinking Time (p. 7)			Thinking Time (p. 6) Thinking Time (p. 7) Ex 1A Q 15(iv) (p. 12)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
1		**1.3 Other Forms of Direct Proportion** (pp. 12 – 18)	• Explain the concept of direct proportion using tables, equations and graphs • Solve problems involving direct proportion		Investigation – Other Forms of Direct Proportion (pp. 12 – 14) Thinking Time (p. 15)			Investigation – Other Forms of Direct Proportion (pp. 12 – 14) Thinking Time (p. 15)
2		**1.4 Inverse Proportion** (pp. 19 – 22)	• Explain the concept of inverse proportion • Solve problems involving inverse proportion	Demonstrate an understanding of ratio and proportion	Investigation – Inverse Proportion (p. 19) Class Discussion – Real-Life Examples of Quantities in Inverse Proportion (p. 20)			Investigation – Inverse Proportion (p. 19) Class Discussion – Real-Life Examples o f Quantities in Inverse Proportion (p. 20)
2		**1.5 Algebraic and Graphical Representations of Inverse Proportion** (pp. 23 – 29)	• Explain the concept of inverse proportion using tables, equations and graphs • Solve problems involving inverse proportion	Express inverse variation in algebraic terms and use this form of expression to find unknown quantities Construct tables of values and draw graphs for functions of the form ax^n where a is a rational constant, $n = -2, -1, 0, 1, 2, 3$, and simple sums of not more than three of these and for functions of the form ka^x where a is a positive integer	Thinking Time (p. 23) Investigation – Graphical Representation of Inverse Proportion (pp. 23 – 25) Thinking Time (p. 26)			Thinking Time (p. 23) Thinking Time (p. 26) Just For Fun (p. 26)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
2		**1.6 Other Forms of Inverse Proportion** (pp. 29 – 33)	• Explain the concept of inverse proportion using tables, equations and graphs • Solve problems involving inverse proportion		Worked Example 14 (p.32)			
2		**Miscellaneous**					Solutions for Challenge Yourself	
3	**2 Linear Graphs and Simultaneous Linear Equations**	**2.1 Gradient of a Straight Line** (pp. 39 – 50)	• Find the gradient of a straight line • State the y-intercept of a straight line	Find the gradient of a straight line Calculate the gradient of a straight line from the coordinates of two points on it	Investigation – Equation of a Straight Line (p. 39) Class Discussion – Gradients of Straight Lines (p. 44) Class Discussion – Gradients in the Real World (p. 44) Investigation – Gradient of a Horizontal Line (p. 45) Investigation – Gradient of a Vertical Line (p. 46)	Investigation – Equation of a Straight Line (p. 39)		

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
		2.2 Further Applications of Linear Graphs in Real-World Contexts (pp. 50 – 54)		Apply the idea of rate of change to easy kinematics involving distance-time and speed-time graphs	Worked Example 2 (p. 51)			
		2.3 Horizontal and Vertical Lines (pp. 55 – 58)	• State the equation of a horizontal line and of a vertical line		Investigation – Equation of a Horizontal Line (pp. 55 – 56) Investigation – Equation of a Vertical Line (p. 57)			Investigation – Equation of a Horizontal Line (pp. 55 – 56) Investigation – Equation of a Vertical Line (p. 57)
3		**2.4 Graphs of Linear Equations in the form $ax + by = k$** (pp. 58 – 62)	• Draw graphs of linear equations in the form $ax + by = k$	Draw graphs from given data	Investigation – Graphs of $ax + by = k$ (p. 59)	Investigation – Graphs of $ax + by = k$ (p. 59)		Investigation – Graphs of $ax + by = k$ (p. 59)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
3		**2.5 Solving Simultaneous Linear Equations Using Graphical Method** (pp. 62 – 67)	• Solve simultaneous linear equations in two variables using the graphical method	Solve simultaneous linear equations in two unknowns Solve associated equations approximately by graphical methods	Investigation – Solving Simultaneous Linear Equations Graphically (p. 63)	Investigation – Solving Simultaneous Linear Equations Graphically (p. 63) Class Discussion – Coincident Lines and Parallel Lines (p. 66)		Investigation – Solving Simultaneous Linear Equations Graphically (p. 63) Class Discussion – Choices of Appropriate Scales for Graphs and Accuracy of Graphs (p. 64) Class Discussion – Coincident Lines and Parallel Lines (p. 66) Thinking Time (p. 66)
					Class Discussion – Choices of Appropriate Scales for Graphs and Accuracy of Graphs (p. 64) Class Discussion – Coincident Lines and Parallel Lines (p. 66) Thinking Time (p. 66)			
4		**2.6 Solving Simultaneous Linear Equations Using Algebraic Methods** (pp. 67 – 76)	• Solve simultaneous linear equations in two variables using the elimination method • Solve simultaneous linear equations in two variables using the substitution method	Construct and transform formulae and equations Solve simultaneous linear equations in two unknowns	Thinking Time (p. 68) Thinking Time (p. 71) Thinking Time (p. 73) Thinking Time (p. 74)			Thinking Time (p. 73) Thinking Time (p. 74)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
4		**2.7 Applications of Simultaneous Equations in Real-World Contexts** (pp. 77 – 83)	• Formulate a pair of linear equations in two variables to solve mathematical and real-life problems		Thinking Time (p. 78)	Just For Fun (p. 81)		Just For Fun (p. 81)
4		**Miscellaneous**					Solutions for Challenge Yourself	
5	**3 Expansion and Factorisation of Quadratic Expressions**	**3.1 Quadratic Expressions** (pp. 91 – 96)	• Recognise quadratic expressions			Practise Now (p. 93) Practise Now (p. 94) Practise Now (p. 96)		
5		**3.2 Expansion and Simplification of Quadratic Expressions** (pp. 97 – 105)	• Expand and simplify quadratic expressions		Class Discussion – Expansion of Quadratic Expressions of the Form $(a + b)(c + d)$ (p. 102)	Practise Now (p. 98) Class Discussion – Expansion of Quadratic Expressions of the Form $(a + b)(c + d)$ (p. 102)		
6		**3.3 Factorisation of Quadratic Expressions** (pp. 105 – 111)	• Use a multiplication frame to factorise quadratic expressions	Factorise where possible expressions of the form: $a^2 + 2ab + b^2$ $ax^2 + bx + c$	Main Text (pp. 105 – 107)	Practise Now (p. 107)		
6		**Miscellaneous**					Solutions for Challenge Yourself	

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
6	**4 Further Expansion and Factorisation of Algebraic Expressions**	**4.1 Expansion and Factorisation of Algebraic Expressions** (pp. 115 – 120)	• Expand and simplify algebraic expressions • Use a multiplication frame to factorise algebraic expressions	Expand product of algebraic expressions	Thinking Time (p. 118)			
7		**4.2 Expansion Using Special Algebraic Identities** (pp. 121 – 125)	• Recognise and apply the three special algebraic identities to expand algebraic expressions		Class Discussion – Special Algebraic Identities (p. 121)			
7		**4.3 Factorisation Using Special Algebraic Identities** (pp. 125 – 128)	• Recognise and apply the three special algebraic identities to factorise algebraic expressions	Factorise where possible expressions of the form: $a^2 + 2ab + b^2$ $ax^2 + bx + c$ $ax + bx + kay + kby$ $a^2x^2 - b^2y^2$				
8		**4.4 Factorisation by Grouping** (pp. 128 – 132)	• Factorise algebraic expressions by grouping		Thinking Time (p. 130) Class Discussion – Equivalent Expressions (p. 131)			Class Discussion – Equivalent Expressions (p. 131)
8		**Miscellaneous**					Solutions for Challenge Yourself	

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
8	**5 Quadratic Equations and Graphs**	**5.1 Solving Quadratic Equations by Factorisation** (pp. 139 – 142)	• Solve quadratic equations by factorisation	Solve quadratic equations by factorisation	Thinking Time (p. 141)			Thinking Time (p. 141)
9		**5.2 Applications of Quadratic Equations in Real-World Contexts** (pp. 143 – 146)	• Solve mathematical and real-life problems involving quadratic equations algebraically					
9		**5.3 Graphs of Quadratic Functions** (pp. 129 – 139)	• Draw graphs of quadratic functions • State the properties of the graphs of quadratic functions • Solve mathematical and real-life problems involving quadratic equations graphically	Interpret graphs of quadratic functions	Investigation – Relationship Between the Area of a Square and the Length of its Side (p. 147) Investigation – Graphs of $y = x^2$ and $y = -x^2$ (p. 148) Investigation – Graphs of $y = ax^2 + bx + c$, where $a \neq 0$ (pp. 149 – 150)	Investigation – Relationship Between the Area of a Square and the Length of its Side (p. 147) Internet Resources (p. 148) Investigation – Graphs of $y = x^2$ and $y = -x^2$ (p. 148) Investigation – Graphs of $y = ax^2 + b + c$, where $a \neq 0$ (pp. 149 – 150)		Investigation – Relationship Between the Area of a Square and the Length of its Side (p. 147) Investigation – Graphs of $y = x^2$ and $y = -x^2$ (p. 148) Investigation – Graphs of $y = ax^2$, where $a \neq 0$ (pp. 149 – 150)
9		**Miscellaneous**					Solutions for Challenge Yourself	

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
10	**6 Algebraic Fractions and Formulae**	**6.1 Algebraic Fractions** (pp. 163 – 165)	• Simplify simple algebraic fractions	Manipulate algebraic fractions				
10		**6.2 Multiplication and Division of Algebraic Fractions** (pp. 165 – 167)	• Multiply and divide simple algebraic fractions					
10		**6.3 Addition and Subtraction of Algebraic Fractions** (pp. 168 – 170)	• Add and subtract algebraic fractions with linear or quadratic denominators					
11		**6.4 Manipulation of Algebraic Formulae** (pp. 171 – 177)	• Change the subject of a formula • Find the value of an unknown in a formula		Class Discussion – Finding the Value of an Unknown in a Formula (p. 173)			Class Discussion – Finding the Value of an Unknown in a Formula (p. 173)
11		**Miscellaneous**					Solutions for Challenge Yourself	
12	**7 Relations and Functions**	**7.1 Relations** (p. 183)						
12		**7.2 Functions** (pp. 184 – 189)	• Define a function • Verify if a given relation is a function • Solve problems on functions involving linear expressions	Use function notation, e.g. $f(x) = 3x - 5$; $f : x \mapsto 3x - 5$, to describe simple functions	Thinking Time (p.184)			Thinking Time (p.184)
12		**Miscellaneous**					Solutions for Challenge Yourself	

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
13	**8 Congruence and Similarity**	**8.1 Congruent Figures** (pp. 193 – 202)	• Examine whether two figures are congruent • Solve simple problems involving congruence	Use and interpret the geometrical term: congruence Solve problems and give simple explanations involving congruence	Investigation – Properties of Congruent Figures (p. 193) Thinking Time (p. 194)	Internet Resources (p. 195)		Investigation – Properties of Congruent Figures (p. 193) Thinking Time (p. 194) Class Discussion – Congruence in the Real World (p. 195)
					Main Text (p. 194) Class Discussion – Congruence in the Real World (p. 195)			
13		**8.2 Similar Figures** (pp. 203 – 212)	• Examine whether two figures are similar • State the properties of similar triangles and polygons • Solve simple problems involving similarity	Use and interpret the geometrical term: similarity Solve problems and give simple explanations involving similarity Calculate lengths of similar figures	Class Discussion – Similarity in the Real World (p. 203) Investigation – Properties of Similar Polygons (p. 204) Thinking Time (p. 205)			Class Discussion – Similarity in the Real World (p. 203) Investigation – Properties of Similar Polygons (p. 204) Thinking Time (p. 205) Class Discussion – Identifying Similar Triangles (p. 206)
					Class Discussion – Identifying Similar Triangles (p. 206)			

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
14		**8.3 Similarity, Enlargement and Scale Drawings** (pp. 213 – 228)	• Make simple scale drawings with appropriate scales • Interpret scales on maps	Read and make scale drawings	Just For Fun (p. 213) Main Text (p. 215) Main Text (p. 222) Main Text (p. 223) Performance Task (p. 224)			Performance Task (p. 224)
14		**Miscellaneous**					Solutions for Challenge Yourself	
15	**9 Geometrical Transformation**	**9.1 Reflection** (pp. 235 – 240)	• Reflect an object and find the line of reflection by construction	Use the following transformations of the plane: reflection (M), rotation (R) and translation (T) Identify and give precise descriptions of transformations connecting given figures	Thinking Time (p. 237)			Attention (p. 236) Thinking Time (p. 237)
15		**9.1 Rotation** (pp. 241 – 246)	• Rotate an object and find the centre of rotation by construction		Thinking Time (p. 242) Thinking Time (p. 243)			Thinking Time (p.242) Thinking Time (p. 243)
16		**9.1 Translation** (pp. 246 – 250)	• Translate an object		Thinking Time (p. 246) Thinking Time (p. 248) Journal Writing (p. 248)			Thinking Time (p. 246) Thinking Time (p. 248)
16		**Miscellaneous**					Solutions for Challenge Yourself	

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
16	**10 Pythagoras' Theorem**	**10.1 Pythagoras' Theorem** (pp. 259 – 268)	• Solve problems using Pythagoras' Theorem	Apply Pythagoras' theorem to the calculation of a side or an angle of a right-angled triangle	Investigation – Pythagoras' Theorem – The Secret of the Rope-Stretchers (pp. 260 – 261)	Investigation – Pythagoras' Theorem – The Secret of the Rope-Stretchers (pp. 260 – 261)		Investigation – Pythagoras' Theorem – The Secret of the Rope-Stretchers (pp. 260 – 261)
					Performance Task (p. 262)	Performance Task (p. 262) Internet Resources (p. 262)		Performance Task (p. 262)
17		**10.2 Applications of Pythagoras' Theorem in Real-World Contexts** (pp. 269 – 275)	• Solve problems using Pythagoras' Theorem					
17		**10.3 Converse of Pythagoras' Theorem** (pp. 276 – 277)	• Determine whether a triangle is a right-angled triangle given the lengths of three sides					
17		**Miscellaneous**					Solutions for Challenge Yourself	

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
18	**11 Trigonometric Ratios**	**11.1 Trigonometric Ratios** (pp. 283 – 288)	• Explain what trigonometric ratios of acute angles are		Investigation – Trigonometric Ratios (p. 283) Thinking Time (p. 284)			Investigation – Trigonometric Ratios (p. 283) Thinking Time (p. 284)
				Use of an electronic calculator efficiently	Worked Example 1 (p. 287)			
18		**11.2 Applications of Trigonometric Ratios to Find Unknown Sides of Right-Angled Triangles** (pp. 289 – 295)	• Find the unknown sides in right-angled triangles	Apply the sine, cosine and tangent ratios for acute angles to the calculation of a side of a right-angled triangle				
19		**11.3 Applications of Trigonometric Ratios to Find Unknown Angles in Right-Angled Triangles** (pp. 296 – 301)	• Find the unknown angles in right-angled triangles	Apply the sine, cosine and tangent ratios for acute angles to the calculation of an angle of a right-angled triangle				
19		**11.4 Applications of Trigonometric Ratios in Real-World Contexts** (pp. 302 – 310)	• Apply trigonometric ratios to solve problems in real-world contexts	Solve trigonometrical problems in two dimensions involving angles of elevation and depression	Investigation – Using a Clinometer to Find the Height of an Object (pp. 302 – 303)			
19		**Miscellaneous**					Solutions for Challenge Yourself	

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
20	**12** **Volume and Surface Area of Pyramids, Cones and Spheres**	**12.1 Volume and Surface Area of Pyramids** (pp. 317 – 328)	• Identify and sketch pyramids • Draw and use nets of pyramids to visualise their surface area • Use formulae to calculate the volume and the surface area of pyramids	Solve problems involving the surface area and volume of a pyramid	Class Discussion – What are Pyramids? (p. 317) Thinking Time (p. 319)	Internet Resources (p. 321)		Class Discussion – What are Pyramids? (p. 317) Thinking Time (p. 319) Journal Writing (p. 319)
					Journal Writing (p. 319) Investigation – Volume of Pyramids (pp. 320 – 321)			
20		**12.2 Volume and Surface Area of Cones** (pp. 328 – 337)	• Identify and sketch cones • Draw and use nets of cones to visualise their surface area • Use formulae to calculate the volume and the surface area of cones	Solve problems involving the surface area and volume of a cone	Class Discussion – What are Cones? (p. 328)			Class Discussion – What are Cones? (p. 328) Journal Writing (p. 329) Investigation – Comparison between a Cone and a Pyramid (p. 330) Thinking Time (p. 331) Investigation – Curved Surface Area of Cones (pp. 333 – 334)
					Journal Writing (p. 329) Investigation – Comparison between a Cone and a Pyramid (p. 330)			
					Thinking Time (p. 331) Investigation – Curved Surface Area of Cones (pp. 333 – 334) Thinking Time (p. 334)			

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
21		**12.3 Volume and Surface Area of Spheres** (pp. 338 – 343)	• Identify and sketch spheres • Use formulae to calculate the volume and the surface area of spheres	Solve problems involving the surface area and volume of a sphere	Thinking Time (p. 338) Class Discussion – Is the King's Crown Made of Pure Gold? (pp. 338 – 339)			Thinking Time (p. 338) Class Discussion – Is the King's Crown Made of Pure Gold? (pp. 338 – 339) Investigation – Surface Area of Spheres (p. 341)
					Investigation – Volume of Spheres (pp. 339 – 340) Investigation – Surface Area of Spheres (p. 341) Thinking Time (p. 342)			
21		**12.4 Volume and Surface Area of Composite Solids** (pp. 344 – 349)	• Solve problems involving the volume and the surface area of composite solids made up of pyramids, cones, spheres, prisms and cylinders	Solve problems involving the surface area and volume of compound shapes				
21		**Miscellaneous**					Solutions for Challenge Yourself	

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
22	**13 Geometrical Transformation**	**13.1 Line symmetry** (p. 357 – 367)	• Identify line symmetry of plane figures	Recognise rotational and line symmetry (including order of rotational symmetry) in two dimensions	Investigation – Line Symmetry in Two Dimensions (p. 357) Thinking Time (p. 358) Thinking Time (p. 359)			Thinking Time (p. 358) Thinking Time (p. 359)
22		**13.2 Rotational Symmetry in Plane Figures** (p. 368 – 372)	• Identify rotational symmetry of plane figures		Investigation – Rotational Symmetry in Two Dimensions (p. 368) Class Discussion – Line and Rotational Symmetry in Circles (p. 369)			Class Discussion – Line and Rotational Symmetry in Circles (p. 369)
23		**13.3 Symmetry in Triangles, Quadrilaterals and Polygons** (p. 372 – 378)	• Make use of the symmetrical properties of triangles, quadrilaterals and regular polygons		Investigation – Symmetry in Triangles (pp. 372 – 373) Investigation – Symmetry in Special Quadrilaterals (pp. 374 – 375) Investigation – Symmetry in Regular Polygons (pp. 377 – 378)			Investigation – Symmetry in Triangles (pp. 372 – 373) Investigation – Symmetry in Special Quadrilaterals (pp. 374 – 375) Investigation – Symmetry in Regular Polygons (pp. 377 – 378)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
23		**13.4 Symmetry in Three Dimensions** (p. 379 – 383)	• Use symmetrical properties of simple solids	Recognise symmetry properties of the prism (including cylinder) and the pyramid (including cone)	Thinking Time (p. 379) Thinking Time (p. 380) Investigation – Symmetry in Cylinders and Cones (p. 380)			Thinking Time (p. 379) Thinking Time (p. 380) Investigation – Symmetry in Cylinders and Cones (p. 380)
23		**Miscellaneous**					Solutions for Challenge Yourself	
24	**14 Sets**	**14.1 Introduction to Set Notations** (p. 393 – 398)	• Describe a set in words, list all the elements in a set, and describe the elements in a set • State and use the terms ‘set’, ‘element’, ‘equal sets’, ‘empty set’	Use set language, set notation and Venn diagrams to describe sets and represent relationships between sets Definition of sets: e.g. $A = \{x : x \text{ is a natural number}\}$, $B = \{(x, y): y = mx + c\}$, $C = \{x : a \le x \le b\}$, $D = \{a, b, c, \ldots\}$	Class Discussion – Well-defined and Distinct Objects in a Set (p. 394) Thinking Time (p. 396)			Class Discussion – Well-defined and Distinct Objects in a Set (p. 394) Attention (p. 394) Thinking Time (p. 396)
24		**14.2 Venn Diagrams, Universal Set and Complement of a Set** (p. 399 – 405)	• State and use the terms ‘universal set’, ‘complement of a set’, ‘subset’, ‘proper subset’ • Use Venn diagrams to represent sets, including universal sets, complement of a set and proper subsets		Thinking Time (p. 401) Class Discussion – Understanding Subsets (p. 402)			Thinking Time (p. 401) Class Discussion – Understanding Subsets (p. 402)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
24		**14.3 Intersection of Two Sets** (p. 406 – 407)	• State and use the term 'intersection of two sets', • Use Venn diagrams to represent sets, including intersection of two sets					
25		**14.4 Union of Two Sets** (p. 408 – 409)	• State and use the term 'union of two sets' • Use Venn diagrams to represent sets, including union of two sets					
25		**14.5 Combining Universal Set, Complement of a Set, Subset, Intersection and Union of Sets** (p. 409 – 416)	• Solve problems using set notations and Venn diagrams		Worked Example 9 (pp. 411 – 412) Thinking Time (p. 412) Performance Task (p. 413)			Thinking Time (p. 412) Performance Task (p. 413)
25		**Miscellaneous**					Solutions for Challenge Yourself	
26	**15 Probability of Single Events**	**15.1 Introduction to Probability** (p. 421)	• Define probability as a measure of chance	Understand relative frequency as an estimate of probability	Thinking Time (p. 421)			Thinking Time (p. 421)
26		**15.2 Sample Space** (pp. 422 – 424)	• List the sample space of a probability experiment		Main Text (p. 422)			

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
26		**15.3 Probability of Single Events** (pp. 425 – 434)	• Find the probability of a single event	Calculate the probability of a single event as either a fraction or a decimal Understanding that the probability of an event occurring = 1 – probability of the event not occurring	Investigation – Tossing a Coin (pp. 425 – 426) Investigation – Rolling a Die (pp. 426 – 427) Thinking Time (pp. 429) Performance Task (pp. 429) Just for Fun (pp. 431)	Internet Resources (p. 426) Investigation – Rolling a Die (pp. 426 – 427) Internet Resources (p. 430)		Investigation – Tossing a Coin (pp. 425 – 426) Investigation – Rolling a Die (pp. 426 – 427) Thinking Time (p. 429) Performance Task (p. 429)
27		**15.4 Further Examples on Probability of Single Events** (pp. 323 – 326)	• Solve problems involving the probability of single events					
27		**Miscellaneous**					Solutions for Challenge Yourself	
27	**16 Statistical Diagrams**	**16.1 Statistical Diagrams** (p. 445)						
27		**16.2 Dot Diagrams** (pp. 446 – 448)	• Construct and interpret data from dot diagrams		Main Text (pp. 446 – 448)			
28		**16.3 Stem-and-Leaf Diagrams** (pp. 448 – 451)	• Construct and interpret data from stem-and-leaf diagrams		Main Text (pp. 448 – 451)			

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Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
28		**16.4 Scatter Diagrams** (pp. 452 – 464)	• Construct and interpret data from scatter diagrams	Construct and interpret scatter diagrams Understand what is meant by positive, negative and zero correlation with reference to a scatter diagram Draw a straight line of best fit by eye	Main Text (pp. 452 – 453) Thinking Time (p. 454) Class Discussion – Scatter Diagram with No Correlation (p. 459) Thinking Time (p. 459) Journal Writing (p. 459)			Thinking Time (p. 454) Class Discussion – Scatter Diagram with No Correlation (p. 459) Thinking Time (p. 459) Journal Writing (p. 459)
29		**16.5 Histograms for Ungrouped Data** (pp. 465 – 469)	• Construct and interpret data from histograms • Evaluate the purposes and appropriateness of the use of different statistical diagrams • Explain why some statistical diagrams can lead to a misinterpretation of data	Construct and interpret simple frequency distributions	Main Text (pp. 465 – 468)			Journal Writing (p. 466) Class Discussion – Evaluation of Statistical Diagrams (pp. 468 – 469)
					Journal Writing (p. 466) Class Discussion – Evaluation of Statistical Diagrams (pp. 468 – 469)			

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
29		**16.6 Histograms for Grouped Data** (pp. 469 – 483)	• Construct and interpret data from histograms • Evaluate the purposes and appropriateness of the use of different statis-tical diagrams • Explain why some statistical diagrams can lead to a mis-interpretation of data	Construct and interpret histograms with equal and unequal intervals	Journal Writing (p. 470) Performance Task (p. 472) Class Discussion – Histograms for Grouped Data with Unequal Class Intervals (p. 472 – 474)			Journal Writing (p. 470) Performance Task (p. 472) Class Discussion – Histograms for Grouped Data with Unequal Class Intervals (p. 472 – 474)
				Construct and interpret frequency polygons	Main Text (p. 477)			
29		**Miscellaneous**					Solutions for Challenge Youself	

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
30	**17 Averages of Statistical Data**	**17.1 Mean** (pp. 491 – 500)	• Find the mean of a set of data • Calculate an estimate for the mean	Calculate the mean for individual and discrete data Calculate an estimate of the mean for grouped and continuous data				
30		**17.2 Median** (pp. 501 – 505)	• Find the median of a set of data • Find the class interval where the median lies	Calculate the median for individual and discrete data	Thinking Time (p. 502) Class Discussion – Creating Sets of Data with Given Conditions (p. 503)			Thinking Time (p. 374)
30		**17.3 Mode** (pp. 506 – 508)	• Find the mode of a set of data • State the modal class of a set of grouped data	Calculate the mode for individual and discrete data Identify the modal class from a grouped frequency distribution	Thinking Time (p. 507)			
31		**17.4 Mean, Median and Mode** (p. 509 – 516)	• Evaluate the purposes and appropriateness of the use of mean, median and mode	Distinguish between the purposes for which the mean, median and mode are used	Thinking Time (p. 510) Class Discussion – Comparison of Mean, Median and Mode (p. 511)			Thinking Time (p. 570) Class Discussion – Comparison of Mean, Median and Mode (p. 511)
31		**Miscellaneous**					Solutions for Challenge Yourself	

Chapter 1 Direct and Inverse Proportion

TEACHING NOTES

Suggested Approach

In Secondary One, students have learnt rates such as \$0.25 per egg, or 13.5 km per litre of petrol etc. Teachers may wish to expand this further by asking what the prices of 2, 4 or 10 eggs are, or the distance that can be covered with 2, 4 or 10 litres of petrol, and leading to the introduction of direct proportion. After students are familiar with direct proportion, teachers can show the opposite scenario that is inverse proportions.

Section 1.1: Direct Proportion

When introducing direct proportion, rates need not be stated explicitly. Rates can be used implicitly (see Investigation: Direct Proportion). By showing how one quantity increases proportionally with the other quantity, the concept should be easily relatable. More examples of direct proportion should be discussed and explored to test and enhance thinking and analysis skills (see Class Discussion: Real-Life Examples of Quantities in Direct Proportion). Teachers should discuss the linkages between direct proportion, algebra, rates and ratios to assess and improve students' understanding at this stage (see page 4 of the textbook). Teachers should also show the unitary method and proportion method in the worked example and advise students to adopt the method that is most comfortable for them.

Section 1.2: Algebraic and Graphical Representations of Direct Proportion

By recapping what was covered in the previous section, teachers should easily state the direct proportion formula between two quantities and the constant k. It is important to highlight the condition $k \neq 0$ as the relation would not hold if $k = 0$ (see Thinking Time on page 6).

Through studying how direct proportion means graphically (see Investigation: Graphical Representation of Direct Proportion), students will gain an understanding on how direct proportion and linear functions are related, particularly the positive gradient of the straight line and the graph passing through the origin. The graphical representation will act as a test to determine if two variables are directly proportional.

Section 1.3: Other Forms of Direct Proportion

Direct proportion does not always involve two linear variables. If one variable divided by another gives a constant, then the two variables are directly proportional (see Investigation: Other Forms of Direct Proportion). In this case, although the graph of y against x will be a hyperbola, the graph of one variable against the other will be a straight line passing through the origin. Teachers may wish to illustrate the direct proportionality clearly by replacing variables with Y and X and showing $Y = kX$, which is in the form students learnt in the previous section.

Section 1.4: Inverse Proportion

The other form of proportion, inverse proportion, can be explored and studied by students (see Investigation: Inverse Proportion). When one variable increases, the other variable decreases proportionally. It is the main difference between direct and inverse proportion and must be emphasised clearly.

Students should be tasked with giving real-life examples of inverse proportion and explaining how they are inversely proportional (see Class Discussion: Real-Life Examples of Quantities in Inverse Proportion).

Teachers should present another difference between both kinds of proportions by reminding students that $\frac{y}{x}$ is a constant in direct proportion while xy is a constant in inverse proportion (see page 20 of the textbook).

Section 1.5: Algebraic and Graphical Representations of Inverse Proportion

Similar to direct proportion teachers can write the inverse proportion formula between two quantities and the constant k. It is important to highlight the condition $k \neq 0$ as the relation would not hold if $k = 0$ (see Thinking Time on page 23).

Although plotting y against x gives a hyperbola, and does not provide any useful information, teachers can show by plotting y against $\frac{1}{x}$ and showing direct proportionality between the two variables (see Investigation: Graphical Representation of Inverse Proportion).

Section 1.6: Other Forms of Inverse Proportion

Inverse proportion, just like direct proportion, may not involve two linear variables all the time. Again, teachers can replace the variables with Y and X and show the inverse proportionality relation $Y = \frac{k}{x}$.

Challenge Yourself

Question 1 involves stating the relations between the variables algebraically and manipulating the equations. For Question 2, teachers may wish to state the equation relating z, x^2 and $\sqrt{y}$ outright initially, and explain that when z is directly proportional to x^2, $\sqrt{y}$ is treated as a constant. The same applies to when z is inversely proportional to $\sqrt{y}$, x^2 will be considered as a constant. Question 3 follows similarly from Question 2.

WORKED SOLUTIONS

Investigation (Direct Proportion)

1. The fine will increase if the number of days a book is overdue increases.

2. $\frac{\text{Fine when a book is overdue for 6 days}}{\text{Fine when a book is overdue for 3 days}} = \frac{90}{45}$

$= 2$

The fine will be doubled if the number of days a book is overdue is doubled.

3. $\frac{\text{Fine when a book is overdue for 6 days}}{\text{Fine when a book is overdue for 2 days}} = \frac{90}{30}$

$= 3$

The fine will be tripled if the number of days a book is overdue is tripled.

4. $\frac{\text{Fine when a book is overdue for 5 days}}{\text{Fine when a book is overdue for 10 days}} = \frac{75}{150}$

$= \frac{1}{2}$

The fine will be halved if the number of days a book is overdue is halved.

5. $\frac{\text{Fine when a book is overdue for 3 days}}{\text{Fine when a book is overdue for 9 days}} = \frac{45}{135}$

$= \frac{1}{3}$

The fine will be reduced to $\frac{1}{3}$ of the original number if the number of days a book is overdue is reduced to $\frac{1}{3}$ of the original number.

Class Discussion (Real-Life Examples of Quantities in Direct Proportion)

The following are some real-life examples of quantities that are in direct proportion and why they are directly proportional to each other.

- In an hourly-rated job, one gets paid by the number of hours he worked. The longer one works, the more wages he will get.
  The wages one gets is directly proportional to the number of hours he worked.
- A Singapore dollar coin weights approximately 6 g. As the number of coins increases, the total mass of the coins will increase proportionally. The total mass of the coins is directly proportional to the number of coins.
- The circumference of a circle is equivalent to the product of π and the diameter of the circle. As the diameter increases, the circumference increases proportionally. The circumference of the circle is directly proportional to the diameter of the circle.
- The speed of a moving object is the distance travelled by the object per unit time. If the object is moving at a constant speed, as the distance travelled increases, then the time spent in travelling increases proportionally. The distance travelled by the object is directly proportional to the time spent in travelling for an object moving at constant speed.
- The length of a spring can be compressed or extended depending on the force applied on it. The force required to compress or extend a spring is directly proportional to the change in the length of the spring. This is known as Hooke's Law, which has many practical applications in science and engineering.

Teachers may wish to note that the list is not exhaustive.

Thinking Time (Page 6)

If we substitute $k = 0$ into $y = kx$, then $y = 0$.
This implies that for all values of x, $y = 0$.
y cannot be directly proportional to x in this case.

Investigation (Graphical Representation of Direct Proportion)

$y = 15x$ in this context means that for any additional number of a day a book is overdue, the fine will increase by 15 cents.

1.

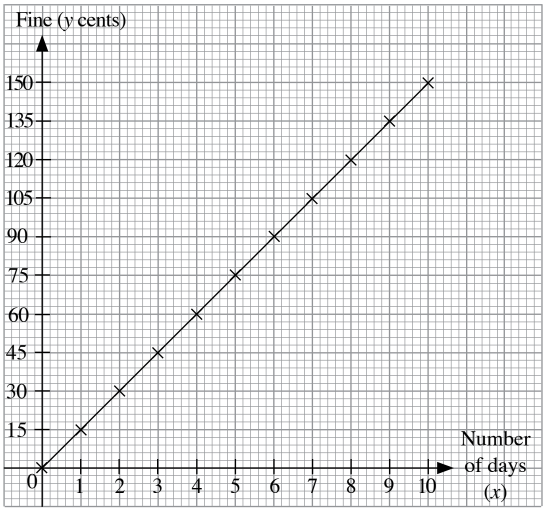


Fig. 1.1

2. The graph is a straight line.

3. The graph passes through the origin.

Thinking Time (Page 7)

1. Since y is directly proportional to x, $y = kx$

$$x = \left(\frac{1}{k}\right)y$$

Since $k \neq 0$, then we can rename $\frac{1}{k} = k_1$ where k_1 is another constant.

Hence, $x = k_1y$, where $k_1 \neq 0$ and x is directly proportional to y.

2. $x = k_1y$ is the equation of a straight line. When $y = 0$, $x = 0$.
We will get a straight line of x against y that passes through the origin.

3. If the graph of y does not pass through the origin, then $y = kx + c$, when $c \neq 0$. Since x and y are not related in the form $y = kx$, y is not directly proportional to x.

4. As x increases, y also increases. This does not necessarily conclude that y is directly proportional to x. It is important that when x increases, y increases **proportionally**. Also, when $x = 0$, $y = 0$.
$y = kx + c$ is an example of how x increases and y increases, but y is not directly proportional to x.

Investigation (Other Forms of Direct Proportion)

1. y is not directly proportional to x. The graph of y against x is not a straight line that passes through the origin.

2.

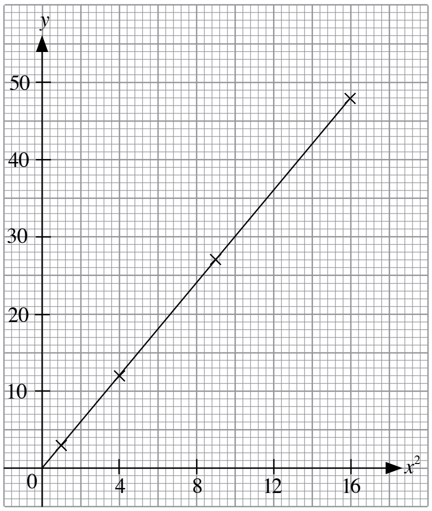


Fig. 1.3

y is directly proportional to x^2. The graph of y against x^2 is a straight line that passes through the origin.

Thinking Time (Page 15)

1.
$$C = 0.2n + 20$$
$$C - 20 = 0.2n$$
$$\frac{C-20}{n} = 0.2$$
$$\frac{n}{C-20} = 5$$

Since $\frac{n}{C-20} = 5$ is a constant, then n is directly proportional to $C - 20$.

The variable is $C - 20$.

2.
$$y - 1 = 4x$$
$$\frac{y-1}{x} = 4$$

Since $\frac{y-1}{x} = 4$ is a constant, then $y - 1$ is directly proportional to x.

Investigation (Inverse Proportion)

1. The time taken decreases when the speed of the car increases.

2. $$\frac{\text{Time taken when speed of the car is 40 km/h}}{\text{Time taken when speed of the car is 20 km/h}} = \frac{3}{6} = \frac{1}{2}$$

The time taken will be halved when the speed of the car is doubled.

3. $$\frac{\text{Time taken when speed of the car is 60 km/h}}{\text{Time taken when speed of the car is 20 km/h}} = \frac{2}{6} = \frac{1}{3}$$

The time taken will be reduced to $\frac{1}{3}$ of the original number when the speed of the car is tripled.

4. $$\frac{\text{Time taken when speed of the car is 30 km/h}}{\text{Time taken when speed of the car is 60 km/h}} = \frac{4}{2} = 2$$

The time taken will be doubled when the speed of the car is halved.

5. $$\frac{\text{Time taken when speed of the car is 40 km/h}}{\text{Time taken when speed of the car is 120 km/h}} = \frac{3}{1} = 3$$

The time taken will be tripled when the speed of the car is reduced to $\frac{1}{3}$ of its original speed.

Class Discussion (Real-Life Examples of Quantities in Inverse Proportion)

The following are some real-life examples of quantities that are in inverse proportion and why they are inversely proportional to each other.

- Soldiers often dig trenches while serving in the army. The more soldiers there are digging the same trench, the faster it will take. The time to dig a trench is therefore inversely proportional to the number of soldiers.
- The area of a rectangle is the product of its length and breadth. Given a rectangle with a fixed area, if the length increases, then the breadth decreases proportionally. Therefore, the length of the rectangle is inversely proportional to the breadth of the rectangle.
- The density of a material is the mass of the material per unit volume. For an object of a material with a fixed mass, the density increases when the volume decreases proportionally. The density of the material is inversely proportional to the volume of the material.
- The speed of a moving object is the distance travelled by the object per unit time. For the same distance, when the speed of the object increases, the time to cover the distance is decreased proportionally. The speed of the object is inversely proportional to the time to cover a fixed distance.
- For a fixed amount of force applied on it, the acceleration of the object is dependent on the mass of the object. When the mass of the object increases or decreases, the acceleration of the object decreases or increases proportionally. This is known as Newton's Second Law and has helped to explain many physical phenomena occurring around us.

Teachers may wish to note that the list is not exhaustive.

Thinking Time (Page 23)

If we substitute $k = 0$ into $y = \frac{k}{x}$, then $y = 0$.

This implies that for all values of x, $y = 0$

y cannot be inversely proportional to x in this case.

Investigation (Graphical Representation of Inverse Proportion)

1. We would obtain a graph of a hyperbola.

2.

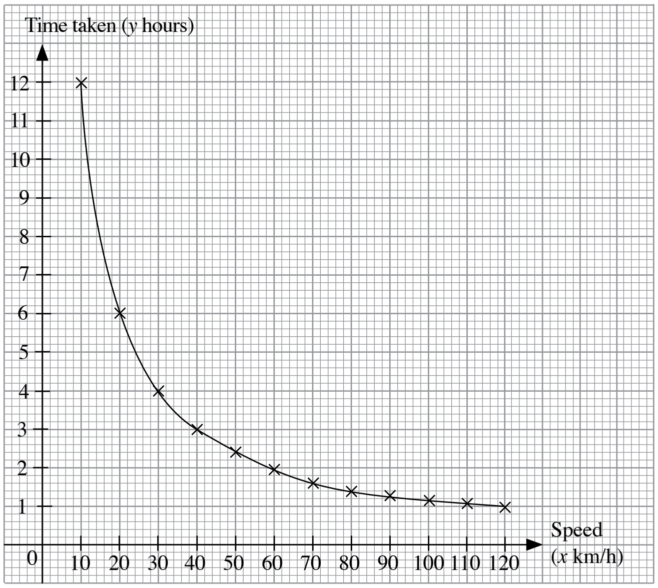


Fig. 1.4

3. When $x = 20$, $y = 6$.
When $x = 40$, $y = 3$.

Change in value of $y = \frac{3}{6}$

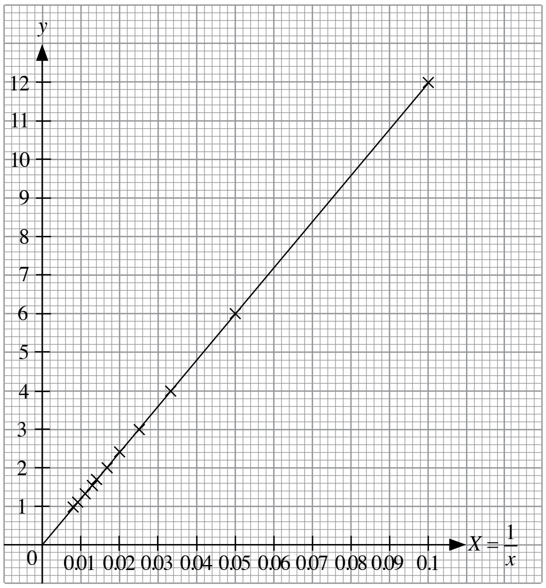
$= \frac{1}{2}$

The value of y will be halved when the value of x is doubled.

Speed (x km/h)	10	20	30	40	50	60
$X = \frac{1}{x}$	0.1	0.05	0.033	0.025	0.02	0.017
Time taken (y hours)	12	6	4	3	2.4	2

Speed (x km/h)	70	80	90	100	110	120
$X = \frac{1}{x}$	0.014	0.013	0.011	0.01	0.009	0.008
Time taken (y hours)	1.7	1.5	1.3	1.2	1.1	1

4.



5. The graph is a straight line that passes through the origin.

$\frac{y}{X}$ is a constant.

6. y is directly proportional to X.

7. $y = kX$, where k is a constant. $\frac{y}{X}$ is a constant and y is directly proportional to X.

Thinking Time (Page 26)

Since y is inversely proportional to x, $y = \frac{k}{x}$

$x = \frac{k}{y}$

Hence, $x = \frac{k}{y}$, where $k \neq 0$ and x is inversely proportional to y.

Practise Now 1

(a) The cost of the sweets is directly proportional to the mass of the sweets.

Method 1: Unitary Method

50 g of sweets cost \$2.10.

1 g of sweets cost $\frac{\$2.10}{50}$.

380 g of sweets cost $\frac{\$2.10}{50} \times 380 = \$15.96$.

\$15.96 = \$15.95 (to the nearest 5 cents)

Method 2: Proportion Method

Let the cost of 380 g of sweets be \$$x$.

Then $\frac{x}{380} = \frac{2.1}{50}$. $\left(\frac{x_1}{y_1} = \frac{x_2}{y_2}\right)$

$x = \frac{2.1}{50} \times 380$

$= 15.96$

$= 15.95$ (to the nearest 5 cents)

Alternatively,

$\frac{x}{2.1} = \frac{380}{50} \left(\frac{x_1}{x_2} = \frac{y_1}{y_2} \right)$

$x = \frac{380}{50} \times 2.1$

$= 15.96$

$= 15.95$ (to the nearest 5 cents)

$\therefore$ 380 g of sweets cost \$15.95.

(b) The amount of metal is directly proportional to the mass of the metal.

Method 1: Unitary Method

$\frac{3}{4}$ of a piece of metal weighs 15 kg.

A whole piece of metal weighs $\frac{15}{\frac{3}{4}}$ kg.

$\frac{2}{5}$ of a piece of metal weighs $\frac{15}{\frac{3}{4}} \times \frac{2}{5} = 8$ kg.

Method 2: Proportion Method

Let the mass of $\frac{2}{5}$ of the piece of metal be x kg.

Then $\frac{x}{\frac{2}{5}} = \frac{15}{\frac{3}{4}} \cdot \left(\frac{x_1}{y_1} = \frac{x_2}{y_2} \right)$

$x = \frac{15}{\frac{3}{4}} \times \frac{2}{5}$

$= 8$

Alternatively,

$\frac{x}{15} = \frac{\frac{2}{5}}{\frac{3}{4}} \left(\frac{x_1}{x_2} = \frac{y_1}{y_2} \right)$

$x = \frac{\frac{2}{5}}{\frac{3}{4}} \times 15$

$= 8$

$\therefore$ The mass of $\frac{2}{5}$ of the piece of metal is 8 kg.

Practise Now 2

1. (i) Since y is directly proportional to x,

then $y = kx$, where k is a constant.

When $x = 2$, $y = 10$,

$10 = k \times 2$

$\therefore k = 5$

$\therefore y = 5x$

(ii) When $x = 10$,

$y = 5 \times 10$

$= 50$

Alternatively,

when $x = 10$, (x increased by 5 times)

$y = 5 \times 10$ (y increased by 5 times)

$= 50$

We can also use $\frac{y_2}{y_1} = \frac{x_2}{x_1}$,

i.e. $\frac{y}{10} = \frac{10}{2}$

$y = 5 \times 10$

$= 50$

(iii) When $y = 60$,

$60 = 5x$

$\therefore x = \frac{60}{5}$

$= 12$

2. Since y is directly proportional to x,

$\frac{y_2}{y_1} = \frac{x_2}{x_1}$

$\frac{y}{5} = \frac{7}{2}$

$y = \frac{7}{2} \times 5$

$= 17.5$

3.

x	4	5	7	8	9.5
y	24	30	42	48	57

Since y is directly proportional to x,

then $y = kx$, where k is a constant.

When $x = 5$, $y = 30$,

$30 = k \times 5$

$\therefore k = 6$

$\therefore y = 6x$

When $y = 48$,

$48 = 6 \times x$

$x = \frac{48}{6}$

$= 8$

When $y = 57$,

$57 = 6 \times x$

$x = \frac{57}{6}$

$= 9.5$

When $x = 4$,

$y = 6 \times 4$

$= 24$

When $x = 7$,

$y = 6 \times 7$

$= 42$

Practise Now 3

(i) Since C is directly proportional to d,

then $C = kd$, where k is a constant.

When $d = 60$, $C = 100$,

$100 = k \times 60$

$\therefore k = \frac{5}{3}$

$\therefore C = \frac{5}{3} d$

(ii) When $d = 45$,

$C = \frac{5}{3} \times 45$

$= 75$

$\therefore$ The cost of transporting goods is \$75.

(iii) When $C = 120$,

$120 = \frac{5}{3} \times d$

$d = 120 \times \frac{3}{5}$

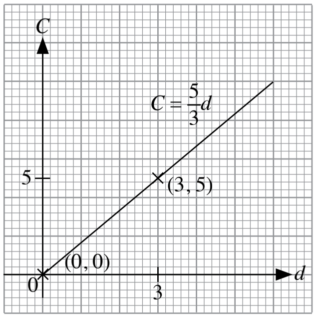
$= 72$

$\therefore$ The distance covered is 72 km.

(iv) $C = \frac{5}{3}d$

When $d = 0$, $C = 0$.

When $d = 3$, $C = 5$.



Practise Now 4

(i) Total monthly cost of running the kindergarten

= \$5000 + 200 × \$41

= \$13 200

(ii) Variable amount = \$20 580 – \$5000

= \$15 580

Number of children enrolled $= \frac{15\,580}{41}$

$= 380$

(iii) Variable amount $= n \times \$41$

$= \$41n$

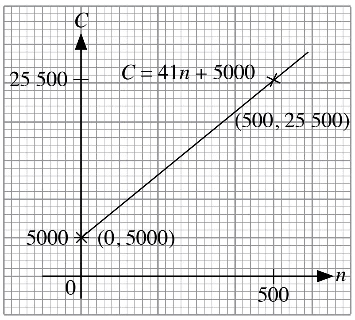
Total monthly cost = variable amount + fixed amount

$\therefore C = 41n + 5000$

(iv) $C = 41n + 5000$

When $n = 0$, $C = 5000$.

When $n = 500$, $C = 25\,500$.



C is *not* directly proportional to n because the line does not pass through the origin.

Practise Now 5

(a) Since $y = 6x^2$, i.e. $\frac{y}{x^2} = 6$ is a constant, then y is directly proportional to x^2.

(b) Since $\sqrt{y} = x^3$, i.e. $\frac{\sqrt{y}}{x^3} = 1$ is a constant, then $\sqrt{y}$ is directly proportional to x^3.

Practise Now 6

1. (i) Since y is directly proportional to x^2,

then $y = kx^2$, where k is a constant.

When $x = 3$, $y = 18$,

$18 = k \times 3^2$

$18 = 9k$

$\therefore k = 2$

$\therefore y = 2x^2$

(ii) When $x = 5$,

$y = 2 \times 5^2$

$= 50$

(iii) When $y = 32$,

$32 = 2x^2$

$x^2 = 16$

$\therefore x = \pm\sqrt{16}$

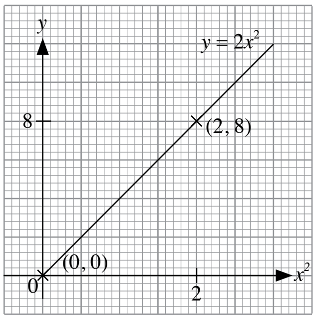
$= \pm 4$

(iv) Since y is directly proportional to x^2, then the graph of y against x^2 is a straight line that passes through the origin.

$y = 2x^2$

When $x = 0$, $y = 0$.

When $x = 2$, $y = 8$.



2. Since y is directly proportional to x^2,

then $y = kx^2$, where k is a constant.

When $x = 2$, $y = 21$,

$21 = k \times 2^2$

$21 = 4k$

$\therefore k = \frac{21}{4}$

$\therefore y = \frac{21}{4}x^2$

When $x = 4$,

$y = \frac{21}{4} \times 4^2$

$= 84$

3.

x	2	2.5	3	5	7
y	36	56.25	81	225	441

Since y is directly proportional to x^2,
then $y = kx^2$, where k is a constant.
When $x = 3$, $y = 81$,
$81 = k \times 3^2$
$\therefore k = 9$
$\therefore y = 9x^2$
When $y = 56.25$,
$56.25 = 9 \times x^2$
$x^2 = 6.25$
$x = \sqrt{6.25}$ $(x > 0)$
$= 2.5$
When $y = 441$,
$441 = 9 \times x^2$
$x^2 = 49$
$x = \sqrt{49}$ $(x > 0)$
$= 7$
When $x = 2$,
$y = 9 \times 2^2$
$= 36$
When $x = 5$,
$y = 9 \times 5^2$
$= 225$

Practise Now 7

(i) Since l is directly proportional to T^2,
then $l = kT^2$, where k is a constant.
When $T = 3$, $l = 220.5$,
$220.5 = k \times 3^2$
$220.5 = 9k$
$\therefore k = 24.5$
$\therefore l = 24.5T^2$

(ii) When $T = 5$,
$l = 24.5 \times 5^2$
$= 612.5$
$\therefore$ The length of the pendulum is 612.5 cm.

(iii) 0.98 m = 98 cm
When $l = 98$,
$98 = 24.5T^2$
$T^2 = 4$
$\therefore T = \sqrt{4}$ $(T > 0)$
$= 2$
$\therefore$ The period of the pendulum is 2 s.

Practise Now 8

The time taken to fill the tank is inversely proportional to the number of taps used.

Method 1: Unitary Method
4 taps can fill the tank in 70 minutes.
1 tap can fill the tank in (70×4) minutes.
7 taps can fill the tank in $\frac{70 \times 4}{7} = 40$ minutes.

Method 2: Proportion Method
Let the time taken for 7 taps to fill the tank by y minutes.
Then $7y = 4 \times 70$ $(x_1y_1 = x_2y_2)$
$y = \frac{4 \times 70}{7}$
$= 40$
$\therefore$ 7 taps can fill the tank in 40 minutes.

Practise Now 9

(a) The three variables are 'number of men', 'number of trenches' and 'number of hours'.
First, we keep the number of trenches constant.

Number of men	Number of trenches	Number of hours
3	2	5
1	2	5×3
5	2	$\frac{5 \times 3}{5} = 3$

Next, we keep the number of men constant.

Number of men	Number of trenches	Number of hours
5	2	3
5	1	$\frac{3}{2}$
5	7	$\frac{3}{2} \times 7 = 10.5$

$\therefore$ 5 men will take 10.5 hours to dig 7 trenches.

(b) The three variables are 'number of taps', 'number of tanks' and 'number of minutes'.
First, we keep the number of tanks constant.

Number of taps	Number of tanks	Number of minutes
7	3	45
1	3	45×7
5	3	$\frac{45 \times 7}{5} = 6.3$

Next, we keep the number of taps constant.

Number of taps	Number of tanks	Number of minutes
5	3	63
5	1	$\frac{63}{3} = 21$

$\therefore$ 5 taps will take 21 minutes to fill one tank.

Practise Now 10

1. (i) When $x = 8$, (x increased by 4 times)

$y = \frac{5}{4}$ (y decreased by 4 times)

$= 1.25$

Alternatively,

$x_2y_2 = x_1y_1$

$8 \times y = 2 \times 5$

$y = \frac{10}{8}$

$= 1.25$

(ii) Since y is inversely proportional to x,

then $y = \frac{k}{x}$, where k is a constant.

When $x = 2$, $y = 5$,

$5 = \frac{k}{2}$

$\therefore k = 10$

$\therefore y = \frac{10}{x}$

(iii) When $y = 10$,

$10 = \frac{10}{x}$

$\therefore x = \frac{10}{10}$

$= 1$

2. Since y is inversely proportional to x,

$x_2y_2 = x_1y_1$

$3 \times y = 2 \times 9$

$y = \frac{18}{3}$

$= 6$

3.

x	0.5	1	2	3	5
y	8	4	2	$1\frac{1}{3}$	0.8

Since y is inversely proportional to x,

then $y = \frac{k}{x}$, where k is a constant.

When $x = 2$, $y = 2$,

$2 = \frac{k}{2}$

$\therefore k = 4$

$\therefore y = \frac{4}{x}$

When $y = 4$,

$4 = \frac{4}{x}$

$x = \frac{4}{4}$

$= 1$

When $y = 0.8$,

$0.8 = \frac{4}{x}$

$x = \frac{4}{0.8}$

$= 5$

When $x = 0.5$,

$y = \frac{4}{0.5}$

$= 8$

When $x = 3$,

$y = \frac{4}{3}$

$= 1\frac{1}{3}$

Practise Now 11

(i) Since I is inversely proportional to R,

then $I = \frac{k}{R}$, where k is a constant.

When $R = 0.5$, $I = 12$,

$12 = \frac{k}{0.5}$

$\therefore k = 6$

$\therefore I = \frac{6}{R}$

When $R = 3$,

$I = \frac{6}{3}$

$= 2$

$\therefore$ The current flowing through the wire is 2 A.

(ii) When $I = 3$,

$3 = \frac{6}{R}$

$R = \frac{6}{3}$

$= 2$

$\therefore$ The resistance of the wire is 2 Ω.

Practise Now 12

(a) Since $y = \frac{4}{x^2}$, i.e. $x^2y = 4$ is a constant, then y is inversely proportional to x^2.

(b) Since $y^2 = \frac{1}{\sqrt[3]{x}}$, i.e. $\sqrt[3]{x}y^2 = 1$ is a constant, then y^2 is inversely proportional to $\sqrt[3]{x}$.

(c) Since $y = \frac{5}{x+2}$, i.e. $(x + 2)y = 5$ is a constant, then y is inversely proportional to $x + 2$.

Practise Now 13

1. (i) When $x = 8 = 2 \times 4$, (8 is 2 times of 4)

$y = \frac{1}{2^2} \times 2$ (y is $\frac{1}{2^2}$ times of 2 since y is inversely proportional to x^2)

$= \frac{1}{2}$

(ii) Since y is inversely proportional to x^2,

then $y = \frac{k}{x^2}$, where k is a constant.

When $x = 4$, $y = 2$,

$2 = \frac{k}{4^2}$

$\therefore k = 32$

$\therefore y = \frac{32}{x^2}$

(iii) When $y = 8$,

$8 = \frac{32}{x^2}$

$x^2 = \frac{32}{8}$

$= 4$

$x = \pm\sqrt{4}$

$= \pm 2$

2. Since y is inversely proportional to $\sqrt{x}$,

then $y = \frac{k}{\sqrt{x}}$, where k is a constant.

When $x = 9$, $y = 6$,

$6 = \frac{k}{\sqrt{9}}$

$\therefore k = 18$

$\therefore y = \frac{18}{\sqrt{x}}$

When $x = 25$,

$y = \frac{18}{\sqrt{25}}$

$= 3.6$

3.

x	0.25	1	4	16	36
y	16	8	2	2	$1\frac{1}{3}$

Since y is inversely proportional to $\sqrt{x}$,

then $y = \frac{k}{\sqrt{x}}$, where k is a constant.

When $x = 1$, $y = 8$,

$8 = \frac{k}{\sqrt{1}}$

$\therefore k = 8$

$\therefore y = \frac{8}{\sqrt{x}}$

When $y = 16$,

$16 = \frac{8}{\sqrt{x}}$

$\sqrt{x} = \frac{8}{16}$

$= \frac{1}{2}$

$x = \left(\frac{1}{2}\right)^2$

$= 0.25$

When $y = 1\frac{1}{3}$,

$1\frac{1}{3} = \frac{8}{\sqrt{x}}$

$\sqrt{x} = \frac{8}{1\frac{1}{3}}$

$= 6$

$x = 6^2$

$= 36$

When $x = 4$,

$y = \frac{8}{\sqrt{4}}$

$= 4$

When $x = 16$,

$y = \frac{8}{\sqrt{16}}$

$= 2$

Practise Now 14

(i) Since F is inversely proportional to d,

then $F = \frac{k}{d^2}$, where k is a constant.

When $d = 2$, $F = 10$,

$10 = \frac{k}{2^2}$

$10 = \frac{k}{4}$

$\therefore k = 40$

$\therefore F = \frac{40}{d^2}$

When $d = 5$,

$F = \frac{40}{5^2}$

$= 1.6$

$\therefore$ The force between the particles is 1.6 N.

(ii) When $F = 25$,

$$25 = \frac{40}{d^2}$$

$$d^2 = \frac{40}{25}$$

$$= \frac{8}{5}$$

$$d = \sqrt{\frac{8}{5}} \quad (d > 0)$$

$$= 1.26 \text{ (to 3 s.f.)}$$

$\therefore$ The distance between the particles is 1.26 m.

Exercise 1A

1. **(i)** The number of books is directly proportional to the mass of books.

108 books have a mass of 30 kg.

1 book has a mass of $\frac{30}{108}$ kg.

150 books have a mass of $\frac{30}{108} \times 150 = 41\frac{2}{3}$ kg.

(ii) The mass of books is directly proportional to the number of books.

30 kg is the mass of 108 books.

1 kg is the mass of $\frac{108}{30}$ books.

20 kg is the mass of $\frac{108}{30} \times 20 = 72$ books.

2. **(i)** The number of books is directly proportional to the length occupied by the books.

60 books occupy a length of 1.5 m.

1 book occupies a length of $\frac{1.5}{60}$ m.

50 books occupy a length of $\frac{1.5}{60} \times 50 = 1.25$ m.

(ii) The length occupied by the books is directly proportional to the number of books.

1.5 m $= (1.5 \times 100)$ cm

$= 150$ m

150 cm is the length occupied by 60 books.

1 cm is the length occupied by $\frac{60}{150}$ books.

80 cm is the length occupied by $\frac{60}{150} \times 80 = 32$ books.

3. **(i)** Since y is directly proportional to x,

then $y = kx$, where k is a constant.

When $x = 4.5$, $y = 3$,

$$3 = k \times 4.5$$

$$\therefore k = \frac{2}{3}$$

$$\therefore y = \frac{2}{3}x$$

(ii) When $y = 6$,

$$6 = \frac{2}{3}x$$

$$x = 6 \times \frac{3}{2}$$

$$= 9$$

(iii) When $x = 12$,

$$y = \frac{2}{3} \times 12$$

$$= 8$$

4. **(i)** Since Q is directly proportional to P,

then $Q = kP$, where k is a constant.

When $P = 4$, $Q = 28$,

$$28 = k \times 4$$

$$\therefore k = 7$$

$$\therefore Q = 7P$$

(ii) When $P = 5$,

$$Q = 7 \times 5$$

$$= 35$$

(iii) When $Q = 42$,

$$42 = 7 \times P$$

$$P = 6$$

5. **(a)** The mass of tea leaves is directly proportional to the cost of tea leaves.

3 kg of tea leaves cost \$18.

1 kg of tea leaves cost $\$\left(\frac{18}{3}\right)$.

10 kg of tea leaves cost $\$\left(\frac{18}{3} \times 10\right) = \$60$.

(b) The mass of sugar is directly proportional to the cost.

b kg of sugar cost $\$c$.

1 kg of sugar cost $\$\frac{c}{b}$.

a kg of sugar cost $\$\left(\frac{c}{b} \times a\right) = \$\frac{ac}{b}$.

6. The amount of metal is directly proportional to the mass of the metal.

$\frac{5}{9}$ of a piece of metal has a mass of 7 kg.

A whole piece of metal has a mass of $\frac{7}{\frac{5}{9}}$ kg.

$\frac{2}{7}$ of a piece of metal has a mass of $\frac{7}{\frac{5}{9}} \times \frac{2}{7} = 3\frac{3}{5}$ kg.

7. Since z is directly proportional to x,

$$\frac{x_2}{z_2} = \frac{x_1}{z_1}$$

$$\frac{x}{18} = \frac{3}{12}$$

$$x = \frac{3}{12} \times 18$$

$$= 4.5$$

8. Since B is directly proportional to A,

$$\frac{B_2}{A_2} = \frac{B_1}{A_1}$$

$$\frac{B}{24} = \frac{3}{18}$$

$$B = \frac{3}{18} \times 24$$
$$= 4$$

9. (a)

x	4	20	24	36	44
y	1	5	6	9	11

Since y is directly proportional to x, then $y = kx$, where k is a constant.

When $x = 24$, $y = 6$,

$6 = k \times 24$

$\therefore k = \frac{1}{4}$

$\therefore y = \frac{1}{4}x$

When $y = 9$,

$9 = \frac{1}{4} \times x$

$x = 9 \times 4$
$= 36$

When $y = 11$,

$11 = \frac{1}{4} \times x$

$x = 11 \times 4$
$= 44$

When $x = 4$,

$y = \frac{1}{4} \times 4$
$= 1$

When $x = 20$,

$y = \frac{1}{4} \times 20$
$= 5$

(b)

x	2	3	5.5	8	9.5
y	2.4	3.6	6.6	9.6	11.4

Since y is directly proportional to x,
then $y = kx$, where k is a constant.

When $x = 3$, $y = 3.6$,

$3.6 = k \times 3$

$\therefore k = 1.2$

$\therefore y = 1.2x$

When $y = 9.6$,

$9.6 = 1.2 \times x$

$x = \frac{9.6}{1.2}$
$= 8$

When $y = 11.4$,

$11.4 = 1.2 \times x$

$x = \frac{11.4}{1.2}$
$= 9.5$

When $x = 2$,

$y = 1.2 \times 2$
$= 2.4$

When $x = 5.5$,

$y = 1.2 \times 5.5$
$= 6.6$

10. (i) Since y is directly proportional to x,
then $y = kx$, where k is a constant.

When $x = 5$, $y = 20$,

$20 = k \times 5$

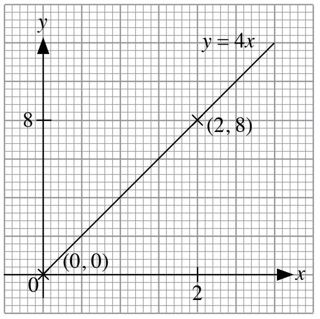
$\therefore k = 4$

$\therefore y = 4x$

(ii) $y = 4x$

When $x = 0$, $y = 0$.

When $x = 2$, $y = 8$.



11. (i) Since z is directly proportional to y,
then $z = ky$, where k is a constant.

When $y = 6$, $z = 48$,

$48 = k \times 6$

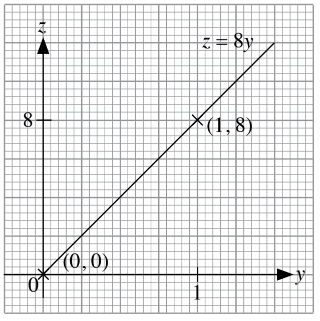
$\therefore k = 8$

$\therefore z = 8y$

(ii) $z = 8y$

When $y = 0$, $y = 0$.

When $x = 1$, $y = 8$.



12. (i) Since F is directly proportional to m,
then $F = km$, where k is a constant.

When $m = 5$, $F = 49$,

$49 = k \times 5$

$\therefore k = 9.8$

$\therefore F = 9.8m$

(ii) When $m = 14$,

$F = 9.8 \times 14$
$= 137.2$

(iii) When $F = 215.6$,

$215.6 = 9.8 \times m$

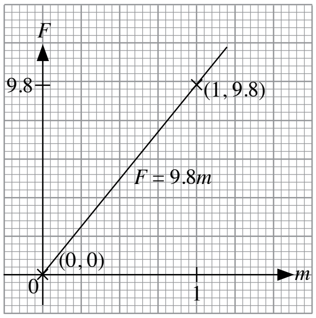
$m = \dfrac{215.6}{9.8}$

$= 22$

(iv) $F = 9.8m$

When $m = 0$, $F = 0$.

When $m = 1$, $F = 9.8$.



13. (i) Since P is directly proportional to T, then $P = kT$, where k is a constant.

When $T = 10$, $y = 25$,

$25 = k \times 10$

$\therefore k = 2.5$

$\therefore P = 2.5T$

(ii) When $T = 24$,

$P = 2.5 \times 24$

$= 60$

(iii) When $P = 12$,

$12 = 2.5 \times T$

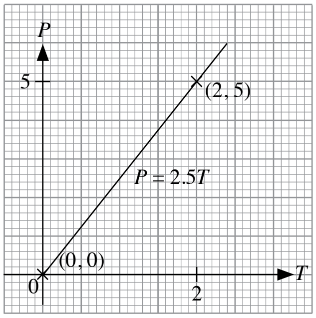
$T = \dfrac{12}{2.5}$

$= 4.8$

(iv) $P = 2.5T$

When $T = 0$, $P = 0$.

When $T = 2$, $P = 5$.



14. (i) Since V is directly proportional to R, then $V = kR$, where k is a constant.

When $R = 6$, $V = 9$,

$9 = k \times 6$

$\therefore k = 1.5$

$\therefore V = 1.5R$

(ii) When $R = 15$,

$V = 1.5 \times 15$

$= 22.5$

(iii) When $V = 15$,

$15 = 1.5 \times R$

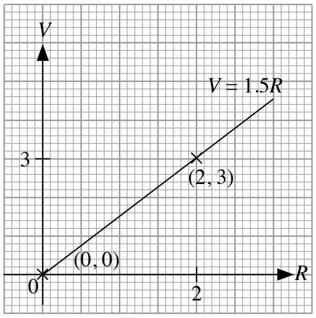
$R = \dfrac{15}{1.5}$

$= 10$

(iv) $V = 1.5R$

When $R = 0$, $V = 0$.

When $R = 2$, $V = 3$.



15. (i) Total income for that month

$= \$600 + \$8 \times 95$

$= \$1360$

(ii) Variable amount $= \$1680 - \$600$

$= \$1080$

Number of tyres he sells in that month $= \dfrac{1080}{8}$

$= 135$

(iii) Variable amount $= n \times \$8$

$= \$8n$

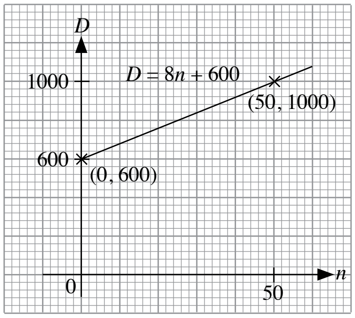
Total income = variable amount + fixed amount

$\therefore D = 8n + 600$

(iv) $D = 8n + 600$

When $n = 0$, $D = 600$.

When $n = 50$, $D = 1000$.



D is *not* directly proportional to n because the line does not pass through the origin.

16. Let the mass of ice produced be m tonnes,
the number of hours of production be T hours.
Since m is directly proportional to T,
then $m = kT$, where k is a constant.

When $T = \frac{30}{60} - \frac{10}{60} = \frac{1}{3}$, $m = 20$,

$20 = k \times \frac{1}{3}$

$\therefore k = 60$

$\therefore m = 60T$

When $T = 1.75 - \frac{10}{60}$,

$m = 60\left(1.75 - \frac{1}{6}\right)$

$= 95$

$\therefore$ The mass of ice manufactured is 95 tonnes.

Exercise 1B

1. **(i)** Since x is directly proportional to y^3,
then $x = ky^3$, where k is a constant.
When $y = 2$, $x = 32$,

$32 = k \times 2^3$

$32 = 8k$

$\therefore k = 4$

$\therefore x = 4y^3$

(ii) When $y = 6$,

$x = 4 \times 6^3$

$= 864$

(iii) When $x = 108$,

$108 = 4 \times y^3$

$y^3 = \frac{108}{4}$

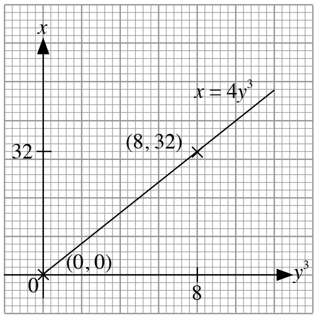
$= 27$

$y = 3$

(iv) $x = 4y^3$

When $y = 0$, $x = 0$.

When $y = 2$, $x = 32$.



2. **(i)** Since z^2 is directly proprotional to w,
then $z^2 = kw$, where k is a constant.
When $w = 8$, $z = 4$,

$4^2 = k \times 8$

$16 = 8k$

$\therefore k = 2$

$\therefore z^2 = 2w$

(ii) When $w = 18$,

$z^2 = 2 \times 18$

$= 36$

$z = \pm\sqrt{36}$

$= \pm 6$

(iii) When $z = 5$,

$5^2 = 2 \times w$

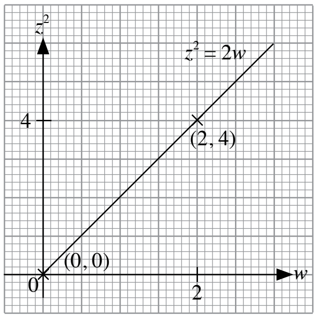
$w = \frac{25}{2}$

$= 12.5$

(iv) $z^2 = 2w$

When $w = 0$, $z^2 = 0$.

When $w = 2$, $z^2 = 4$.



3. **(i)** Since y is directly proportional to x^n,
then $y = kx^n$, where k is a constant.
Since y m^2 is the area of a square of length x m, then $y = x^2$.

$kx^n = x^2$

$\therefore n = 2$

(ii) Since y is directly proportional to x^n,
then $y = kx^n$, where k is a constant.
Since y cm^3 is the volume of a cube of length x cm, then $y = x^3$.

$kx^n = x^3$

$\therefore n = 3$

4. **(a)** Since $y = 4x^2$, i.e. $\frac{y}{x^2} = 4$ is a constant, then y is directly proportional to x^2.

(b) Since $y = 3\sqrt{x}$, i.e. $\frac{y}{\sqrt{x}} = 3$ is a constant, then y is directly proportional to $\sqrt{x}$.

(c) Since $y^2 = 5x^3$, i.e. $\frac{y^2}{x^3} = 5$ is a constant, then y^2 is directly proportional to x^3.

(d) Since $p^3 = q^2$, i.e. $\frac{p^3}{q^2} = 1$ is a constant, then p^3 is directly proportional to q^2.

5. Since z^2 is directly proportional to x^3,

$$\frac{z_2^2}{x_2^3} = \frac{z_1^2}{x_1^3}$$

$$\frac{z^2}{9^3} = \frac{8^2}{4^3}$$

$$z^2 = \frac{8^2}{4^3} \times 9^3$$

$$= 729$$

$$z = \pm\sqrt{729}$$

$$= \pm 27$$

6. Since q is directly proportional to $(p - 1)^2$,

$$\frac{(p_2 - 1)^2}{q_2} = \frac{(p_1 - 1)^2}{q_1}$$

$$\frac{(p - 1)^2}{80} = \frac{(3 - 1)^2}{20}$$

$$(p - 1)^2 = \frac{(3 - 1)^2}{20} \times 80$$

$$= 16$$

$p - 1 = -4$ or $p - 1 = 4$

$p = -3$ $\quad$ $p = 5$

$\therefore p = -3$ or 5

7.

x	3	4	5	6	7
y	81	192	375	648	1029

Since y is directly proportional to x^3,

then $y = kx^3$, where k is a constant.

When $x = 6$, $y = 648$,

$648 = k \times 6^3$

$\therefore k = 3$

$\therefore y = 3x^3$

When $y = 375$,

$375 = 3 \times x^3$

$x^3 = 125$

$x = \sqrt[3]{125}$

$= 5$

When $y = 1029$,

$1029 = 3 \times x^3$

$x^3 = 343$

$x = \sqrt[3]{343}$

$= 7$

When $x = 3$,

$y = 3 \times 3^3$

$= 81$

When $x = 4$,

$y = 3 \times 4^3$

$= 192$

8.

r	0.2	0.5	0.7	1.5	1.8
m	0.016	0.25	0.686	6.75	11.664

Since m is directly proportional to r^3,

then $m = kr^3$, where k is a constant.

When $r = 1.5$, $m = 6.75$,

$6.75 = k \times 1.5^3$

$\therefore k = 2$

$\therefore m = 2r^3$

When $m = 0.25$,

$0.25 = 2 \times r^3$

$r^3 = 0.125$

$r = \sqrt[3]{0.125}$

$= 0.5$

When $m = 11.664$,

$11.664 = 2 \times r^3$

$r^3 = 5.832$

$r = \sqrt[3]{5.832}$

$= 1.8$

When $r = 0.2$,

$m = 2 \times 0.2^3$

$= 0.016$

When $r = 0.7$,

$m = 2 \times 0.7^3$

$= 0.686$

9. **(i)** Since L is directly proportional to $\sqrt{N}$,

then $L = k\sqrt{N}$, where k is a constant.

When $N = 1$, $L = 2.5$,

$2.5 = k\sqrt{1}$

$\therefore k = 2.5$

$\therefore L = 2.5\sqrt{N}$

(ii) When $N = 4$,

$L = 2.5 \times \sqrt{4}$

$= 5$

$\therefore$ The length 4 hours after its birth is 5 cm.

(iii) When $L = 15$,

$15 = 2.5 \times \sqrt{N}$

$\sqrt{N} = 6$

$N = 6^2$

$= 36$

$\therefore$ It will take 36 hours for the earthworm to grow to a length of 15 cm.

10. Since y is directly proportional to x^2,
then $y = kx^2$, where k is a constant.
When $x = 1$,
$y = k \times 1^2$
$= k$
When $x = 3$,
$y = k \times 3^2$
$= 9k$
Since the difference in the values of y is 32,
$9k - k = 32$
$8k = 32$
$\therefore k = 4$
$\therefore y = 4x^2$
When $x = -2$,
$y = 4 \times (-2)^2$
$= 16$

11. Since y is directly proportional to x^2,
$$\frac{y_2}{x_2^2} = \frac{y_1}{x_1^2}$$
$$\frac{y}{(2x)^2} = \frac{a}{x^2}$$
$$y = \frac{a}{x^2} \times (2x)^2$$
$$= \frac{a}{x^2} \times 4x^2$$
$$= 4a$$

12. Let the braking distance of a vehicle be D m,
the speed of the vehicle be B m/s.
Since D is directly proportional to B^2, then $D = kB^2$, where k is a constant.
When $B = b, D = d$,
$d = k \times b^2$
$$\therefore k = \frac{d}{b^2}$$
$$\therefore D = \frac{d}{b^2} B^2$$
When the speed of the vehicle is increased by 200%,
$B = (100\% + 200\%) \times b$
$$= \frac{100 + 200}{100} \times b$$
$= 3b$
When $B = 3b$,
$$D = \frac{d}{b^2} (3b)^2$$
$$= \frac{d}{b^2} (9b^2)$$
$= 9d$
Percentage increase in its braking distance
$$= \frac{9d - d}{d} \times 100\%$$
$= 800\%$

Exercise 1C

1. **(a)** The number of pencils is directly proportional to the total cost of the pencils.
Assumption: All pencils are identical and cost the same each.
(b) The number of taps filling a tank is inversely proportional to the time taken to fill the tank.
Assumption: All taps are identical and each tap takes the same time to fill the tank.
(c) The number of men laying a road is inversely proportional to the taken to finish laying the road.
Assumption: All the men work at the same rate in laying the road.
(d) The number of cattle to be fed is directly proportional to the amount of fodder.
Assumption: All the cattle eat the same amount of fodder.
(e) The number of cattle to be fed is inversely proportional to the time taken to finish a certain amount of the fodder.
Assumption: All the cattle eat the fodder at the same rate.
$\therefore$ **(b)**, **(c)** and **(e)** are in inverse proportion.

2. The number of men to build a bridge is inversely proportional to the number of days to build the bridge.
8 men can build a bridge in 12 days.
1 man can build the bridge in (12×8) days.
6 men can build the bridge in $\frac{12 \times 8}{6} = 16$ days.
The assumption made is that all the men work at the same rate in building the bridge.

3. **(i)** Since x is inversely proportional to y,
$y_2x_2 = y_1x_1$
$25 \times x = 5 \times 40$
$$x = \frac{5 \times 40}{25}$$
$= 8$
(ii) Since x is inversely proportional to y,
then $x = \frac{k}{y}$, where k is a constant.
When $y = 5, x = 40$,
$$40 = \frac{k}{5}$$
$\therefore k = 200$
$$\therefore x = \frac{200}{y}$$
(iii) When $x = 400$,
$$400 = \frac{200}{y}$$
$$y = \frac{200}{400}$$
$= 0.5$

4. **(i)** Since Q is inversely proportional to P,

then $Q = \frac{k}{P}$, where k is a constant.

When $P = 2$, $Q = 0.25$,

$0.25 = \frac{k}{2}$

$\therefore k = 0.5$

$\therefore Q = \frac{0.5}{P}$

$= \frac{1}{2P}$

(ii) When $P = 5$,

$Q = \frac{1}{2(5)}$

$= 0.1$

(iii) When $Q = 0.2$,

$0.2 = \frac{1}{2P}$

$2P = \frac{1}{0.2}$

$= 5$

$P = 2.5$

5. The number of days is inversely proportional to the number of workers employed.

16 days are needed for 35 workers to complete the projet.

1 day is needed for (35×16) workers to complete the project.

14 days are needed for $\frac{35 \times 16}{14} = 40$ workers to complete the project.

Number of additional workers to employ $= 40 - 35$

$= 5$

6. **(i)** The number of days is inversely proportional to the number of cattle to consume a consignment of fodder.

50 days are needed for 1260 cattle to consume a consignment of fodder.

1 day is needed for (1260×50) cattle to consume a consignment of fodder.

75 days are needed for $\frac{1260 \times 50}{75} = 840$ cattle to consume a consignment of fodder.

(ii) 1260 cattle consume a consignment of fodder in 50 days.

1 cattle consume a consignment of fodder in (50×1260) days.

1575 cattle consume a consignment of fodder in

$\frac{50 \times 1260}{1575} = 40$ days.

7. The number of athletes is inversely proportional to the number of days the food can last.

72 athletes take 6 days to consume the food.

1 athlete takes (6×72) days to consume the food.

$72 - 18 = 54$ athletes take $\frac{6 \times 72}{54} = 8$ days to consume the food.

Number of additional days the food can last $= 8 - 6$

$= 2$ days

The assumption made is that all athletes consume the same amount of food every day.

8. Since z is inversely proportional to x,

$x_2 z_2 = x_1 z_1$

$x \times 70 = 7 \times 5$

$x = \frac{7 \times 5}{70}$

$= 0.5$

9. Since B is inversely proportional to A,

$A_2 B_2 = A_1 B_1$

$1.4 \times B = 2 \times 3.5$

$B = \frac{2 \times 3.5}{1.4}$

$= 5$

10. **(a)**

x	0.5	2	2.5	3	8
y	24	6	4.8	4	1.5

Since y is inversely proportional to x,

then $y = \frac{k}{x}$, where k is a constant.

When $x = 3$, $y = 4$,

$4 = \frac{k}{3}$

$\therefore k = 12$

$\therefore y = \frac{12}{x}$

When $y = 24$,

$24 = \frac{12}{x}$

$x = \frac{12}{24}$

$= 0.5$

When $y = 1.5$,

$1.5 = \frac{12}{x}$

$x = \frac{12}{1.5}$

$= 8$

When $x = 2$,

$y = \frac{12}{2}$

$= 6$

When $x = 2.5$,

$y = \frac{12}{2.5}$

$= 4.8$

(b)

x	3	4	4.5	14.4	25
y	12	9	8	2.5	1.44

Since y is inversely proportional to x,

then $y = \frac{k}{x}$, where k is a constant.

When $x = 4$, $y = 9$,

$9 = \frac{k}{4}$

$\therefore k = 36$

$\therefore y = \frac{36}{x}$

When $y = 8$,

$8 = \frac{36}{x}$

$x = \frac{36}{8}$

$= 4.5$

When $y = 2.5$,

$2.5 = \frac{36}{x}$

$x = \frac{36}{2.5}$

$= 14.4$

When $x = 3$,

$y = \frac{36}{3}$

$= 12$

When $x = 25$,

$y = \frac{36}{25}$

$= 1.44$

11. (i) Since f is inversely proportional to λ,

then $f = \frac{k}{\lambda}$, where k is a constant.

When $\lambda = 3000$, $f = 100$,

$100 = \frac{k}{3000}$

$\therefore k = 300\,000$

$\therefore f = \frac{300\,000}{\lambda}$

When $\lambda = 500$,

$f = \frac{300\,000}{500}$

$= 600$

$\therefore$ The frequency of the radio wave is 600 kHz.

(ii) When $f = 800$,

$800 = \frac{300\,000}{\lambda}$

$\lambda = \frac{300\,000}{800}$

$= 375$

$\therefore$ The wavelength of the radio wave is 375 m.

12. (i) Since t is inversely proportional to N,

then $t = \frac{k}{N}$, where k is a constant.

When $N = 3$, $t = 8$,

$8 = \frac{k}{3}$

$\therefore k = 24$

$\therefore t = \frac{24}{N}$

(ii) When $N = 6$,

$t = \frac{24}{6}$

$= 4$

$\therefore$ The number of hours needed by 6 men is 4 hours.

(iii) When $t = \frac{3}{4}$,

$\frac{3}{4} = \frac{24}{N}$

$N = 24 \times \frac{4}{3}$

$= 32$

$\therefore$ 32 men need to be employed.

13. The three variables are 'number of glassblowers', 'number of vases' and 'number of minutes'.

First, we keep the number of vases constant.

Number of glassblowers	Number of vases	Number of minutes
12	12	9
1	12	9×12
8	12	$\frac{9 \times 12}{8} = 13.5$

Next, we keep the nmber of glassblowers constant.

Number of glassblowers	Number of vases	Number of minutes
8	12	13.5
8	1	$\frac{13.5}{12}$
8	32	$\frac{13.5}{12} \times 32 = 36$

$\therefore$ 8 glassblowers will take 36 minutes to make 32 vases.

14. The three variables are 'number of sheep', 'number of consignments' and 'number of days'.

First, we keep the number of consignments constant.

Number of sheep	Number of consignments	Number of days
100	1	20
1	1	20×1000
550	1	$\frac{20 \times 1000}{550} = 36\frac{4}{11}$

Next, we keep the number of sheep constant.

Number of sheep	Number of consignments	Number of days
550	1	$36\frac{4}{11}$
550	$400 \div 36\frac{4}{11} = 11$	400

$\therefore$ 11 consignments of fodder are needed.

15. In 1 minute, tap A alone fills up $\frac{1}{6}$ of the tank.

In 1 minute, tap B alone fills up $\frac{1}{9}$ of the tank.

In 1 minute, pipe C alone empties $\frac{1}{15}$ of the tank.

In 1 minute, when both taps and the pipe are turned on,

$\frac{1}{6} + \frac{1}{9} - \frac{1}{15} = \frac{19}{90}$ of the tank is filled up.

$$\text{Time to fill up the tank} = \frac{90}{19} = 4\frac{14}{19} \text{ minutes}$$

16. Total number of hours worked on the road after 20 working days

$= 20 \times 50 \times 8$

$= 8000$ hours

The length of the road laid is directly proportional to the number of hours.

1200 m of road is laid in 8000 hours.

1 m of road is laid in $\frac{8000}{1200}$ hours.

$3000 - 1200 = 1800$ m of road is laid in $\frac{8000}{1200} \times 1800 = 12\,000$ hours.

Let the number of additional men to employ be x.

$$\begin{aligned}(30 - 20) \times (50 + x) \times 10 &= 12\,000\\ 100(50 + x) &= 12\,000\\ 50 + x &= 120\\ x &= 70\end{aligned}$$

$\therefore$ 70 more men needs to be employed.

Exercise 1D

1. **(i)** Since x is inversely proportional to y^3,

$$\begin{aligned}y_2^3 x_2 &= y_1^3 x_1\\ 4^3 \times x &= 2^3 \times 50\\ x &= \frac{2^3 \times 50}{4^3}\\ &= 6.25\end{aligned}$$

(ii) Since x is inversely proportional to y^3,

then $x = \frac{k}{y^3}$, where k is a constant.

When $y = 2$, $x = 50$,

$$\begin{aligned}50 &= \frac{k}{2^3}\\ \therefore k &= 400\\ \therefore x &= \frac{400}{y^3}\end{aligned}$$

(iii) When $x = 3.2$,

$$\begin{aligned}3.2 &= \frac{400}{y^3}\\ y^3 &= \frac{400}{3.2}\\ &= 125\\ y &= \sqrt[3]{125}\\ &= 5\end{aligned}$$

2. **(i)** Since z is inversely proportional to $\sqrt{w}$,

then $z = \frac{k}{\sqrt{w}}$, where k is a constant.

When $w = 9$, $z = 9$,

$$\begin{aligned}9 &= \frac{k}{\sqrt{9}}\\ \therefore k &= 27\\ \therefore z &= \frac{27}{\sqrt{w}}\end{aligned}$$

(ii) When $w = 16$,

$$\begin{aligned}z &= \frac{27}{\sqrt{16}}\\ &= 6.75\end{aligned}$$

(iii) When $z = 3$,

$$\begin{aligned}3 &= \frac{27}{\sqrt{w}}\\ \sqrt{w} &= \frac{27}{3}\\ &= 9\\ w &= 9^2\\ &= 81\end{aligned}$$

3. **(a)** Since $y = \frac{3}{x^2}$, i.e. $yx^2 = 3$ is a constant, then y is inversely proportional to x^2.

(b) Since $y = \frac{1}{\sqrt{x}}$, i.e. $y\sqrt{x} = 1$ is a constant, then y is inversely proportional to $\sqrt{x}$.

(c) Since $y^2 = \frac{5}{x^3}$, i.e. $y^2x^3 = 5$ is a constant, then y^2 is inversely proportional to x^3.

(d) Since $n = \frac{7}{m-1}$, i.e. $n(m - 1) = 7$ is a constant, then n is inversely proportional to $m - 1$.

(e) Since $q = \frac{4}{(p+1)^2}$, i.e. $q(p + 1)^2 = 4$ is a constant, then q is inversely proportional to $(p + 1)^2$.

4. Since z is inversely proportional to $\sqrt[3]{x}$.

$$\begin{aligned}\sqrt[3]{x_2}\, z_2 &= \sqrt[3]{x_1}\, z_1\\ \sqrt[3]{216} \times z &= \sqrt[3]{64} \times 5\\ z &= \frac{\sqrt[3]{64} \times 5}{\sqrt[3]{216}}\\ &= 3\frac{1}{3}\end{aligned}$$

5. Since q^2 is inversely proportional to $p + 3$,

$$(p_2 + 3)q_2^2 = (p_1 + 3)q_1^2$$
$$(17 + 3) \times q^2 = (2 + 3) \times 5^2$$
$$20q^2 = 125$$
$$q^2 = \frac{125}{20}$$
$$= 6.25$$
$$q = \pm\sqrt{6.25}$$
$$= \pm 2.5$$

6.

s	1	2	4	10	20
t	80	10	1.25	0.08	0.01

Since t is inversely proportional to s^3,

then $t = \frac{k}{s^3}$, where k is a constant.

When $s = 1$, $t = 80$,

$$80 = \frac{k}{1^3}$$
$$\therefore k = 80$$
$$\therefore t = \frac{80}{s^3}$$

When $t = 0.08$,

$$0.08 = \frac{80}{s^3}$$
$$s^3 = \frac{80}{0.08}$$
$$= 1000$$
$$s = \sqrt[3]{1000}$$
$$= 10$$

When $t = 0.01$,

$$0.01 = \frac{80}{s^3}$$
$$s^3 = \frac{80}{0.01}$$
$$= 8000$$
$$s = \sqrt[3]{8000}$$
$$= 20$$

When $s = 2$,

$$t = \frac{80}{2^3}$$
$$= 10$$

When $s = 4$,

$$t = \frac{80}{4^3}$$
$$= 1.25$$

7. (i) Since F is inversely proportional to d^2,

then $F = \frac{k}{d^2}$, where k is a constant.

(ii) When $F = 20$, let the distance between the particles be x m.

$$20 = \frac{k}{x^2}$$
$$\therefore k = 20x^2$$
$$\therefore F = \frac{20x^2}{d^2}$$

When the distance is halved, i.e. $d = \frac{1}{2}x$,

$$F = \frac{20x^2}{\left(\frac{1}{2}x\right)^2}$$
$$= 20x^2 \times \frac{4}{x^2}$$
$$= 80$$

$\therefore$ The force is 80 N.

8. (i) For a fixed volume, since h is inversely proportional to r^2, then

$h = \frac{k}{r^2}$, where k is a constant.

When $r = 6$, $h = 5$,

$$5 = \frac{k}{6^2}$$
$$\therefore k = 180$$
$$\therefore h = \frac{180}{r^2}$$

When $r = 3$,

$$h = \frac{180}{3^2}$$
$$= 20$$

$\therefore$ The height of cone B is 20 cm.

(ii) When $h = 1.25$,

$$1.25 = \frac{180}{r^2}$$
$$r^2 = \frac{180}{1.25}$$
$$= 144$$
$$r = \sqrt{144} \quad (r > 0)$$
$$= 12$$

$\therefore$ The base radius of cone C is 12 cm.

9. Since y is inversely proportional to $2x + 1$,

then $y = \frac{k}{2x+1}$, where k is a constant.

When $x = 0.5$,

$$y = \frac{k}{2(0.5)+1}$$
$$= \frac{k}{2}$$

When $x = 2$,

$$y = \frac{k}{2(2)+1}$$
$$= \frac{k}{5}$$

Since the difference in the values of y is 0.9,

$$\frac{k}{2} - \frac{k}{5} = 0.9$$
$$0.3k = 0.9$$
$$\therefore k = 3$$
$$\therefore y = \frac{3}{2x+1}$$

When $x = -0.25$,

$$y = \frac{3}{2(-0.25)+1}$$
$$= 6$$

10. Since y is inversely proportional to x^2,

$$x_2^2 y_2 = x_1^2 y_1$$
$$(3x)^2 y = x^2 b$$
$$9x^2 y = bx^2$$
$$y = \frac{bx^2}{9x^2}$$
$$= \frac{1}{9}b$$

11. Let the force of attraction between two magnets be X N, the distance between two magnets be Y cm.

Since X is inversely proportional to Y^2,

then $X = \frac{k}{Y^2}$, where k is a constant.

When $X = F$, $Y = r$,

$$F = \frac{k}{r^2}$$
$$\therefore k = Fr^2$$
$$\therefore X = \frac{Fr^2}{Y^2}$$

When the distance between the magnets is increased by 400%,

$$Y = (100\% + 400\%) \times r$$
$$= \frac{100 + 400}{100} \times r$$
$$= 5r$$

When $Y = 5r$,

$$X = \frac{Fr^2}{(5r)^2}$$
$$= \frac{Fr^2}{25r^2}$$
$$= 0.04F$$

Comparing with $X = cF$,

$\therefore$ The value of c is 0.04.

Review Exercise 1

1. **(i)** Since y is directly proportional to x, then $y = kx$, where k is a constant.

When $x = 2$, $y = 6$,

$$6 = k \times 2$$
$$\therefore k = 3$$
$$\therefore y = 3x$$

(ii) When $x = 11$,

$$y = 3 \times 11$$
$$= 33$$

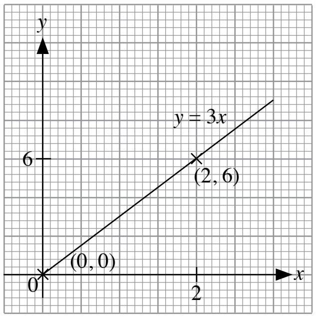
(iii) When $y = 12$,

$$12 = 3 \times x$$
$$x = 4$$

(iv) $y = 3x$

When $x = 0$, $y = 0$.

When $x = 2$, $y = 6$.



2. **(i)** Since A is directly proportional to B, then $A = kB$, where k is a constant.

When $B = \frac{5}{6}$, $A = 1\frac{2}{3}$,

$$1\frac{2}{3} = k \times \frac{5}{6}$$
$$\therefore k = 2$$
$$\therefore A = 2B$$

When $B = \frac{1}{3}$,

$$A = 2 \times \frac{1}{3}$$
$$= \frac{2}{3}$$

(ii) When $A = 7\frac{1}{2}$,

$$7\frac{1}{2} = 2 \times B$$
$$B = 3\frac{3}{4}$$

3. **(i)** Since y is directly proportional to x^3, then $y = kx^3$, where k is a constant.

When $x = 3$, $y = 108$,

$$108 = k \times 3^3$$
$$108 = 27k$$
$$\therefore k = 4$$
$$\therefore y = 4x^3$$

(ii) When $x = 7$,

$$y = 4 \times 7^3$$
$$= 1372$$

(iii) When $y = 4000$,

$4000 = 4 \times x^3$

$x^3 = \frac{4000}{4}$

$= 1000$

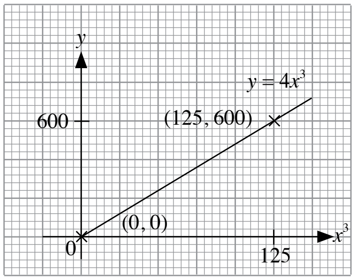
$x = \sqrt[3]{1000}$

$= 10$

(iv) $y = 4x^3$

When $x = 0$, $y = 0$.

When $x = 5$, $y = 600$.



4. **(i)** Since n is directly proportional to m^2,

then $n = km^2$, where k is a constant.

When $m = 2.5$, $n = 9.375$,

$9.375 = k \times 2.5^2$

$\therefore k = 1.5$

$\therefore n = 1.5m^2$

When $m = 3$,

$n = 1.5 \times 3^2$

$= 13.5$

(ii) When $n = 181.5$,

$181.5 = 1.5 \times m^2$

$m^2 = \frac{181.5}{1.5}$

$= 121$

$m = \pm\sqrt{121}$

$= \pm 11$

5. **(i)** Since t is directly proportional to $\sqrt[3]{s}$,

then $t = k\sqrt[3]{s}$, where k is a constant.

When $s = 64$, $t = 4$,

$4 = k \times \sqrt[3]{64}$

$= 4k$

$\therefore k = 1$

$\therefore t = \sqrt[3]{s}$

When $s = 125$,

$t = \sqrt[3]{125}$

$= 5$

(ii) When $t = 2$,

$2 = \sqrt[3]{s}$

$s = 2^3$

$= 8$

6. **(i)** Since y is inversely proportional to x,

then $y = \frac{k}{x}$, where k is a constant.

When $x = 3$, $y = 4$,

$4 = \frac{k}{3}$

$\therefore k = 12$

$\therefore y = \frac{12}{x}$

(ii) When $x = 6$,

$y = \frac{12}{6}$

$= 2$

(iii) When $y = 24$,

$24 = \frac{12}{x}$

$x = \frac{12}{24}$

$= 0.5$

7. **(i)** Since q is inversely proportional to p^2,

then $q = \frac{k}{p^2}$, where k is a constant.

When $p = 5$, $q = 3$,

$3 = \frac{k}{5^2}$

$\therefore k = 75$

$\therefore q = \frac{75}{p^2}$

(ii) When $p = 10$,

$q = \frac{75}{10^2}$

$= 0.75$

(iii) When $q = \frac{1}{3}$,

$\frac{1}{3} = \frac{75}{p^2}$

$p^2 = 75 \times 3$

$= 225$

$p = -\sqrt{225}$ $(p < 0)$

$= -15$

8. **(i)** Since z is inversely proportional to $w + 3$,

then $z = \frac{k}{w+3}$, where k is a constant.

When $w = 3$, $z = 4$,

$4 = \frac{k}{3+3}$

$\therefore k = 24$

$\therefore z = \frac{24}{w+3}$

When $w = 9$,

$z = \frac{24}{9+3}$

$= 2$

(ii) When $z = 2.4$,

$$2.4 = \frac{24}{w+3}$$

$$w + 3 = \frac{24}{2.4}$$

$$= 10$$

$$w = 7$$

9.

x	0.2	0.5	1	1.25	2
y	37.5	6	1.5	0.96	0.375

Since y is inversely proportional to $2x^2$,

then $y = \frac{k}{2x^2}$, where k is a constant.

When $x = 2$, $y = 0.375$,

$$0.375 = \frac{k}{2(2)^2}$$

$$\therefore k = 3$$

$$\therefore y = \frac{3}{2x^2}$$

When $y = 1.5$,

$$1.5 = \frac{3}{2x^2}$$

$$2x^2 = \frac{3}{1.5}$$

$$= 2$$

$$x^2 = 1$$

$$x = 1 \ (x > 0)$$

When $y = 0.96$,

$$0.96 = \frac{3}{2x^2}$$

$$2x^2 = \frac{3}{0.96}$$

$$= 3.125$$

$$x^2 = 1.5625$$

$$x = \sqrt{1.5625} \ \ (x > 0)$$

$$= 1.25$$

When $x = 0.2$

$$y = \frac{3}{2(0.2)^2}$$

$$= 37.5$$

When $x = 0.5$,

$$y = \frac{3}{2(0.5)^2}$$

$$= 6$$

10. (i) Total monthly charges

$= \$9.81 + \$0.086 \times 300$

$= \$35.61$

(ii) Variable amount $= \$20.56 - \$9.81$

$= \$10.75$

Duration of usage $= \frac{10.75}{0.086}$

$= 125$ minutes

(iii) Variable amount $= n \times \$0.086$

$= \$0.086n$

Total income = variable amount + fixed amount

$$\therefore C = 0.086n + 9.81$$

$$C - 9.81 = 0.086n$$

Since $\frac{C - 9.81}{n} = 0.086$ is a constant, then $C - 9.81$ is directly proportional to n.

11. (i) Since G is directly proportional to h,

then $G = kh$, where k is a constant.

When $h = 40$, $G = 2200$,

$$2200 = k \times 40$$

$$\therefore k = 55$$

$$\therefore G = 55h$$

(ii) When $h = 22$,

$$G = 55 \times 22$$

$$= 1210$$

$\therefore$ The gravitational potential energy of the objects is 1210 J.

(iii) When $G = 3025$,

$$3025 = 55 \times h$$

$$h = \frac{3025}{55}$$

$$= 55$$

$\therefore$ The height of the object above the surface of the Earth is 55 m.

12. Let the donations Kate makes be $\$d$,

the savings of Kate be $\$s$.

Since d is directly proportional to s^2,

then $d = ks^2$, where k is a constant.

When $s = 900$,

$$d = k \times 900^2$$

$$= 810\,000k$$

When $s = 1200$,

$$d = k \times 1200^2$$

$$= 1\,440\,000k$$

Since Kate's donation increases by \$35,

$$1\,440\,000k - 810\,000k = 35$$

$$63\,000k = 35$$

$$k = \frac{35}{630\,000}$$

$$= \frac{1}{18\,000}$$

Amount Kate donates in January $= \frac{1}{18\,000} \times 900^2$

$= \$45$

Amount Kate donates in February $= \frac{1}{18\,000} \times 1200^2$ or $45 + 35$

$= \$80$

13. (i) Since P is inversely proportional to V,

then $P = \frac{k}{V}$, where k is a constant.

When $V = 4000$, $P = 250$,

$250 = \frac{k}{4000}$

$\therefore k = 1\,000\,000$

$\therefore P = \frac{1\,000\,000}{V}$

When $V = 5000$,

$P = \frac{1\,000\,000}{5000}$

$= 200$

$\therefore$ The pressure of the gas is 200 Pa.

(ii) When $P = 125$,

$125 = \frac{1\,000\,000}{V}$

$V = \frac{1\,000\,000}{125}$

$= 8000$

$\therefore$ The volume of the gas is 8000 dm^3.

14. Let the number of days for 5 men to complete the job be x.

The number of men is inversely proportional to the number of days to complete the job.

5 men take x days to complete the job.

1 man takes $x \times 5$ days to complete the job.

6 men take $\frac{x \times 5}{6}$ days to complete the job.

Since the job can be completed 8 days earlier when 1 more man is hired,

$\frac{x \times 5}{6} = x - 8$

$\frac{5x}{6} = x - 8$

$5x = 6(x - 8)$

$= 6x - 48$

$x = 48$

It takes (48×5) days for 1 man to complete the job.

It takes 1 day for $(1 \times 48 \times 5)$ men to complete the job.

It takes $48 - 28 = 20$ days for $\frac{1 \times 48 \times 5}{20} = 12$ men to complete the job.

Additional number of men to hire $= 12 - 5$

$= 7$

$\therefore$ 7 more men should be hired.

Challenge Yourself

1. (a) Since A is directly proportional to C,

then $A = k_1C$, where k_1 is a constant.

Since B is directly proportional to C,

then $A = k_2C$, where k_2 is a constant.

$A + B = k_1C + k_2C$

$= (k_1 + k_2)C$

Since $\frac{A + B}{C} = k_1 + k_2$ is a constant, then $A + B$ is directly proportional to C.

(b) From **(a)**,

$A - B = k_1C - k_2C$

$= (k_1 - k_2)C$

Since $\frac{A + B}{C} = k_1 - k_2$ is a constant, then $A - B$ is directly proportional to C.

(c) $AB = (k_1C)(k_2C)$

$= k_1k_2C^2$

$\sqrt{AB} = \sqrt{k_1k_2C^2}$

$= \sqrt{k_1k_2}C$

Since $\frac{\sqrt{AB}}{C} = \sqrt{k_1k_2}$ is a constant, then $\sqrt{AB}$ is directly proportional to C.

2. (i) Since z is directly proportional to x^2 and inversely proportional to $\sqrt{y}$, then $z = \frac{kx^2}{\sqrt{y}}$, where k is a constant.

(ii) When $x = 2$, $y = 9$, $z = 16$,

$16 = \frac{k(2)^2}{\sqrt{9}}$

$= \frac{4k}{3}$

$\therefore k = 16 \times \frac{3}{4}$

$= 12$

$\therefore z = \frac{12x^2}{\sqrt{y}}$

When $x = 5$, $y = 4$,

$z = \frac{12(5)^2}{\sqrt{4}}$

$= 150$

3. (i) Since T is directly proportional to B and inversely proportional to P, then

$T = \frac{kB}{P}$, where k is a constant.

When $B = 3$, $P = 18$, $T = 20$,

$20 = \frac{k \times 3}{18}$

$= \frac{k}{6}$

$\therefore k = 120$

$\therefore T = \frac{120B}{P}$

(ii) When $B = 4$, $P = 16$,

$$T = \frac{120 \times 4}{16}$$

$= 30$

$\therefore$ The number of days needed is 30.

(iii) When $B = 10$, $T = 24$,

$$24 = \frac{120 \times 10}{P}$$

$$P = \frac{120 \times 10}{24}$$

$= 50$

$\therefore$ 50 painters need to be employed.

Chapter 2 Linear Graphs and Simultaneous Linear Equations

TEACHING NOTES

Suggested Approach

Students have learnt the graphs of straight lines in the form $y = mx + c$ in Secondary One. In this chapter, this will be expanded to cover linear equations in the form $ax + by = k$.

They have also learnt how to solve simple linear equations. Here, they will be learning how to solve simultaneous linear equations, where a pair of values of x and of y satisfies two linear equations simultaneously, or at the same time. Students are expected to know how to solve them graphically and algebraically and apply this to real-life scenarios by the end of the chapter.

Teachers can build up on past knowledge learnt by students when covering this chapter.

Section 2.1: Gradient of a Straight Line

Teachers should teach students how to take two points on the line and use it to calculate the vertical change (rise) and horizontal change (run), and then the gradient of the straight line.

To make learning more interactive, students can explore how the graph of a straight line in the form $y = mx + c$ changes when either m or c varies (see Investigation: Equation of a Straight Line). Through this investigation, students should be able to observe what happens to the line when m varies. Students should also learn how to differentiate between lines with a positive value of m, a negative value of m and when the value of m is 0.

Section 2.2: Further Applications of Linear Graphs in Real-World Contexts

Teachers can give examples of linear graphs used in many daily situations and explain what each of the graphs is used for. Through Worked Example 2, students will learn how the concepts of gradient and y-intercept can be applied and about their significance in real-world contexts and hence solve similar problems.

Section 2.3: Horizontal and Vertical Lines

Teachers should bring students' attention to the relationship between the graphs of $y = mx + c$ where $m = 0$, i.e. c units up or down parallel to the x-axis depending on whether $c > 0$ or $c < 0$. Hence, teachers can lead students to the conclusion that the graphs of $y = mx + c$ for various values of c are parallel and cut the y-axis at different points corresponding to different values of c. Students also need to know that vertical lines parallel to the x-axis have the equation $x = a$ and how this is related to the graphs of $y = mx + c$.

Section 2.4: Graphs of Linear Equations in the form $ax + by = k$

Before students start plotting the functions, they should revise the choice of scales and labelling of scales on both axes. Students are often weak in some of these areas. Many errors in students' work arise from their choice of scales. Teachers should spend some time to ensure students learn how to choose an appropriate scale. At this stage however, the choice of scales are specified in most questions.

Section 2.5: Solving Simultaneous Linear Equations Using Graphical Method

It is important teachers state the concept clearly that the point(s) of intersection of two graphs given the solution of a pair of simultaneous equations and this can be illustrated by solving a pair of linear simultaneous equations and then plotting the graphs of these two linear equations to verify the results (see Investigation: Solving Simultaneous Linear Equations Graphically)

Teachers should show clearly that a pair of simultaneous linear equations may have an infinite number of solutions or no solution (see Class Discussion: Coincident Lines and Parallel Lines, and Thinking Time on page 66).

Section 2.6: Solving Simultaneous Linear Equations Using Algebraic Methods

The ability to solve equations is crucial to the study of mathematics. The concept of solving simultaneous linear equations by adding or subtracting both sides of equations can be illustrated using physical examples. An example is drawing a balance and adding or removing coins from both sides of the balance.

Some students make common errors when they are careless in the multiplication or division of both sides of an equation and they may forget that all terms must be multiplied or divided by the same number throughout. The following are some examples.

- $x + 3y = 5$ is taken to imply $2x + 6y = 5$
- $5x + 15y = 14$ is taken to imply $x + 3y = 14$, and then $x = 14 - 3y$

Section 2.7: Applications of Simultaneous Equations in Real-World Contexts

Weaker students may have problems translating words into simultaneous linear equations. Teachers may wish to show more examples and allow more practice for students. Teachers may also want to group students of varying ability together, so that the better students can help the weaker students.

Challenge Yourself

Question 1 can be solved if the Thinking Time activity on page 88 has been discussed. The simultaneous equations in Question 2 can be converted to a familiar form by substituting $\frac{1}{x}$ with a and $\frac{1}{y}$ with b.

Teachers can slowly guide the students for Question 3 if they need help in forming the simultaneous equations.

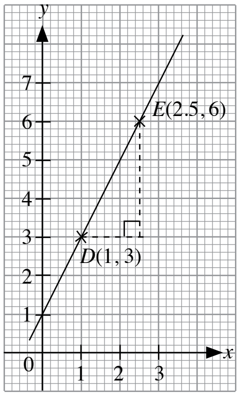
For Questions 4 and 5, teachers can advise students to eliminate one unknown variable and then applying the guess and check method which they have learnt in Primary Six.

WORKED SOLUTIONS

Investigation (Equation of a Straight Line)

1. As the value of c changes, the y-coordinate of the point of intersection of the line with the y-axis changes. The coordinates of the point where the line cuts the y-axis are $(0, c)$.
2. As the value of m increases from 0 to 5, the steepness of the line increases.
3. As the value of m decreases from 0 to –5, the steepness of the line increases.
4. A line with a positive value for m slopes upwards from the left to the right while a line with a negative value for m slopes downwards from the left to the right.

Class Discussion (Gradients of Straight Lines)



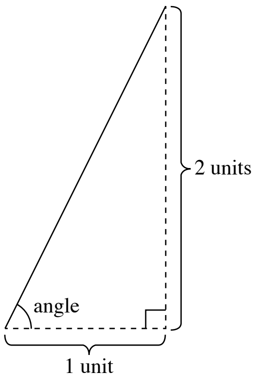
(i) Gradient of $DE = \frac{3}{1.5}$

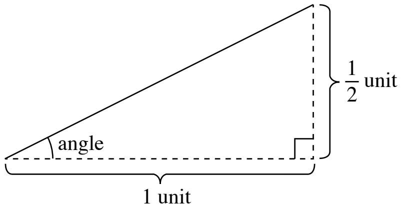
$= 2$

(ii) Yes, gradient of DE = gradient of AB.

(iii) Hence, we can choose *any* two points on a line to find its gradient because the gradient of a straight line is constant.

Class Discussion (Gradients in the Real World)

1. Angle of inclination = 45°
2. 

   Angle of inclination = 63°
3. 

   Angle of inclination = 27°
4. A road with a gradient of 1 is generally considered to be steep.

   Teachers may wish to get students to name some roads in Pakistan which they think may have an approximate gradient of 1 and to ask students how they can determine the gradients of the roads they have named.
5. A road with a gradient of $\frac{1}{2}$ is generally considered to be steep.

Investigation (Gradient of a Horizontal Line)

1. $B(-1, 2)$, $D(4, 2)$
2. In the line segment AC, rise = 0 and run = 3.
3. In the line segment BD, rise = 0 and run = 5.
4. Gradient of $AC = \frac{\text{rise}}{\text{run}}$

   $= \frac{0}{3}$

   $= 0$

   Gradient of $BD = \frac{\text{rise}}{\text{run}}$

   $= \frac{0}{5}$

   $= 0$

   $\therefore$ The gradient of a horizontal line is 0.

Investigation (Gradient of a Vertical Line)

1. $Q(3, 2)$, $S(3, -3)$
2. In the line segment PR, rise = 4 and run = 0.
3. In the line segment QS, rise = 5 and run = 0.
4. Gradient of $PR = \frac{\text{rise}}{\text{run}}$

   $= \frac{4}{0}$

   = undefined

   Gradient of $QS = \frac{\text{rise}}{\text{run}}$

   $= \frac{5}{0}$

   $\therefore$ The gradient of QS is undefined.

   $\therefore$ The gradient of a vertical line is undefined.

Investigation (Equation of a Horizontal Line)

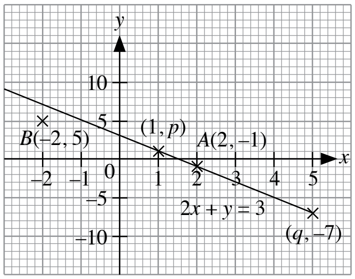
1. The gradient of the horizontal line is 0.
2. $B(-2, 3)$, $D(3, 3)$
3. The y-coordinates of all the four points are equal to 3.
4. $y = 3$

Investigation (Equation of a Vertical Line)

1. The gradient of the horizontal line is undefined.
2. $Q(2, 1)$, $S(2, -4)$
3. The x-coordinates of all the four points are equal to 2.
4. $x = 2$

Investigation (Graphs of $ax + by = k$)

1. (i)



(ii) The point $A(2, -1)$ lies on the graph. The point $B(-2, 5)$ does not lie on the graph.

When $x = 2$, $2(2) + y = 3$

$4 + y = 3$

$y = -1$

When $x = -2$, $2(-2) + y = 3$

$-4 + y = 3$

$y = 7 \neq 5$

$A(2, -1)$ satisfies the equation $2x + y = 3$. $B(-2, 5)$ does not satisfy the equation $2x + y = 3$.

(iii) When $x = 1$, $y = p = 1$.

(iv) When $y = -7$, $x = q = 5$.

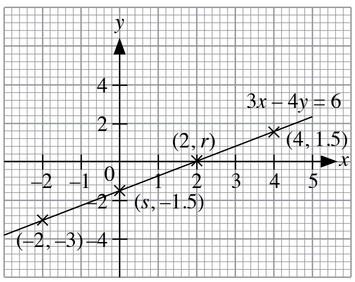
(v) The graph of $y = -2x + 3$ coincides with the graph of $2x + y = 3$.

$2x + y = 3$

$2x - 2x + y = -2x + 3$ (Subtract $2x$ from both sides)

$y = -2x + 3$

2. (i)



(ii) When $x = 2$, $y = r = 0$

(iii) When $y = -1.5$, $x = s = 0$

(iv) The coordinates of two other points are $(-2, -3)$ and $(4, 1.5)$.

Other points can be used, as long as they lie on the line.

(v) The graph of $y = \frac{3}{4}x - \frac{3}{2}$ coincides with the graph of $3x - 4y = 6$.

$3x - 4y = 6$

$3x - 3x - 4y = -3x + 6$ (Subtract $3x$ from both sides)

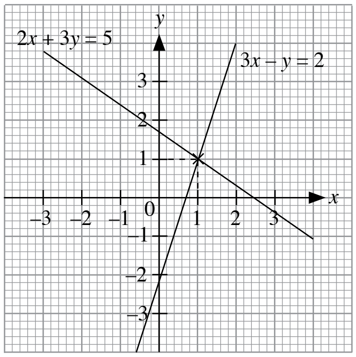
$-4y = -3x + 6$

$\frac{-4y}{-4} = \frac{-3x + 6}{-4}$ (Divide both sides by -4)

$y = \frac{3}{4}x - \frac{3}{2}$

Investigation (Solving Simultaneous Linear Equation Graphically)

1. (i)



(ii) The coordinates of the point of intersection of the two graphs are $(1, 1)$.

(iii) For $2x + 3y = 5$

When $x = -2$, $2(-2) + 3y = 5$

$y = 3$

When $x = 0$, $2(0) + 3y = 5$

$y = 1\frac{2}{3} \neq -2$

When $x = 1$, $2(1) + 3y = 5$

$y = 1$

When $x = 2$, $2(2) + 3y = 5$

$y = \frac{1}{3} \neq 4$

When $x = 4$, $2(4) + 3y = 5$

$y = -1$

For $3x - y = 2$

When $x = -2$, $3(-2) - y = 2$

$y = -8 \neq 3$

When $x = 0$, $3(0) - y = 2$

$y = -2$

When $x = 1$, $3(1) - y = 2$

$y = 1$

When $x = 2$, $3(2) - y = 2$

$y = 4$

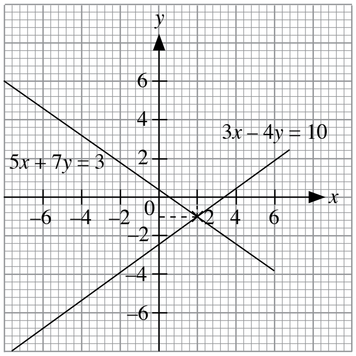
When $x = 4$, $3(4) - y = 2$

$y = 10$

The pair of values satisfying both equations is $x = 1$, $y = 1$.

The pair of values is the same as the point of intersection of the two graphs.

2. **(i)**



(ii) The coordinates of the point of intersection of the two graphs are $(2, -1)$

(iii) The pair of values of x and y that satisfies both equations are $x = 2$ and $y = -1$.

The coordinates of the point of intersection of the two graphs is the pair of values of x and y that satisfies both the equations.

A coordinates that lies on one line will satisfy the equation of that line. The same applies to the second line. Hence, the coordinates of the point of intersection is the same as the point that lies on both lines and that satisfy both equations.

Class Discussion (Choice of Appropriate Scales for Graphs and Accuracy of Graphs)

1. The graphs should look different to students who have used different scales in both axes.

Teachers should remind students to make a table of values, with at least 3 points, so as to construct the graph of a linear equation.

Though two points are sufficient to draw a straight line, the third point will act as a check for the accuracy of the straight line.

It is likely that most students will use 1 cm to 1 unit for both scales.

For the better students, prompt them to experiment with other scales, such as 2 cm to 1 unit, 4 cm to 1 unit or 5 cm to 1 unit.

2. **(i)** $y = 2.9$

(ii) $x = -0.6$

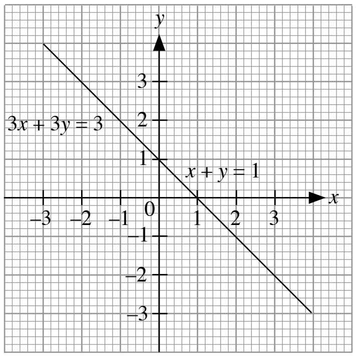
If students use 1 cm to 1 unit for both scales, they would discover that the point in **(i)** *lies between squares on the graph paper.*

3. By substituting the given value into the linear equation, one can check for the accuracy of the answers.

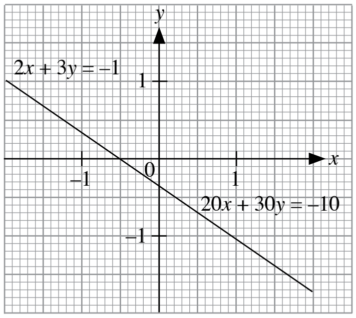
4. Use a larger scale (from 1 cm to 1 unit to 2 cm to 1 unit) and redraw the graph.

Class Discussion (Coincident Lines and Parallel Lines)

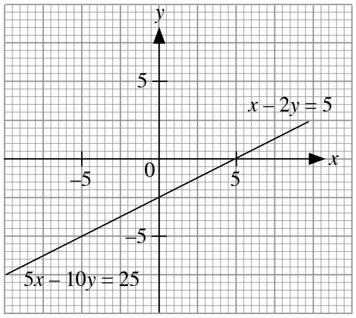
1. **(a)** **(i)**



(ii)



(iii)

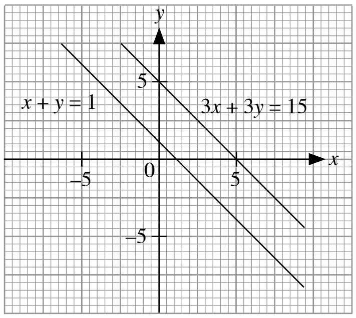


(b) The graphs of each pair of simultaneous equations are a pair of lines that coincide.

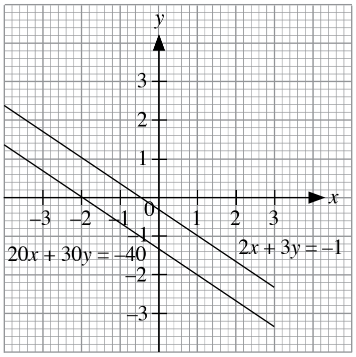
(c) Yes, each pair of simultaneous equations has solutions.

The solutions are all the points that lie on the line.

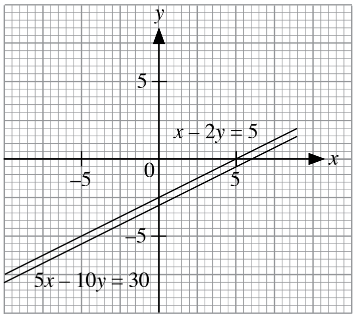
2. **(a)** **(i)**



(ii)



(iii)



(b) The graphs of each pair of simultaneous equations are a pair of parallel lines.

(c) No, each pair of simultaneous equations does not have any solution since they do not intersect and have any point of intersection.

Thinking Time (Page 66)

(a) A pair of simultaneous equations where one equation can be obtained from the other equation through multiplication or division, that is, both equations are equivalent, has infinitely many solutions.

(b) A pair of simultaneous equations where one equation can be contradicted by the other equation has no solution.

Besides the equations in the Class Discussion on the same page, teachers may wish to ask students to come up with their own pairs of simultaneous equations with infinitely many solutions or no solutions.

Thinking Time (Page 68)

The solutions to a linear equation in two variables are the set of x values and y values that satisfy the linear equation. There are infinitely many solutions for all real values of x and y.

For example, the solutions to the equation $2x + y = 13$ is the set $\{(x, y): 2x + y = 13\}$. Some solutions in the set are (1, 11), (2, 9), (3, 7) etc.

Thinking Time (Page 71)

$13x - 6y = 20$ — (1)

$7x + 4y = 18$ — (2)

$7 \times (1)$: $91x - 42y = 140$ — (3)

$13 \times (2)$: $91x + 52y = 234$ — (4)

(3) – (4):

$$(91x - 42y) - (91x + 52y) = 140 - 234$$
$$-94y = -94$$
$$y = 1$$

Substitute $y = 1$ into (1): $13x - 61(1) = 20$

$$13x = 26$$
$$x = 2$$

$\therefore$ The solution is $x = 2$ and $y = 1$.

No. it is not easier to eliminate x first as the LCM of 13 and 7 is larger than 12.

Thinking Time (Page 73)

$7x - 2y = 21$ — (1)

$4x + y = 57$ — (2)

From (2), $x = \dfrac{57 - y}{4}$ — (3)

Substitute (3) into (1):

$$\left(\frac{57 - y}{4}\right) - 2y = 21$$
$$7(57 - y) - 8y = 84$$
$$399 - 7y - 8y = 84$$
$$15y = 315$$
$$y = 21$$

Substitute $y = 21$ into (3): $x = \dfrac{57 - 21}{4}$

$$= 9$$

$\therefore$ The solution is $x = 9$ and $y = 21$.

If x is made the subject of equation (1) or (2), we will get the same solution. Making y as the subject of equation is easier since algebraic fractions will not be introduced then.

Thinking Time (Page 74)

$2x + y = 6$ — (1)

$x = 1 - \dfrac{1}{2}y$ — (2)

$2 \times (2)$: $2x = 2 - y$

$2x + y = 2$ — (3)

Comparing (1) and (3), we notice that the gradients of the 2 equations are the same but with different constants; i.e. they are parallel lines with no solution.

Thinking Time (Page 78)

Let the smaller number be x. Then the greater number is $67 - x$.

$\therefore (67 - x) - x = 3$

$67 - 2x = 3$

$2x = 64$

$\therefore x = 32$

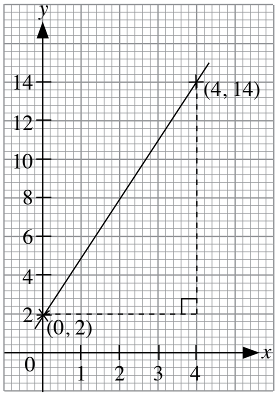
Greater number $= 67 - 32$

$= 35$

The two numbers are 32 and 35.

Practice Now 1

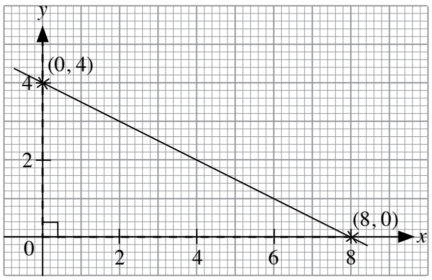
(a)



Gradient $= \frac{12}{4}$

$= 3$

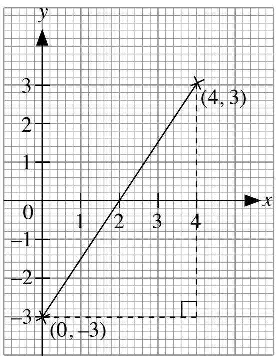
(b)



Gradient $= -\frac{4}{8}$

$= -\frac{1}{2}$

(c)

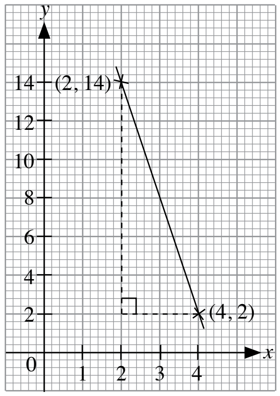


Gradient $= \frac{6}{4}$

$= \frac{3}{2}$

$= 1\frac{1}{2}$

(d)



Gradient $= -\frac{12}{2}$

$= -6$

Practise Now 2

(a) Time taken for the technician to repair each computer = 20 minutes

(b) Distance between the technician's workshop and his first customer = 9 km

(c) **(i)** Gradient of $OA = \frac{9}{10}$

The average speed of the technician was $\frac{9}{10}$ km/min.

(ii) Gradient of $AB = 0$

The average speed of the technician was 0 km/min.

(iii) Gradient of $BC = -\frac{4}{5}$

The average speed of the technician was $\frac{4}{5}$ km/min.

(iv) Gradient of $CD = 0$

The average speed of the technician was 0 km/min.

(v) Gradient of $DE = -\frac{5}{7}$

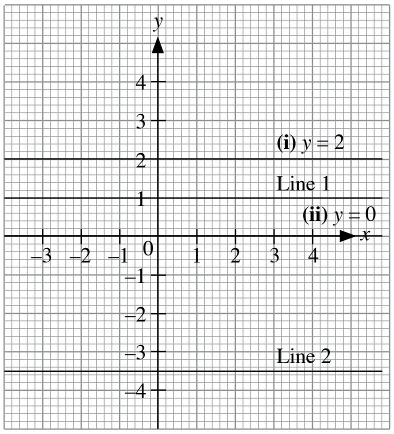
The average speed of the technician was $\frac{5}{7}$ km/min.

Practise Now (Page 56)

(a) Line 1: $y = 1$

Line 2: $y = -3.5$

(b)



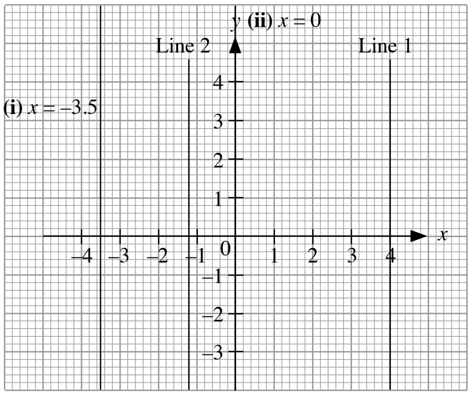
The lines are horizontal. The y-coordinates of all the points on the lines are a constant.

Practise Now (Page 58)

(a) Line 1: $x = 4$

Line 2: $x = -1.2$

(b)



The lines are vertical. The x-coordinates of all the points on the lines are a constant.

Practise Now 3

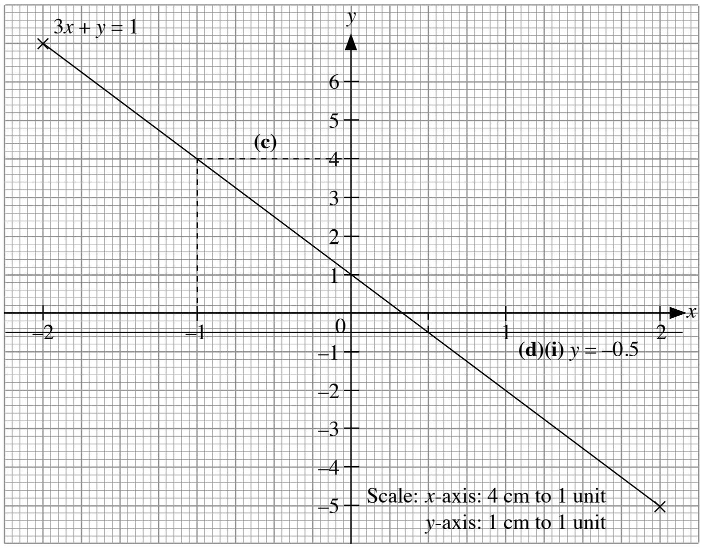
(a) When $x = -2$, $y = p$,

$3(-2) + p = 1$

$-6 + p = 1$

$\therefore p = 7$

(b)



(c) From the graph in **(b)**,

When $x = -1$,

$q = y = 4$

(d) **(ii)** x-coordinate = 0.5

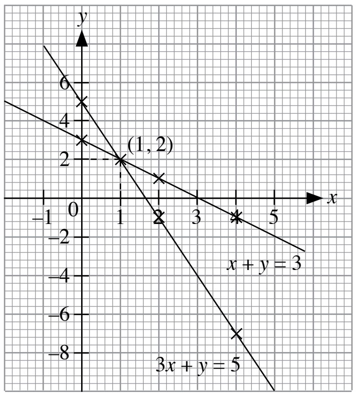
Practise Now 4

1. $x + y = 3$

x	0	2	4
y	3	1	–1

$3x + y = 5$

x	0	2	4
y	5	–1	–7



Scale: x-axis: 1 cm to 1 unit

y-axis: 1 cm to 2 units

The graphs intersect at the point (1, 2).

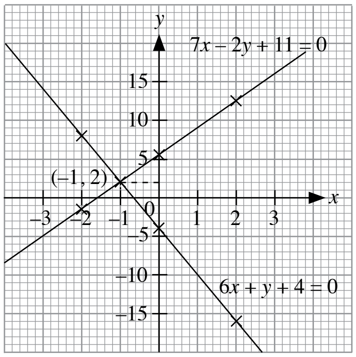
$\therefore$ The solution is $x = 1$ and $y = 2$.

2. $7x - 2y + 11 = 0$

x	-2	0	2
y	-1.5	5.5	12.5

$6x + y + 4 = 0$

x	-2	0	2
y	8	-4	-16



Scale: x-axis: 1 cm to 1 unit
y-axis: 1 cm to 5 units

The graphs intersect at the point $(-1, 2)$.
$\therefore$ The solution is $x = -1$ and $y = 2$.

Practise Now 5

1. (a) $x - y = 3$ — (1)
$4x + y = 17$ — (2)
(2) + (1):
$(4x + y) + (x - y) = 17 + 3$
$4x + y + x - y = 20$
$5x = 20$
$x = 4$
Substitute $x = 4$ into (2):
$4(4) + y = 17$
$16 + y = 17$
$y = 1$
$\therefore$ The solution is $x = 4$ and $y = 1$.

(b) $7x + 2y = 19$ — (1)
$7x + 8y = 13$ — (2)
(2) – (1):
$(7x + 8y) - (7x + 2y) = 13 - 19$
$7x + 8y - 7x - 2y = -6$
$6y = -6$
$y = -1$
Substitute $y = -1$ into (1):
$7x + 2(-1) = 19$
$7x - 2 = 19$
$7x = 21$
$x = 3$
$\therefore$ The solution is $x = 3$ and $y = -1$.

(c) $13x + 9y = 4$ — (1)
$17x - 9y = 26$ — (2)
(1) + (2):
$(13x + 9y) + (17x - 9y) = 4 + 26$
$13x + 9y + 17x - 9y = 30$
$30x = 30$
$x = 1$
Substitute $x = 1$ into (1):
$13(1) + 9y = 4$
$13 + 9y = 4$
$9y = -9$
$y = -1$
$\therefore$ The solution is $x = 1$ and $y = -1$.

(d) $4x - 5y = 17$ — (1)
$x - 5y = 8$ — (2)
(1) – (2):
$(4x - 5y) - (x - 5y) = 17 - 8$
$4x - 5y - x + 5y = 9$
$3x = 9$
$x = 3$
Substitute $x = 3$ into (2):
$3 - 5y = 8$
$-5y = 5$
$y = -1$
$\therefore$ The solution is $x = 3$ and $y = -1$.

2. $3x - y + 14 = 0$ — (1)
$2x + y + 1 = 0$ — (2)
(1) + (2):
$(3x - y + 14) + (2x + y + 1) = 0 + 0$
$3x - y + 14 + 2x + y + 1 = 0$
$5x + 15 = 0$
$5x = -15$
$x = -3$
Substitute $x = -3$ into (2):
$2(-3) + y + 1 = 0$
$y - 5 = 0$
$y = 5$
$\therefore$ The solution is $x = -3$ and $y = 5$.

Practise Now 6

(a) $2x + 3y = 18$ — (1)
$3x - y = 5$ — (2)
$3 \times$ (2): $9x - 3y = 15$ — (3)
(1) + (3):
$(2x + 3y) + (9x - 3y) = 18 + 15$
$2x + 3y + 9x - 3y = 33$
$11x = 33$
$x = 3$
Substitute $x = 3$ into (2):
$3(3) - y = 5$
$9 - y = 5$
$y = 4$
$\therefore$ The solution is $x = 3$ and $y = 4$.

(b) $4x + y = 11$ — (1)

$3x + 2y = 7$ — (2)

$2 \times (1)$: $8x + 2y = 22$ — (3)

(3) – (2):

$(8x + 2y) - (3x + 2y) = 22 - 7$

$8x + 2y - 3x - 2y = 15$

$5x = 15$

$x = 3$

Substitute $x = 3$ into (1):

$4(3) + y = 11$

$12 + y = 11$

$y = -1$

$\therefore$ The solution is $x = 3$ and $y = -1$.

Practise Now 7

(a) $9x + 2y = 5$ — (1)

$7x - 3y = 13$ — (2)

$3 \times (1)$: $27x + 6y = 15$ — (3)

$2 \times (2)$: $14x - 6y = 26$ — (4)

(3) + (4):

$(27x + 6y) + (14x - 6y) = 15 + 26$

$27x + 6y + 14x - 6y = 41$

$41x = 41$

$x = 1$

Substitute $x = 1$ into (1):

$9(1) + 2y = 5$

$9 + 2y = 5$

$2y = -4$

$y = -2$

$\therefore$ The solution is $x = 1$ and $y = -2$.

(b) $5x - 4y = 17$ — (1)

$2x - 3y = 11$ — (2)

$2 \times (1)$: $10x - 8y = 34$ — (3)

$5 \times (2)$: $10x - 15y = 55$ — (4)

(3) – (4):

$(10x - 8y) - (10x - 15y) = 34 - 55$

$10x - 8y - 10x + 15y = -21$

$7y = -21$

$y = -3$

Substitute $y = -3$ into (2):

$2x - 3(-3) = 11$

$2x + 9 = 11$

$2x = 2$

$x = 1$

$\therefore$ The solution is $x = 1$ and $y = -3$.

Practise Now 8

Method 1:

$\frac{x}{2} - \frac{y}{3} = 4$ — (1)

$\frac{2}{5}x - \frac{y}{6} = 3\frac{1}{2}$ — (2)

$\frac{1}{2} \times (1)$: $\frac{x}{4} - \frac{y}{6} = 2$ — (3)

(2) – (3):

$\left(\frac{2}{5}x - \frac{y}{6}\right) - \left(\frac{x}{4} - \frac{y}{6}\right) = 3\frac{1}{2} - 2$

$\frac{2}{5}x - \frac{y}{6} - \frac{x}{4} + \frac{y}{6} = 1\frac{1}{2}$

$\frac{3}{20}x = 1\frac{1}{2}$

$x = 10$

Substitute $x = 10$ into (1):

$\frac{10}{2} - \frac{y}{3} = 4$

$5 - \frac{y}{3} = 4$

$\frac{y}{3} = 1$

$y = 3$

$\therefore$ The solution is $x = 10$ and $y = 3$.

Method 2:

$\frac{x}{2} - \frac{y}{3} = 4$ — (1)

$\frac{2}{5}x - \frac{y}{6} = 3\frac{1}{2}$ — (2)

$30 \times (1)$: $15x - 10y = 120$ — (3)

$60 \times (2)$: $24x - 10y = 210$ — (4)

(4) – (3):

$(24x - 10y) - (15x - 10y) = 210 - 120$

$24x - 10y - 15x + 10y = 90$

$9x = 90$

$x = 10$

Substitute $x = 10$ into (3):

$15(10) - 10y = 120$

$150 - 10y = 120$

$-10y = -30$

$y = 3$

$\therefore$ The solution is $x = 10$ and $y = 3$.

Practise Now 9

$3y - x = 7$ — (1)

$2x + 3y = 4$ — (2)

From (1), $x = 3y - 7$ — (3)

Substitute (3) into (2):

$2(3y - 7) + 3y = 4$

$6y - 14 + 3y = 4$

$9y = 18$

$y = 2$

Substitute $y = 2$ into (3):

$x = 3(2) - 7$

$= -1$

$\therefore$ The solution is $x = -1$ and $y = 2$.

Practise Now 10

$3x - 2y = 8 \quad — (1)$

$4x + 3y = 5 \quad — (2)$

From (1), $3x = 2y + 8$

$$x = \frac{2y+8}{3} \quad — (3)$$

Substitute (3) into (2):

$$4\left(\frac{2y+8}{3}\right) + 3y = 5$$

$$4(2y + 8) + 9y = 15$$

$$8y + 32 + 9y = 15$$

$$17y + 32 = 15$$

$$17y = -17$$

$$y = -1$$

Substitute $y = -1$ into (3):

$$x = \frac{2(-1)+8}{3}$$

$$= 2$$

$\therefore$ The solution is $x = 2$ and $y = -1$.

Practise Now 11

(a) $\frac{x-1}{y-3} = \frac{2}{3} \quad — (1)$

$\frac{x-2}{y-1} = \frac{1}{2} \quad — (2)$

From (1),

$3(x - 1) = 2(y - 3)$

$3x - 3 = 2y - 6$

$3x - 2y = -3 \quad — (3)$

From (2),

$2(x - 2) = y - 1$

$2x - 4 = y - 1$

$y = 2x - 3 \quad — (4)$

Substitute (4) into (3):

$3x - 2(2x - 3) = -3$

$3x - 4x + 6 = -3$

$-x + 6 = -3$

$x = 9$

Substitute $x = 9$ into (4):

$y = 2(9) - 3$

$= 15$

$\therefore$ The solution is $x = 9$ and $y = 15$.

(b) $3x + 2y = 3 \quad — (1)$

$\frac{1}{x+y} = \frac{3}{x+2y} \quad — (2)$

From (2),

$x + 2y = 3(x + y)$

$= 3x + 3y$

$y = -2x \quad — (3)$

Substitute (3) into (1):

$3x + 2(-2x) = 3$

$3x - 4x = 3$

$x = -3$

Substitute $x = -3$ into (3):

$y = -2(-3)$

$= 6$

$\therefore$ The solution is $x = -3$ and $y = 6$.

Practise Now 12

1. Let the smaller number be x and the greater number be y.

$x + y = 36 \quad — (1)$

$y - x = 9 \quad — (2)$

(1) + (2):

$2y = 45$

$y = 22.5$

Substitute $y = 22.5$ into (1):

$x + 22.5 = 36$

$x = 13.5$

$\therefore$ The two numbers are 13.5 and 22.5.

2. Let the smaller angle be x and the greater angle be y.

$\frac{1}{3}(x + y) = 60° \quad — (1)$

$\frac{1}{4}(y - x) = 28° \quad — (2)$

$3 \times (1)$: $x + y = 180° \quad — (3)$

$4 \times (2)$: $y - x = 112° \quad — (4)$

(3) + (4):

$2y = 292°$

$y = 146°$

Substitute $y = 146°$ into (3):

$x + 146° = 180°$

$x = 34°$

$\therefore$ The two angles are 34° and 146°.

3. $x + y + 2 = 2x + 1 \quad — (1)$

$2y = x + 2 \quad — (2)$

From (1),

$y = x - 1 \quad — (3)$

Substitute (3) into (2):

$2(x - 1) = x + 2$

$2x - 2 = x + 2$

$x = 4$

Substitute $x = 4$ into (3):

$y = 4 - 1$

$= 3$

Length of rectangle = 2(4) + 1

= 9 cm

Breadth of rectangle = 2(3)

= 6 cm

Perimeter of rectangle = 2(9 + 6)

= 30 cm

$\therefore$ The perimeter of the rectangle is 30 cm.

Practise Now 13

Let the numerator of the fraction be x and its denominator be y,

i.e. let the fraction be $\frac{x}{y}$.

$\frac{x+1}{y+1} = \frac{4}{5}$ — (1)

$\frac{x-5}{y-5} = \frac{1}{2}$ — (2)

From (1),

$5(x + 1) = 4(y + 1)$

$5x + 5 = 4y + 4$

$5x - 4y = -1$ — (3)

From (2),

$2(x - 5) = y - 5$

$2x - 10 = y - 5$

$y = 2x - 5$ — (4)

Substitute (4) into (3):

$5x - 4(2x - 5) = -1$

$5x - 8x + 20 = 1$

$-3x = -21$

$x = 7$

Substitute $x = 7$ into (4):

$y = 2(7) - 5$

$= 9$

∴ The fraction is $\frac{7}{9}$.

Practise Now 14

1. Let the present age of Kate be x years and that of Kate's father be y years.

Then in 5 years' time, Kate's father will be $(y + 5)$ years old and Kate will be $(x + 5)$ years old.

4 years ago, Kate's father was $(y - 4)$ years old and and Kate was $(x - 4)$ years old.

$y + 5 = 3(x + 5)$ — (1)

$y - 4 = 6(x - 4)$ — (2)

From (1),

$y + 5 = 3x + 15$

$y = 3x + 10$ — (3)

Substitute (3) into (2):

$3x + 10 - 4 = 6(x - 4)$

$= 6x - 24$

$3x = 30$

$x = 10$

Substitute $x = 10$ into (3):

$y = 3(10) + 10$

$= 40$

∴ Kate's present age is 10 years and Kate's father's present age is 40 years.

2. Let the amount an adult has to pay be $\$x$ and the amount a child has to pay be $\$y$.

$11x + 5y = 280$ — (1)

$14x + 9y = 388$ — (2)

$9 \times (1)$: $99x + 45y = 2520$ — (3)

$5 \times (2)$: $70x + 45y = 1940$ — (4)

(3) – (4):

$(99x + 45y) - (70x + 45y) = 2520 - 1940$

$29x = 580$

$x = 20$

Substitute $x = 20$ into (1):

$11(20) + 5y = 280$

$220 + 5y = 280$

$5y = 60$

$y = 12$

Total amount a family of 2 adults and 3 children have to pay

$= \$(2x + 3y)$

$= \$[2(20) + 3(12)]$

$= \$76$

∴ The family has to pay $76.

Practise Now 15

Let the tens digit of the original numer be x and its ones digit be y.
Then the original number is $10x + y$, the number obtained when the digits of the original number are reversed is $10y + x$.

$x + y = 11$ — (1)

$10x + y - (10y + x) = 9$ — (2)

From (2),

$10x + y - 10y - x = 9$

$9x - 9y = 9$

$x - y = 1$ — (3)

(1) + (3):

$2x = 12$

$x = 6$

Substitute $x = 6$ into (1):

$6 + y = 11$

$y = 5$

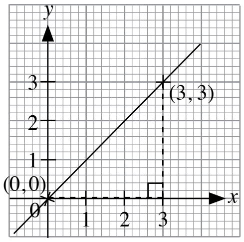
∴ The original number is 65.

Exercise 2A

1. Gradient of Line 1 = 0

The Gradient of Line 2 is undefined.

2. **(a)**

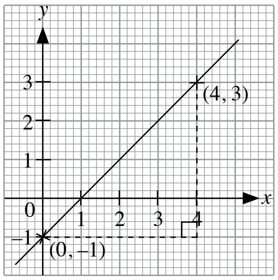


$m = \frac{3}{3}$

$= 1$

$c = 0$

(b)

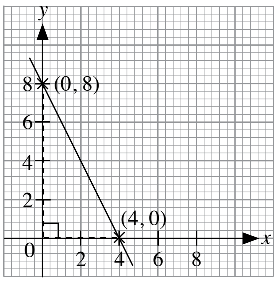


$m = \frac{4}{4}$

$= 1$

$c = -1$

(c)

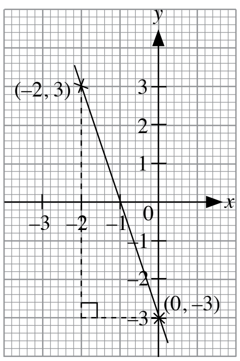


$m = -\frac{8}{4}$

$= -2$

$c = 8$

(d)

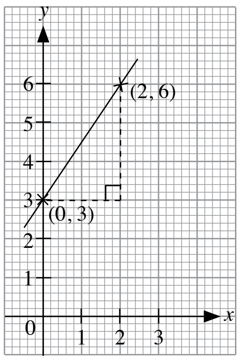


$m = -\frac{6}{2}$

$= -3$

$c = -3$

(e)

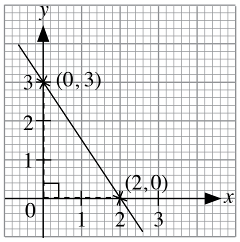


$m = \frac{3}{2}$

$= 1\frac{1}{2}$

$c = 3$

(f)

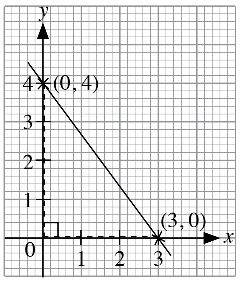


$m = -\frac{3}{2}$

$= -1\frac{1}{2}$

$c = 3$

(g)

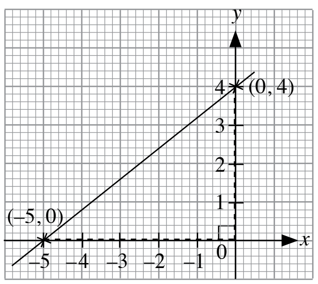


$m = -\frac{4}{3}$

$= -1\frac{1}{3}$

$c = 4$

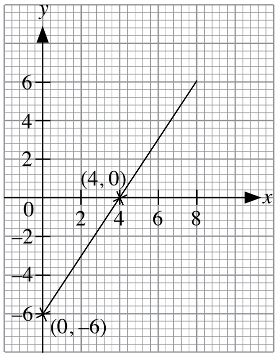
(h)



$m = \frac{4}{5}$

$c = 4$

(i)

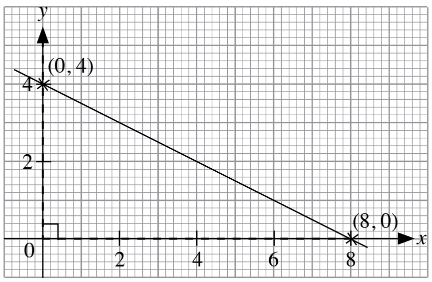


$m = \frac{6}{4}$

$= 1\frac{1}{2}$

$c = -6$

(j)

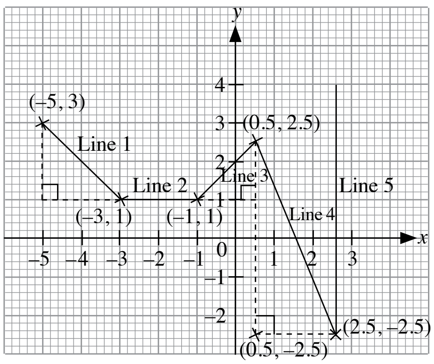


$m = -\frac{4}{8}$

$= -\frac{1}{2}$

$c = 4$

3.



Gradient of Line 1 $= -\frac{2}{2}$

$= -1$

Gradient of Line 2 = 0

Gradient of Line 3 $= \frac{1.5}{1.5}$

$= 1$

Gradient of Line 4 $= -\frac{5}{2}$

$= -2\frac{1}{2}$

The gradient of Line 5 is undefined.

4. Gradient of Line 1 = 0

Gradient of Line 2 = gradient of Line 5

$= -3$

The slope of Line 3 is undefined.

Gradient of Line 4 = gradient of Line 6

$= \frac{1}{2}$

Exercise 2B

1. **(a)** Khairul left home at 1000 hours.

(b) Distance Khairul travelled before he reached the cafeteria

= 50 km

(c) **(i)** Gradient of $OA = \frac{50}{1}$

$= 50$

Khairul's average speed was 50 km/h.

(ii) Gradient of $AB = 0$

Khairul's average speed was 0 km/h.

(iii) Gradient of $BC = \frac{30}{\frac{1}{2}}$

$= 60$

Khairul's average speed was 60 km/h.

2. **(a)** Distance between Ethan's home and the post office = 40 km

(b) Total time Ethan stayed at the post office and at the hawker centre

$= 1 + \frac{1}{2}$

$= 1\frac{1}{2}$ hours

(c) **(i)** Gradient of $OA = \frac{40}{2}$

$= 20$

Ethan's average speed was 20 km/h.

(ii) Gradient of $BC = -\frac{20}{1\frac{1}{2}}$

$= -13\frac{1}{3}$

Ethan's average speed was $13\frac{1}{3}$ km/h.

(iii) Gradient of $DE = -\frac{20}{1}$

$= -20$

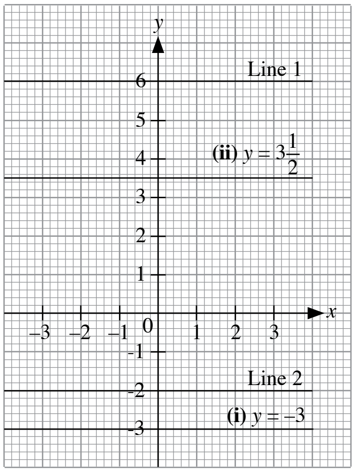
Ethan's average speed was 20 km/h.

Exercise 2C

1. **(a)** Line 1: $y = 6$

Line 2: $y = -2$

(b)

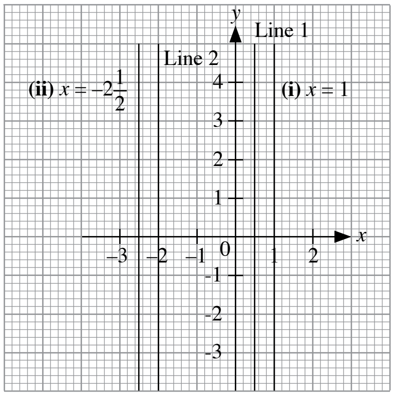


The lines are horizontal. The y-coordinates of all the points on the lines are a constant.

2. **(a)** Line 1: $x = 0.5$

Line 2: $x = -2$

(b)



The lines are vertical. The x-coordinates of all the points on the lines are a constant.

3. **(a)** When $x = -5$, $y = p$,

$-(-5) + 2p = 4$

$5 + 2p = 4$

$2p = -1$

$p = -\frac{1}{2}$

When $x = 5$, $y = q$,

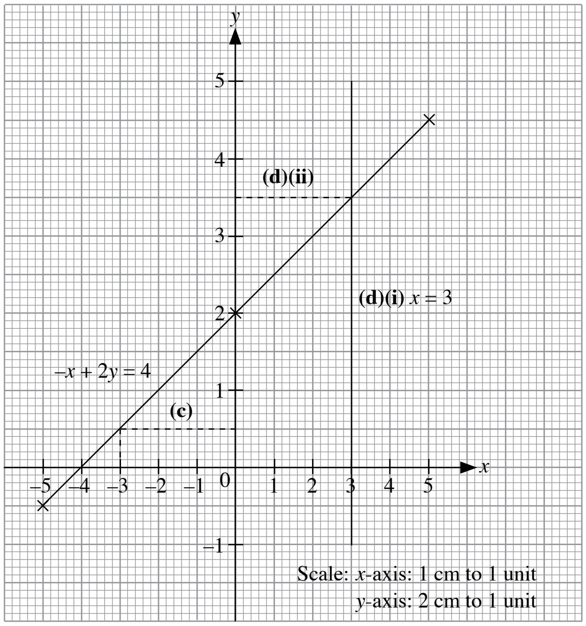
$-5 + 2q = 4$

$2q = 9$

$q = 4\frac{1}{2}$

$\therefore p = -\frac{1}{2}, q = 4\frac{1}{2}$

(b)



(c) From the graph in **(b)**,

When $y = 0.5$,

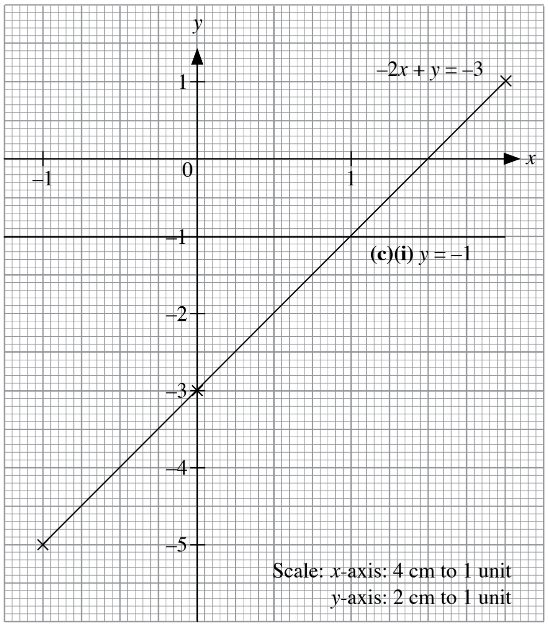
$r = x = -3$

(d) **(ii)** y-coordinate $= 3\frac{1}{2}$

4. **(a)** $-2x + y = -3$

x	–1	0	2
y	–5	–3	1

(b)



(c) **(ii)** Area of trapezium

$= \frac{1}{2} \times \left(1\frac{1}{2} + 1\right) \times 1$

$= 1\frac{1}{4}$ units^2

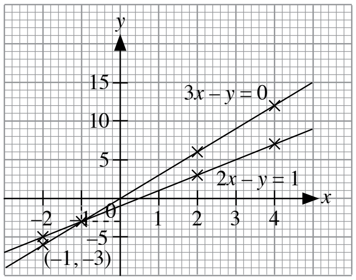
Exercise 2D

1. (a) $3x - y = 0$

x	–2	2	4
y	–6	6	12

$2x - y = 1$

x	–2	2	4
y	–5	3	7



Scale: x-axis: 1 cm to 1 unit
y-axis: 1 cm to 5 units

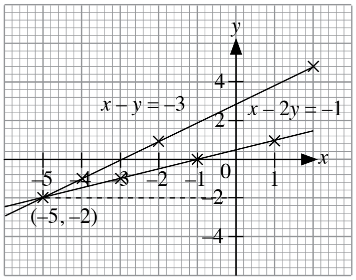
The graphs intersect at the point (–1, –3).
∴ The solution is $x = -1$ and $y = -3$.

(b) $x - y = -3$

x	–4	–2	0
y	–1	1	3

$x - 2y = -1$

x	–3	–1	1
y	–1	0	1



Scale: x-axis: 1 cm to 1 unit
y-axis: 1 cm to 2 units

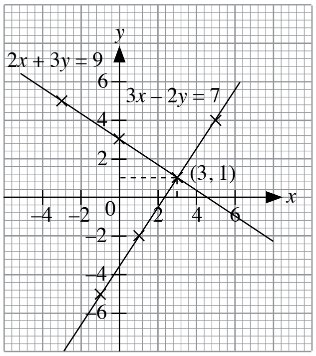
The graphs intersect at the point (–5, –2).
∴ The solution is $x = -5$ and $y = -2$.

(c) $3x - 2y = 7$

x	–1	1	5
y	–5	–2	4

$2x + 3y = 9$

x	–3	0	3
y	5	3	1



Scale: x-axis: 1 cm to 2 units
y-axis: 1 cm to 2 units

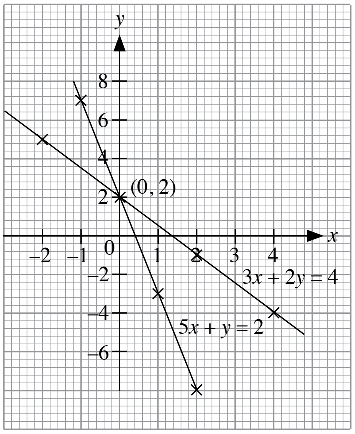
The graphs intersect at the point (3, 1).
∴ The solution is $x = 3$ and $y = 1$.

(d) $3x + 2y = 4$

x	–2	2	4
y	5	–1	–4

$5x + y = 2$

x	–1	1	2
y	7	–3	–8



Scale: x-axis: 1 cm to 1 unit
y-axis: 1 cm to 2 units

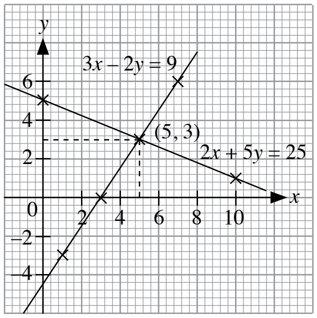
The graphs intersect at the point (0, 2).
∴ The solution is $x = 0$ and $y = 2$.

(e) $2x + 5y = 25$

x	0	5	10
y	5	3	1

$3x - 2y = 9$

x	1	3	7
y	–3	0	6



Scale: x-axis: 1 cm to 2 units
y-axis: 1 cm to 5 units

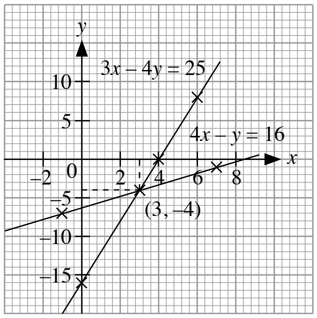
The graphs intersect at the point (5, 3).
∴ The solution is $x = 5$ and $y = 3$.

(f) $3x - 4y = 25$

x	–1	3	7
y	–7	–4	–1

$4x - y = 16$

x	0	4	6
y	–16	0	8



Scale: x-axis: 1 cm to 2 units
y-axis: 1 cm to 5 units

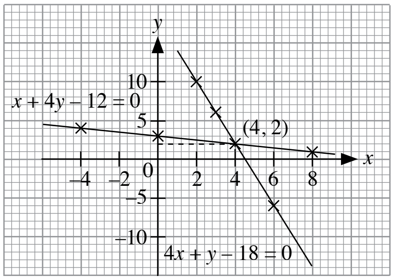
The graphs intersect at the point (3, –4).
∴ The solution is $x = 3$ and $y = –4$.

2. **(a)** $x + 4y - 12 = 0$

x	–4	0	8
y	4	3	1

$4x + y - 18 = 0$

x	2	3	6
y	10	6	–6



Scale: x-axis: 1 cm to 2 units
y-axis: 1 cm to 5 units

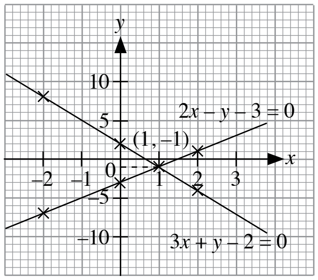
The graphs intersect at the point (4, 2).
∴ The solution is $x = 4$ and $y = 2$.

(b) $3x + y - 2 = 0$

x	–2	0	2
y	8	2	–4

$2x - y - 3 = 0$

x	–2	0	2
y	–7	–3	1



Scale: x-axis: 1 cm to 1 unit
y-axis: 1 cm to 5 units

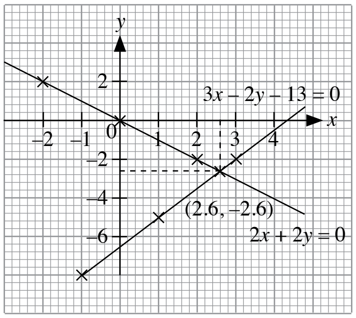
The graphs intersect at the point (1, –1).
∴ The solution is $x = 1$ and $y = –1$.

(c) $3x - 2y - 13 = 0$

x	–1	1	3
y	–8	–5	–2

$2x + 2y = 0$

x	–2	0	2
y	2	0	–2



Scale: x-axis: 1 cm to 1 unit
y-axis: 1 cm to 2 units

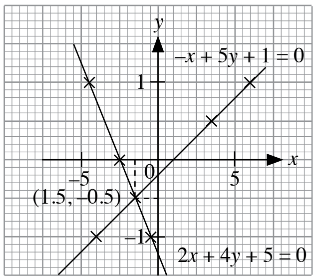
The graphs intersect at the point (2.6, –2.6).
$\therefore$ The solution is $x = 2.6$ and $y = -2.6$.

(d) $2x + 4y + 5 = 0$

x	–4.5	–2.5	–0.5
y	1	0	–1

$-x + 5y + 1 = 0$

x	–4	3.5	6
y	–1	0.5	1



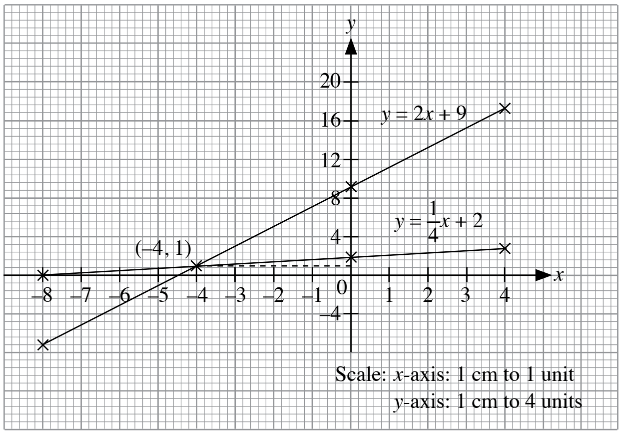
Scale: x-axis: 2 cm to 5 units
y-axis: 2 cm to 1 unit

The graphs intersect at the point (1.5, –0.5).
$\therefore$ The solution is $x = 1.5$ and $y = -0.5$.

3. (a) (i) $y = 2x + 9$

x	–8	0	4
y	–7	9	17

(ii)



(b) (i) $y = \frac{1}{4}x + 2$

x	–8	0	4
y	0	2	3

(c) $2x - y = -9$ — (1)
$x - 4y = -8$ — (2)
From (1), $y = 2x + 9$
From (2), $4y = x + 8$

$$y = \frac{1}{4}x + 2$$

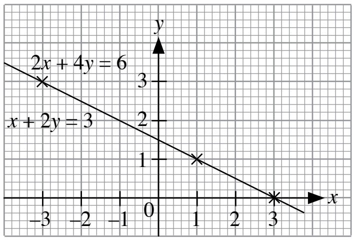
From **(a)(ii)**, the graphs intersect at the point (–4, 1).
$\therefore$ The solution is $x = -4$ and $y = 1$.

4. (a) $x + 2y = 3$

x	–3	1	3
y	3	1	0

$2x + 4y = 6$

x	–3	1	3
y	3	1	0



Scale: x-axis: 1 cm to 1 unit
y-axis: 1 cm to 1 unit

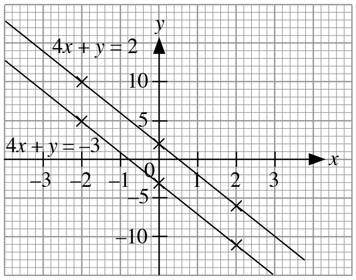
The graphs of each pair of simultaneous equations are identical.
The simultaneous equations have an infinite number of solutions.

(b) $4x + y = 2$

x	–2	0	2
y	10	2	–6

$4x + y = -3$

x	–2	0	2
y	5	–3	–11



Scale: x-axis: 1 cm to 1 unit
y-axis: 1 cm to 5 units

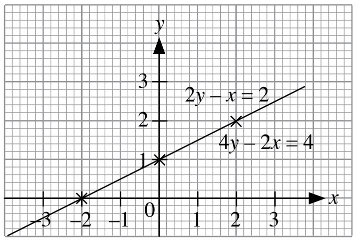
The graphs of each pair of simultaneous equations are parallel and have no intersection point.
The simultaneous equations have no solution.

(c) $2y - x = 2$

x	–2	0	2
y	0	1	2

$4y - 2x = 4$

x	–2	0	2
y	0	1	2



Scale: x-axis: 1 cm to 1 unit
y-axis: 1 cm to 1 unit

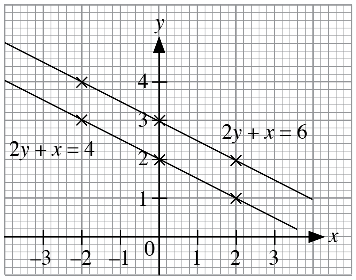
The graphs of each pair of simultaneous equations are identical.
The simultaneous equations have an infinite number of solutions.

(d) $2y + x = 4$

x	–2	0	2
y	3	2	1

$2y + x = 6$

x	–2	0	2
y	4	3	2



Scale: x-axis: 1 cm to 1 unit
y-axis: 1 cm to 1 unit

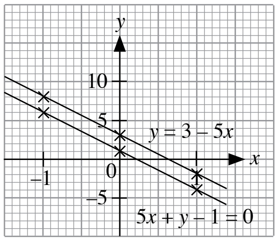
The graphs of each pair of simultaneous equations are parallel and have no intersection point.
The simultaneous equations have no solution.

5. **(a)** $y = 3 - 5x$

x	–1	0	1
y	8	3	–2

$5x + y - 1 = 0$

x	–1	0	1
y	6	1	–4



Scale: x-axis: 2 cm to 1 unit
y-axis: 1 cm to 5 units

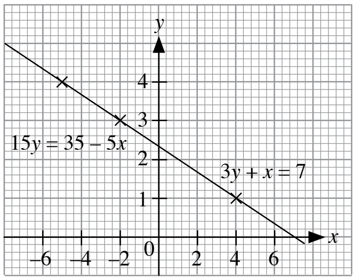
The graphs of each pair of simultaneous equations are parallel and have no intersection point.
The simultaneous equations have no solutions.

(b) $3y + x = 7$

x	–5	–2	4
y	4	3	1

$15y = 35 - 5x$

x	–5	–2	4
y	4	3	1



Scale: x-axis: 1 cm to 2 units
y-axis: 1 cm to 1 unit

The graphs of each pair of simultaneous equations are identical. The simultaneous equations have an infinite number of solutions.

Exercise 2E

1. **(a)** $x + y = 16$ — (1)
$x - y = 0$ — (2)
(1) + (2):
$(x + y) + (x - y) = 16 + 0$
$x + y + x - y = 16$
$2x = 16$
$x = 8$
Substitute $x = 8$ into (1):
$8 + y = 16$
$y = 8$
∴ The solution is $x = 8$ and $y = 8$.

(b) $x - y = 5$ — (1)
$x + y = 19$ — (2)
(2) + (1):
$(x + y) + (x - y) = 19 + 5$
$x + y + x - y = 24$
$2x = 24$
$x = 12$
Substitute $x = 12$ into (2):
$12 + y = 19$
$y = 7$
∴ The solution is $x = 12$ and $y = 7$.

(c) $11x + 4y = 12$ — (1)
$9x - 4y = 8$ — (2)
(1) + (2):
$(11x + 4y) + (9x - 4y) = 12 + 8$
$11x + 4y + 9x - 4y = 20$
$20x = 20$
$x = 1$
Substitute $x = 1$ into (1):
$11(1) + 4y = 12$
$11 + 4y = 12$
$4y = 1$
$y = \frac{1}{4}$
∴ The solution is $x = 1$ and $y = \frac{1}{4}$.

(d) $4y + x = 11$ — (1)
$3y - x = 3$ — (2)
(1) + (2):
$(4y + x) + (3y - x) = 11 + 3$
$4y + x + 3y - x = 14$
$7y = 14$
$y = 2$
Substitute $y = 2$ into (1):
$4(2) + x = 11$
$8 + x = 11$
$x = 3$
∴ The solution is $x = 3$ and $y = 2$.

(e) $3x + y = 5$ — (1)
$x + y = 3$ — (2)
(1) – (2):
$(3x + y) - (x + y) = 5 - 3$
$3x + y - x - y = 2$
$2x = 2$
$x = 1$
Substitute $x = 1$ into (2):
$1 + y = 3$
$y = 2$
∴ The solution is $x = 1$ and $y = 2$.

(f) $2x + 3y = 5$ — (1)
$2x + 7y = 9$ — (2)
(2) – (1):
$(2x + 7y) - (2x + 3y) = 9 - 5$
$2x + 7y - 2x - 3y = 4$
$4y = 4$
$y = 1$
Substitute $y = 1$ into (1):
$2x + 3(1) = 5$
$2x + 3 = 5$
$2x = 2$
$x = 1$
∴ The solution is $x = 1$ and $y = 1$.

(g) $7x - 3y = 15 \quad —(1)$

$11x - 3y = 21 \quad —(2)$

(2) – (1):

$$(11x - 3y) - (7x - 3y) = 21 - 15$$
$$11x - 3y - 7x + 3y = 6$$
$$4x = 6$$
$$x = 1\frac{1}{2}$$

Substitute $x = 1\frac{1}{2}$ into (1):

$$7\left(1\frac{1}{2}\right) - 3y = 15$$
$$10\frac{1}{2} - 3y = 15$$
$$3y = -4\frac{1}{2}$$
$$y = -1\frac{1}{2}$$

∴ The solution is $x = 1\frac{1}{2}$ and $y = -1\frac{1}{2}$.

(h) $3y - 2x = 9 \quad —(1)$

$2y - 2x = 7 \quad —(2)$

(1) – (2):

$$(3y - 2x) - (2y - 2x) = 9 - 7$$
$$3y - 2x - 2y + 2x = 2$$
$$y = 2$$

Substitute $y = 2$ into (1):

$$3(2) - 2x = 9$$
$$6 - 2x = 9$$
$$2x = -3$$
$$x = -1\frac{1}{2}$$

∴ The solution is $x = -1\frac{1}{2}$ and $y = 2$.

(i) $3a - 2b = 5 \quad —(1)$

$2b - 5a = 9 \quad —(2)$

(1) + (2):

$$(3a - 2b) + (2b - 5a) = 5 + 9$$
$$3a - 2b + 2b - 5a = 14$$
$$-2a = 14$$
$$a = -7$$

Substitute $a = -7$ into (2):

$$2b - 5(-7) = 9$$
$$2b + 35 = 9$$
$$2b = -26$$
$$b = -13$$

∴ The solution is $a = -7$ and $b = -13$.

(j) $5c - 2d = 9 \quad —(1)$

$3c + 2d = 7 \quad —(2)$

(1) + (2):

$$(5c - 2d) + (3c + 2d) = 9 + 7$$
$$5c - 2d + 3c + 2d = 16$$
$$8c = 16$$
$$c = 2$$

Substitute $c = 2$ into (2):

$$3(2) + 2d = 7$$
$$6 + 2d = 7$$
$$2d = 1$$
$$d = \frac{1}{2}$$

∴ The solution is $c = 2$ and $d = \frac{1}{2}$.

(k) $3f + 4h = 1 \quad —(1)$

$5f - 4h = 7 \quad —(2)$

(1) + (2):

$$(3f + 4h) + (5f - 4h) = 1 + 7$$
$$3f + 4h + 5f - 4h = 8$$
$$8f = 8$$
$$f = 1$$

Substitute $f = 1$ into (1):

$$3(1) + 4h = 1$$
$$3 + 4h = 1$$
$$4h = -2$$
$$h = -\frac{1}{2}$$

∴ The solution is $f = 1$ and $h = -\frac{1}{2}$.

(l) $6j - k = 23 \quad —(1)$

$3k + 6j = 11 \quad —(2)$

(2) – (1):

$$(3k + 6j) - (6j - k) = 11 - 23$$
$$3k + 6j - 6j + k = -12$$
$$4k = -12$$
$$k = -3$$

Substitute $k = -3$ into (2):

$$3(-3) + 6j = 11$$
$$-9 + 6j = 11$$
$$6j = 20$$
$$j = 3\frac{1}{3}$$

∴ The solution is $j = 3\frac{1}{3}$ and $k = -3$.

2. **(a)** $7x - 2y = 17 \quad —(1)$

$3x + 4y = 17 \quad —(2)$

$2 \times (1)$: $14x - 4y = 34 \quad —(3)$

(3) + (2):

$$(14x - 4y) + (3x + 4y) = 34 + 17$$
$$14x - 4y + 3x + 4y = 51$$
$$17x = 51$$
$$x = 3$$

Substitute $x = 3$ into (2):

$$3(3) + 4y = 17$$
$$9 + 4y = 17$$
$$4y = 8$$
$$y = 2$$

∴ The solution is $x = 3$ and $y = 2$.

(b) $16x + 5y = 39$ — (1)

$4x - 3y = 31$ — (2)

$4 \times (2)$: $16x - 12y = 124$ — (3)

(1) – (3):

$(16x + 5y) - (16x - 12y) = 39 - 124$

$16x + 5y - 16x + 12y = -85$

$17y = -85$

$y = -5$

Substitute $y = -5$ into (2):

$4x - 3(-5) = 31$

$4x + 15 = 31$

$4x = 16$

$x = 4$

$\therefore$ The solution is $x = 4$ and $y = -5$.

(c) $x + 2y = 3$ — (1)

$3x + 5y = 7$ — (2)

$3 \times (1)$: $3x + 6y = 9$ — (3)

(3) – (2):

$(3x + 6y) - (3x + 5y) = 9 - 7$

$3x + 6y - 3x - 5y = 2$

$y = 2$

Substitute $y = 2$ into (1):

$x + 2(2) = 3$

$x + 4 = 3$

$x = -1$

$\therefore$ The solution is $x = -1$ and $y = 2$.

(d) $3x + y = -5$ — (1)

$7x + 3y = 1$ — (2)

$3 \times (1)$: $9x + 3y = -15$ — (3)

(3) – (2):

$(9x + 3y) - (7x + 3y) = -15 - 1$

$9x + 3y - 7x - 3y = -16$

$2x = -16$

$x = -8$

Substitute $x = -8$ into (1):

$3(-8) + y = -5$

$-24 + y = -5$

$y = 19$

$\therefore$ The solution is $x = -8$ and $y = 19$.

(e) $7x - 3y = 13$ — (1)

$2x - y = 3$ — (2)

$3 \times (2)$: $6x - 3y = 9$ — (3)

(1) – (3):

$(7x - 3y) - (6x - 3y) = 13 - 9$

$7x - 3y - 6x + 3y = 4$

$x = 4$

Substitute $x = 4$ into (2):

$2(4) - y = 3$

$8 - y = 3$

$y = 5$

$\therefore$ The solution is $x = 4$ and $y = 5$.

(f) $9x - 5y = 2$ — (1)

$3x - 4y = 10$ — (2)

$3 \times (2)$: $9x - 12y = 30$ — (3)

(1) – (3):

$(9x - 5y) - (9x - 12y) = 2 - 30$

$9x - 5y - 9x + 12y = -28$

$7y = -28$

$y = -4$

Substitute $y = -4$ into (2):

$3x - 4(-4) = 10$

$3x + 16 = 10$

$3x = -6$

$x = -2$

$\therefore$ The solution is $x = -2$ and $y = -4$.

3. **(a)** $7x - 3y = 18$ — (1)

$6x + 7y = 25$ — (2)

$7 \times (1)$: $49x - 21y = 126$ — (3)

$3 \times (2)$: $18x + 21y = 75$ — (4)

(3) + (4):

$(49x - 21y) + (18x + 21y) = 126 + 75$

$49x - 21y + 18x + 21y = 201$

$67x = 201$

$x = 3$

Substitute $x = 3$ into (2):

$6(3) + 7y = 25$

$18 + 7y = 25$

$7y = 7$

$y = 1$

$\therefore$ The solution is $x = 3$ and $y = 1$.

(b) $4x + 3y = -5$ — (1)

$3x - 2y = 43$ — (2)

$2 \times (1)$: $8x + 6y = -10$ — (3)

$3 \times (2)$: $9x - 6y = 129$ — (4)

(3) + (4):

$(8x + 6y) + (9x - 6y) = -10 + 129$

$8x + 6y + 9x - 6y = 119$

$17x = 119$

$x = 7$

Substitute $x = 7$ into (1):

$4(7) + 3y = -5$

$28 + 3y = -5$

$3y = -38$

$y = -11$

$\therefore$ The solution is $x = 7$ and $y = -11$.

(c) $2x + 3y = 8 \quad — (1)$

$5x + 2y = 9 \quad — (2)$

$2 \times (1): \quad 4x + 6y = 16 \quad — (3)$

$3 \times (2): 15x + 6y = 27 \quad — (4)$

$(4) – (3)$:

$(15x + 6y) – (4x + 6y) = 27 – 16$

$15x + 6y – 4x – 6y = 11$

$11x = 11$

$x = 1$

Substitute $x = 1$ into (2):

$5(1) + 2y = 9$

$5 + 2y = 9$

$2y = 4$

$y = 2$

$\therefore$ The solution is $x = 1$ and $y = 2$.

(d) $5x + 4y = 11 \quad — (1)$

$3x + 5y = 4 \quad — (2)$

$3 \times (1): 15x + 12y = 33 \quad — (3)$

$5 \times (2): 15x + 25y = 20 \quad — (4)$

$(4) – (3)$:

$(15x + 25y) – (15x + 12y) = 20 – 33$

$15x + 25y – 15x – 12y = –13$

$13y = –13$

$y = –1$

Substitute $y = –1$ into (1):

$5x + 4(–1) = 11$

$5x – 4 = 11$

$5x = 15$

$x = 3$

$\therefore$ The solution is $x = 3$ and $y = –1$.

(e) $4x – 3y = –1 \quad — (1)$

$5x – 2y = 4 \quad — (2)$

$2 \times (1): 8x – 6y = –2 \quad — (3)$

$3 \times (2): 15x – 6y = 12 \quad — (4)$

$(4) – (3)$:

$(15x – 6y) – (8x – 6y) = 12 – (–2)$

$15x – 6y – 8x + 6x = 14$

$7x = 14$

$x = 2$

Substitute $x = 2$ into (2):

$5(2) – 2y = 4$

$10 – 2y = 4$

$2y = 6$

$y = 3$

$\therefore$ The solution is $x = 2$ and $y = 3$.

(f) $5x – 4y = 23 \quad — (1)$

$2x – 7y = 11 \quad — (2)$

$2 \times (1): 10x – 8y = 46 \quad — (3)$

$5 \times (2): 10x – 35y = 55 \quad — (4)$

$(3) – (4)$:

$(10x – 8y) – (10x – 35y) = 46 – 55$

$10x – 8y – 10x + 35y = –9$

$27y = –9$

$y = -\frac{1}{3}$

Substitute $y = -\frac{1}{3}$ into (1):

$5x - 4\left(-\frac{1}{3}\right) = 23$

$5x + \frac{4}{3} = 23$

$5x = 21\frac{2}{3}$

$x = 4\frac{1}{3}$

$\therefore$ The solution is $x = 4\frac{1}{3}$ and $y = -\frac{1}{3}$.

4. (a) $x + y = 7 \quad — (1)$

$x – y = 5 \quad — (2)$

From (1), $y = 7 – x \quad — (3)$

Substitute (3) into (2):

$x – (7 – x) = 5$

$x – 7 + x = 5$

$2x = 12$

$x = 6$

Substitute $x = 6$ into (3):

$y = 7 – 6$

$= 1$

$\therefore$ The solution is $x = 6$ and $y = 1$.

(b) $3x – y = 0 \quad — (1)$

$2x + y = 5 \quad — (2)$

From (2), $y = 5 – 2x \quad — (3)$

Substitute (3) into (1):

$3x – (5 – 2x) = 0$

$3x – 5 + 2x = 0$

$5x = 5$

$x = 1$

Substitute $x = 1$ into (3):

$y = 5 – 2(1)$

$= 3$

$\therefore$ The solution is $x = 1$ and $y = 3$.

(c) $2x – 7y = 5 \quad — (1)$

$3x + y = –4 \quad — (2)$

From (2), $y = –4 – 3x \quad — (3)$

Substitute (3) into (1):

$2x – 7(–4 – 3x) = 5$

$2x + 28 + 21x = 5$

$23x = –23$

$x = –1$

Substitute $x = –1$ into (3):

$y = –4 – 3(–1)$

$= –1$

$\therefore$ The solution is $x = –1$ and $y = –1$.

(d) $5x - y = 5$ — (1)

$3x + 2y = 29$ — (2)

From (1), $y = 5x - 5$ — (3)

Substitute (3) into (2):

$3x + 2(5x - 5) = 29$

$3x + 10x - 10 = 29$

$13x = 39$

$x = 3$

Substitute $x = 3$ into (3):

$y = 5(3) - 5$

$= 10$

∴ The solution is $x = 3$ and $y = 10$.

(e) $5x + 3y = 11$ — (1)

$4x - y = 2$ — (2)

From (2), $y = 4x - 2$ — (3)

Substitute (3) into (1):

$5x + 3(4x - 2) = 11$

$5x + 12x - 6 = 11$

$17x = 17$

$x = 1$

Substitute $x = 1$ into (3):

$y = 4(1) - 2$

$= 2$

∴ The solution is $x = 1$ and $y = 2$.

(f) $3x + 5y = 10$ — (1)

$x - 2y = 7$ — (2)

From (2), $x = 2y + 7$ — (3)

Substitute (3) into (2):

$3(2y + 7) + 5y = 10$

$6y + 21 + 5y = 10$

$11y = -11$

$y = -1$

Substitute $y = -1$ into (3):

$x = 2(-1) + 7$

$= 5$

∴ The solution is $x = 5$ and $y = -1$.

(g) $x + y = 9$ — (1)

$5x - 2y = 4$ — (2)

From (1), $y = 9 - x$ — (3)

Substitute (3) into (2):

$5x - 2(9 - x) = 4$

$5x - 18 + 2x = 4$

$7x = 22$

$x = 3\frac{1}{7}$

Substitute $x = 3\frac{1}{7}$ into (3):

$y = 9 - 3\frac{1}{7}$

$= 5\frac{6}{7}$

∴ The solution is $x = 3\frac{1}{7}$ and $y = 5\frac{6}{7}$.

(h) $5x + 2y = 3$ — (1)

$x - 4y = -6$ — (2)

From (2), $x = 4y - 6$ — (3)

Substitute (3) into (1):

$5(4y - 6) + 2y = 3$

$20y - 30 + 2y = 3$

$22y = 33$

$y = 1\frac{1}{2}$

Substitute $y = 1\frac{1}{2}$ into (3):

$x = 4\left(1\frac{1}{2}\right) - 6$

$= 0$

∴ The solution is $x = 0$ and $y = 1\frac{1}{2}$.

5. **(a)** $x + y = 0.5$ — (1)

$x - y = 1$ — (2)

(1) + (2):

$(x + y) + (x - y) = 0.5 + 1$

$x + y + x - y = 1.5$

$2x = 1.5$

$x = 0.75$

Substitute $x = 0.75$ into (1):

$0.75 + y = 0.5$

$y = -0.25$

∴ The solution is $x = 0.75$ and $y = -0.25$.

(b) $2x + 0.4y = 8$ — (1)

$5x - 1.2y = 9$ — (2)

$3 \times$ (1): $6x + 1.2y = 24$ — (3)

(3) + (2):

$(6x + 1.2y) + (5x - 1.2y) = 24 + 9$

$6x + 1.2y + 5x - 1.2y = 33$

$11x = 33$

$x = 3$

Substitute $x = 3$ into (1):

$2(3) + 0.4y = 8$

$6 + 0.4y = 8$

$0.4y = 2$

$y = 5$

∴ The solution is $x = 3$ and $y = 5$.

(c) $10x - 3y = 24.5$ — (1)

$3x - 5y = 13.5$ — (2)

$5 \times$ (1): $50x - 15y = 122.5$ — (3)

$3 \times$ (2): $9x - 15y = 40.5$ — (4)

(3) – (4):

$(50x - 15y) - (9x - 15y) = 122.5 - 40.5$

$50x - 15y - 9x + 15y = 82$

$41x = 82$

$x = 2$

Substitute $x = 2$ into (1):

$10(2) - 3y = 24.5$

$20 - 3y = 24.5$

$3y = -4.5$

$y = -1.5$

∴ The solution is $x = 2$ and $y = -1.5$.

(d) $6x + 5y = 10.5$ — (1)
$5x - 3y = -2$ — (2)
$3 \times (1)$: $18x + 15y = 31.5$ — (3)
$5 \times (2)$: $25x - 15y = -10$ — (4)
(4) + (3):
$(25x - 15y) + (18x + 15y) = -10 + 31.5$
$25x - 15y + 18x + 15y = 21.5$
$43x = 21.5$
$x = 0.5$
Substitute $x = 0.5$ into (1):
$6(0.5) + 5y = 10.5$
$3 + 5y = 10.5$
$5y = 7.5$
$y = 1.5$
$\therefore$ The solution is $x = 0.5$ and $y = 1.5$.

6. (a) $4x - y - 7 = 0$ — (1)
$4x + 3y - 11 = 0$ — (2)
(2) – (1):
$(4x + 3y - 11) - (4x - y - 7) = 0 - 0$
$4x + 3y - 11 - 4x + y + 7 = 0$
$4y = 4$
$y = 1$
Substitute $y = 1$ into (1):
$4x - 1 - 7 = 0$
$4x = 8$
$x = 2$
$\therefore$ The solution is $x = 2$ and $y = 1$.

(b) $7x + 2y - 33 = 0$ — (1)
$3y - 7x - 17 = 0$ — (2)
(1) + (2):
$(7x + 2y - 33) + (3y - 7x - 17) = 0 + 0$
$7x + 2y - 33 + 3y - 7x - 17 = 0$
$5y = 50$
$y = 10$
Substitute $y = 10$ into (1):
$7x + 2(10) - 33 = 0$
$7x + 20 - 33 = 0$
$7x = 13$
$x = 1\frac{6}{7}$
$\therefore$ The solution is $x = 1\frac{6}{7}$ and $y = 10$.

(c) $5x - 3y - 2 = 0$ — (1)
$x + 5y - 6 = 0$ — (2)
$5 \times (2)$: $5x + 25y - 30 = 0$ — (3)
(3) – (1):
$(5x + 25y - 30) - (5x - 3y - 2) = 0 - 0$
$5x + 25y - 30 - 5x + 3y + 2 = 0$
$28y = 28$
$y = 1$
Substitute $y = 1$ into (2):
$x + 5(1) - 6 = 0$
$x + 5 - 6 = 0$
$x = 1$
$\therefore$ The solution is $x = 1$ and $y = 1$.

(d) $5x - 3y - 13 = 0$ — (1)
$7x - 6y - 20 = 0$ — (2)
$2 \times (1)$: $10x - 6y - 26 = 0$ — (3)
(3) – (2):
$(10x - 6y - 26) - (7x - 6y - 20) = 0 - 0$
$10x - 6y - 26 - 7x + 6y + 20 = 0$
$3x = 6$
$x = 2$
Substitute $x = 2$ into (1):
$5(2) - 3y - 13 = 0$
$10 - 3y - 13 = 0$
$3y = 3$
$y = 1$
$\therefore$ The solution is $x = 2$ and $y = 1$.

(e) $7x + 3y - 8 = 0$ — (1)
$3x - 4y - 14 = 0$ — (2)
$4 \times (1)$: $28x + 12y - 32 = 0$ — (3)
$3 \times (2)$: $9x - 12y - 42 = 0$ — (4)
(3) + (4):
$(28x + 12y - 32) + (9x - 12y - 42) = 0 + 0$
$28x + 12y - 32 + 9x - 12y - 42 = 0$
$37x = 74$
$x = 2$
Substitute $x = 2$ into (1):
$7(2) + 3y - 8 = 0$
$14 + 3y - 8 = 0$
$3y = -6$
$y = -2$
$\therefore$ The solution is $x = 2$ and $y = -2$.

(f) $3x + 5y + 8 = 0$ — (1)
$4x + 13y - 2 = 0$ — (2)
$4 \times (1)$: $12x + 20y + 32 = 0$ — (3)
$3 \times (2)$: $12x + 39y - 6 = 0$ — (4)
(3) – (4):
$(12x + 20y + 32) - (12x + 39y - 6) = 0 - 0$
$12x + 20y + 32 - 12x - 39y + 6 = 0$
$19y = 38$
$y = 2$
Substitute $y = 2$ into (1):
$3x + 5(2) + 8 = 0$
$3x + 10 + 8 = 0$
$3x = -18$
$x = -6$
$\therefore$ The solution is $x = -6$ and $y = 2$.

7. (a) $\frac{x + 1}{y + 2} = \frac{3}{4}$ — (1)
$\frac{x - 2}{y - 1} = \frac{3}{5}$ — (2)
From (1),
$4(x + 1) = 3(y + 2)$
$4x + 4 = 3y + 6$
$4x - 3y = 2$ — (3)

From (2),

$5(x - 2) = 3(y - 1)$

$5x - 10 = 3y - 3$

$5x - 3y = 7$ — (4)

(4) – (3):

$(5x - 3y) - (4x - 3y) = 7 - 2$

$5x - 3y - 4x + 3y = 5$

$x = 5$

Substitute $x = 5$ into (3):

$4(5) - 3y = 2$

$20 - 3y = 2$

$3y = 18$

$y = 6$

∴ The solution is $x = 5$ and $y = 6$.

(b) $\frac{x}{3} - \frac{y}{2} = \frac{5}{6}$ — (1)

$3x - \frac{2}{5}y = 3\frac{2}{5}$ — (2)

$9 \times (1)$: $3x - \frac{9y}{2} = 7\frac{1}{2}$ — (3)

(2) – (3):

$\left(3x - \frac{2}{5}y\right) - \left(3x - \frac{9y}{2}\right) = 3\frac{2}{5} - 7\frac{1}{2}$

$3x - \frac{2}{5}y - 3x + \frac{9y}{2} = -4\frac{1}{10}$

$4\frac{1}{10}y = -4\frac{1}{10}$

$y = -1$

Substitute $y = -1$ into (2):

$3x - \frac{2}{5}(-1) = 3\frac{2}{5}$

$3x + \frac{2}{5} = 3\frac{2}{5}$

$3x = 3$

$x = 1$

∴ The solution is $x = 1$ and $y = -1$.

(c) $\frac{x}{4} - \frac{3}{8}y = 3$ — (1)

$\frac{5}{3}x - \frac{y}{2} = 12$ — (2)

$8 \times (1)$: $2x - 3y = 24$ — (3)

$6 \times (2)$: $10x - 3y = 72$ — (4)

(4) – (3):

$(10x - 3y) - (2x - 3y) = 72 - 24$

$10x - 3y - 2x + 3y = 48$

$8x = 48$

$x = 6$

Substitute $x = 6$ into (3):

$2(6) - 3y = 24$

$12 - 3y = 24$

$3y = -12$

$y = -4$

∴ The solution is $x = 6$ and $y = -4$.

(d) $\frac{x - 3}{5} = \frac{y - 7}{2}$ — (1)

$11x = 13y$ — (2)

$26 \times (1)$: $\frac{26}{5}(x - 3) = 13(y - 7)$

$\frac{26}{5}x - \frac{78}{5} = 13y - 91$ — (3)

(2) – (3):

$11x - \left(\frac{26}{5}x - \frac{78}{5}\right) = 13y - (13y - 91)$

$11x - \frac{26}{5}x + \frac{78}{5} = 13y - 13y + 91$

$5\frac{4}{5}x = 75\frac{2}{5}$

$x = 13$

Substitute $x = 13$ into (2):

$11(13) = 13y$

$y = 11$

∴ The solution is $x = 13$ and $y = 11$.

8. **(a)** $2x + 5y = 12$ — (1)

$4x + 3y = -4$ — (2)

From (1), $2x = 12 - 5y$

$x = \frac{12 - 5y}{2}$ — (3)

Substitute (3) into (2):

$4\left(\frac{12 - 5y}{2}\right) + 3y = -4$

$24 - 10y + 3y = -4$

$7y = 28$

$y = 4$

Substitute $y = 4$ into (3):

$x = \frac{12 - 5(4)}{2}$

$= -4$

∴ The solution is $x = -4$ and $y = 4$.

(b) $4x - 3y = 25$ — (1)

$6x + 5y = 9$ — (2)

From (1), $4x = 3y + 25$

$x = \frac{3y + 25}{4}$ — (3)

Substitute (3) into (2):

$6\left(\frac{3y + 25}{4}\right) + 5y = 9$

$\frac{9y}{2} + \frac{75}{2} + 5y = 9$

$9\frac{1}{2}y = -28\frac{1}{2}$

$y = -3$

Substitute $y = -3$ into (3):

$x = \frac{3(-3) + 25}{4}$

$= 4$

∴ The solution is $x = 4$ and $y = -3$.

(c) $3x + 7y = 2 \quad — (1)$

$6x - 5y = 4 \quad — (2)$

From (1), $3x = 2 - 7y$

$$x = \frac{2 - 7y}{3} \quad — (3)$$

Substitute (3) into (2):

$$6\left(\frac{2 - 7y}{3}\right) - 5y = 4$$

$$4 - 14y - 5y = 4$$

$$19y = 0$$

$$y = 0$$

Substitute $y = 0$ into (3):

$$x = \frac{2 - 7(0)}{3}$$

$$= \frac{2}{3}$$

$\therefore$ The solution is $x = \frac{2}{3}$ and $y = 0$.

(d) $9x + 2y = 5 \quad — (1)$

$7x - 3y = 13 \quad — (2)$

From (1), $9x = 5 - 2y$

$$x = \frac{5 - 2y}{9} \quad — (3)$$

Substitute (3) into (2):

$$7\left(\frac{5 - 2y}{9}\right) - 3y = 13$$

$$\frac{35}{9} - \frac{14}{9}y - 3y = 13$$

$$4\frac{5}{9}y = -9\frac{1}{9}$$

$$y = -2$$

Substitute $y = -2$ into (3):

$$x = \frac{5 - 2(-2)}{9}$$

$$= 1$$

$\therefore$ The solution is $x = 1$ and $y = -2$.

(e) $2y - 5x = 25 \quad — (1)$

$4x + 3y = 3 \quad — (2)$

From (1), $2y = 5x + 25$

$$y = \frac{5x + 25}{2} \quad — (3)$$

Substitute (3) into (2):

$$4x + 3\left(\frac{5x + 25}{2}\right) = 3$$

$$4x + \frac{15}{2}x + \frac{75}{2} = 3$$

$$11\frac{1}{2}x = -34\frac{1}{2}$$

$$x = -3$$

Substitute $x = -3$ into (3):

$$y = \frac{5(-3) + 25}{2}$$

$$= 5$$

$\therefore$ The solution is $x = -3$ and $y = 5$.

(f) $3x - 5y = 7 \quad — (1)$

$4x - 3y = 3 \quad — (2)$

From (1), $3x = 5y + 7$

$$x = \frac{5y + 7}{3} \quad — (3)$$

Substitute (3) into (2):

$$4\left(\frac{5y + 7}{3}\right) - 3y = 3$$

$$\frac{20}{3}y + \frac{28}{3} - 3y = 3$$

$$3\frac{2}{3}y = -6\frac{1}{3}$$

$$y = -1\frac{8}{11}$$

Substitute $y = -1\frac{8}{11}$ into (3):

$$x = \frac{5\left(-1\frac{8}{11}\right) + 7}{3}$$

$$= -\frac{6}{11}$$

$\therefore$ The solution is $x = -\frac{6}{11}$ and $= -1\frac{8}{11}$.

9. **(a)** $\frac{x}{5} + y + 2 = 0 \quad — (1)$

$\frac{x}{3} - y - 10 = 0 \quad — (2)$

From (1), $y = -\frac{x}{5} - 2 \quad — (3)$

Substitute (3) into (2):

$$\frac{x}{3} - \left(-\frac{x}{5} - 2\right) - 10 = 0$$

$$\frac{x}{3} + \frac{x}{5} + 2 - 10 = 0$$

$$\frac{8}{15}x = 8$$

$$x = 15$$

Substitute $x = 15$ into (3):

$$y = -\frac{15}{5} - 2$$

$$= -5$$

$\therefore$ The solution is $x = 15$ and $y = -5$.

(b) $\frac{x + y}{3} = 3 \quad — (1)$

$\frac{3x + y}{5} = 1 \quad — (2)$

From (1), $x + y = 9$

$$x = 9 - y \quad — (3)$$

Substitute (3) into (2):

$$\frac{3(9 - y) + y}{5} = 1$$

$$27 - 3y + y = 5$$

$$2y = 22$$

$$y = 11$$

Substitute $y = 11$ into (3):

$$x = 9 - 11$$

$$= -2$$

$\therefore$ The solution is $x = -2$ and $y = 11$.

(c) $3x - y = 23$ — (1)

$\frac{x}{3} + \frac{y}{4} = 4$ — (2)

From (1), $y = 3x - 23$ — (3)

Substitute (3) into (2):

$$\frac{x}{3} + \frac{3x - 23}{4} = 4$$

$$4x + 9x - 69 = 48$$

$$13x = 117$$

$$x = 9$$

Substitute $x = 9$ into (3):

$$y = 3(9) - 23$$

$$= 4$$

∴ The solution is $x = 9$ and $y = 4$.

(d) $\frac{x}{3} + \frac{y}{2} = 4$ — (1)

$\frac{2}{3}x - \frac{y}{6} = 1$ — (2)

From (1), $2x + 3y = 24$

$$2x = 24 - 3y$$

$$x = \frac{24 - 3y}{2} \quad \text{— (3)}$$

Substitute (3) into (1):

$$\frac{2}{3}\left(\frac{24 - 3y}{2}\right) - \frac{y}{6} = 1$$

$$48 - 6y - y = 6$$

$$7y = 42$$

$$y = 6$$

Substitute $y = 6$ into (3):

$$x = \frac{24 - 3(6)}{2}$$

$$= 3$$

∴ The solution is $x = 3$ and $y = 6$.

10. (a) $\frac{2}{x + y} = \frac{1}{2x + y}$ — (1)

$3x + 4y = 9$ — (2)

From (1),

$$2(2x + y) = x + y$$

$$4x + 2y = x + y$$

$$y = -3x \quad \text{— (3)}$$

Substitute (3) into (2):

$$3x + 4(-3x) = 9$$

$$3x - 12x = 9$$

$$-9x = 9$$

$$x = -1$$

Substitute $x = -1$ into (3):

$$y = -3(-1)$$

$$= 3$$

∴ The solution is $x = -1$ and $y = 3$.

(b) $\frac{1}{5}(x - 2) = \frac{1}{4}(1 - y)$ — (1)

$\frac{1}{7}\left(x + 2\frac{2}{3}\right) = \frac{1}{3}(3 - y)$ — (2)

$20 \times (1)$:

$$4(x - 2) = 5(1 - y)$$

$$4x - 8 = 5 - 5y$$

$$4x + 5y = 13 \quad \text{— (3)}$$

$21 \times (2)$:

$$3\left(x + 2\frac{2}{3}\right) = 7(3 - y)$$

$$3x + 8 = 21 - 7y$$

$$3x = 13 - 7y$$

$$x = \frac{13 - 7y}{3} \quad \text{— (4)}$$

Substitute (4) into (3):

$$4\left(\frac{13 - 7y}{3}\right) + 5y = 13$$

$$\frac{52}{3} - \frac{28}{3}y + 5y = 13$$

$$4\frac{1}{3}y = 4\frac{1}{3}$$

$$y = 1$$

Substitute $y = 1$ into (4):

$$x = \frac{13 - 7(1)}{3}$$

$$= 2$$

∴ The solution is $x = 2$ and $y = 1$.

(c) $\frac{5x + y}{9} = 2 - \frac{x + y}{5}$ — (1)

$\frac{7x - 3}{2} = 1 + \frac{y - x}{3}$ — (2)

$45 \times (1)$:

$$5(5x + y) = 90 - 9(x + y)$$

$$25x + 5y = 90 - 9x - 9y$$

$$34x + 14y = 90$$

$$17x + 7y = 45 \quad \text{— (3)}$$

$6 \times (2)$:

$$3(7x - 3) = 6 + 2(y - x)$$

$$21x - 9 = 6 + 2y - 2x$$

$$2y = 23x - 15$$

$$y = \frac{23x - 15}{2} \quad \text{— (4)}$$

Substitute (4) into (3):

$$17x + 7\left(\frac{23x - 15}{2}\right) = 45$$

$$17x + \frac{161}{2}x - \frac{105}{2} = 45$$

$$97\frac{1}{2}x = 97\frac{1}{2}$$

$$x = 1$$

Substitute $x = 1$ into (4):

$$y = \frac{23(1) - 15}{2}$$

$$= 4$$

∴ The solution is $x = 1$ and $y = 4$.

(d) $\dfrac{x+y}{3} = \dfrac{x-y}{5}$ — (1)

$\dfrac{x-y}{5} = 2x - 3y + 5$ — (2)

From (1), $5(x + y) = 3(x - y)$

$5x + 5y = 3x - 3y$

$2x = -8y$

$x = -4y$ — (3)

Substitute (3) into (2):

$\dfrac{-4y-y}{5} = 2(-4y) - 3y + 5$

$-y = -8y - 3y + 5$

$10y = 5$

$y = \dfrac{1}{2}$

Substitute $y = \dfrac{1}{2}$ into (3):

$x = -4\left(\dfrac{1}{2}\right)$

$= -2$

∴ The solution is $x = -2$ and $y = \dfrac{1}{2}$.

11. When $x = 3$, $y = -1$,

$3p(3) + q(-1) = 11$

$9p - q = 11$ — (1)

$-q(3) + 5(-1) = p$

$p = -3q - 5$ — (2)

Substitute (2) into (1):

$9(-3q - 5) - q = 11$

$-27q - 45 - q = 11$

$28q = -56$

$q = -2$

Substitute $q = -2$ into (2):

$p = -3(-2) - 5$

$= 1$

∴ The values of p and of q are 1 and –2 respectively.

12. When $x = -11$, $y = 5$,

$p(-11) + 5(5) = q$

$-11p + 25 = q$ — (1)

$q(-11) + 7(5) = p$

$-11q + 35 = p$ — (2)

Substitute (2) into (1):

$-11(-11q + 35) + 25 = q$

$121q - 385 + 25 = q$

$120q = 360$

$q = 3$

Substitute $q = 3$ into (2):

$-11(3) + 35 = p$

$p = 2$

∴ The values of p and of q are 2 and 3 respectively.

13. $8s - 3h = -9$ — (1)

$-29s + 10h = 16$ — (2)

$10 \times (1)$: $80s - 30h = -90$ — (3)

$3 \times (2)$: $-87s + 30h = 48$ — (4)

(3) + (4):

$(80s - 30h) + (-87s + 30h) = -90 + 48$

$7s = 42$

$s = 6$

Substitute $s = 6$ into (1):

$8(6) - 3h = -9$

$48 - 3h = -9$

$3h = 57$

$h = 19$

∴ The height above the ground is 19 m and the time when the cat meets the mouse is 5 s.

Exercise 2F

1. Let the smaller number be x and the greater number be y.

$x + y = 138$ — (1)

$y - x = 88$ — (2)

(1) + (2):

$(x + y) + (y - x) = 138 + 88$

$x + y + y - x = 226$

$2y = 226$

$y = 113$

Substitute $y = 113$ into (1):

$x + 113 = 138$

$x = 25$

∴ The two numbers are 25 and 113.

2. Let the smaller number be x and the greater number be y.

$y - x = 10$ — (1)

$x + y = 4x$ — (2)

From (2), $y = 3x$ — (3)

Substitute (3) into (1):

$3x - x = 10$

$2x = 10$

$x = 5$

Substitute $x = 5$ into (3):

$y = 3(5)$

$= 15$

∴ The two numbers are 5 and 15.

3. Let the cost of a belt be \$$x$ and the cost of a wallet be \$$y$.

$x + y = 42$ — (1)

$7x + 4y = 213$ — (2)

From (1), $y = 42 - x$ — (3)

Substitute (3) into (2):

$7x + 4(42 - x) = 213$

$7x + 168 - 4x = 213$

$3x = 45$

$x = 15$

Substitute $x = 15$ into (3):

$y = 42 - 15$

$= 27$

∴ The cost of a belt is \$15 and the cost of a wallet is \$27.

4. Let the cost of 1 kg of potatoes be \$$x$ and the cost of 1 kg of carrots be \$$y$.

$8x + 5y = 28$ — (1)

$2x + 3y = 11.2$ — (2)

$4 \times (2)$: $8x + 12y = 44.8$ — (3)

(3) – (1):

$(8x + 12y) - (8x + 5y) = 44.8 - 28$

$8x + 12y - 8x - 5y = 16.8$

$7y = 16.8$

$y = 2.4$

Substitute $y = 2.4$ into (2):

$2x + 3(2.4) = 11.2$

$2x + 7.2 = 11.2$

$2x = 4$

$x = 2$

∴ 1 kg of potatoes cost \$2 and 1 kg of carrots cost \$2.40.

5. Let the first number be x and the second number be y.

$x + 7 = 2y$ — (1)

$y + 20 = 4x$ — (2)

From (1), $x = 2y - 7$ — (3)

Substitute (3) into (2):

$y + 20 = 4(2y - 7)$

$= 8y - 28$

$7y = 48$

$y = 6\frac{6}{7}$

Substitute $y = 6\frac{6}{7}$ into (3):

$x = 2\left(6\frac{6}{7}\right) - 7$

$= 6\frac{5}{7}$

∴ The two numbers are $6\frac{5}{7}$ and $6\frac{6}{7}$.

6. Let the smaller number be x and the greater number be y.

$x + y = 48$ — (1)

$x = \frac{1}{5}y$ — (2)

Substitute (2) into (1):

$\frac{1}{5}y + y = 48$

$\frac{6}{5}y = 48$

$y = 40$

Substitute $y = 40$ into (2):

$x = \frac{1}{5}(40)$

$= 8$

∴ The two numbers are 8 and 40.

7. Let the smaller angle be x and the greater angle be y.

$\frac{1}{5}(x + y) = 24°$ — (1)

$\frac{1}{2}(y - x) = 14°$ — (2)

$5 \times (1)$: $x + y = 120°$ — (3)

$2 \times (2)$: $y - x = 28°$ — (4)

(3) + (4):

$(x + y) + (y - x) = 120° + 28°$

$x + y + y - x = 148°$

$2y = 148°$

$y = 74°$

Substitute $y = 74°$ into (3):

$x + 74° = 120°$

$x = 46°$

∴ The two angles are 46° and 74°.

8. The sides of an equilateral triangle are equal.

$x + y - 9 = y + 5$ — (1)

$y + 5 = 2x - 7$ — (2)

From (1), $x = 14$

Length of each side $= 2(14) - 7$

$= 21$ cm

∴ The length of each side of the triangle is 21 cm.

9. $3x - y = 2x + y$ — (1)

$3x - y + 2x + y + 2(2x - 3) = 120$ — (2)

From (2),

$3x - y + 2x + y + 4x - 6 = 120$

$9x = 126$

$x = 14$

Substitute $x = 14$ into (1):

$3(14) - y = 2(14) + y$

$42 - y = 28 + y$

$2y = 14$

$y = 7$

Area of rectangle $= [3(14) - 7] \times [2(14) - 3]$

$= 35 \times 25$

$= 875$ cm^2

∴ The area of the rectangle is 875 cm^2.

10. The sides of a rhombus are equal.

$2x + y + 1 = \frac{3x - y - 2}{2}$ — (1)

$2x + y + 1 = x - y$ — (2)

From (2), $x = -2y - 1$ — (3)

Substitute (3) into (1):

$2(-2y - 1) + y + 1 = \frac{3(-2y - 1) - y - 2}{2}$

$-4y - 2 + y + 1 = \frac{-7y - 5}{2}$

$-6y - 2 = -7y - 5$

$y = -3$

Substitute $y = -3$ into (3):

$x = -2(-3) - 1$

$= 5$

Perimeter of the figure $= 4[5 - (-3)]$

$= 32$ cm

∴ The perimeter of the figure is 32 cm.

11. Let the numerator of the fraction be x and its denominator be y,

i.e. let the fraction be $\frac{x}{y}$.

$\frac{x-1}{y-1} = \frac{1}{2}$ — (1)

$\frac{x+1}{y+1} = \frac{2}{3}$ — (2)

From (1),

$2(x-1) = y-1$

$2x - 2 = y - 1$

$y = 2x - 1$ — (3)

Substitute (3) into (2):

$\frac{x+1}{2x-1+1} = \frac{2}{3}$

$3(x+1) = 4x$

$3x + 3 = 4x$

$x = 3$

Substitute $x = 3$ into (3):

$y = 2(3) - 1$

$= 5$

∴ The fraction is $\frac{3}{5}$.

12. Let the age of Kai Kai in 2013 be x years old and the age of Jia Jia in 2013 be y years old.

$x + y = 11$ — (1)

$x + 9 = 3y$ — (2)

(1) – (2):

$(x + y) - (x + 9) = 11 - 3y$

$x + y - x - 9 = 11 - 3y$

$y - 9 = 11 - 3y$

$4y = 20$

$y = 5$

Substitute $y = 5$ into (1):

$x + 5 = 11$

$x = 6$

In 2014,

Age of Kai Kai = 6 + 1

= 7

Age of Jia Jia = 5 + 1

= 6

∴ In 2014, the ages of Kai Kai and Jia Jia are 7 years and 6 years respectively.

13. Let the amount an adult has to pay be \$$x$ and the amount a senior citizen has to pay be \$$y$.

$6x + 4y = 228$ — (1)

$13x + 7y = 459$ — (2)

From (1), $3x + 2y = 114$ — (3)

$2 \times$ (2): $26x + 14y = 918$ — (4)

$7 \times$ (3): $21x + 14y = 798$ — (5)

(4) – (5):

$(26x + 14y) - (21x + 14y) = 918 - 798$

$26x + 14y - 21x - 14y = 120$

$5x = 120$

$x = 24$

Substitute $x = 24$ into (3):

$3(24) + 2y = 114$

$72 + 2y = 114$

$2y = 42$

$y = 21$

Total amount 2 adults and a senior citizen have to pay

= 2(\$24) + \$21

= \$69

∴ The total amount is \$69.

14. Let the number of gift A to buy be x and the number of gift B to buy be y.

$10x + 8y = 230$ — (1)

$x + y = 2 + 2 + 13 + 10$

$= 27$ — (2)

From (2), $y = 27 - x$ — (3)

Substitute (3) into (1):

$10x + 8(27 - x) = 230$

$10x + 216 - 8x = 230$

$2x = 14$

$x = 7$

Substitute $x = 7$ into (3):

$y = 27 - 7$

$= 20$

∴ Lixin should buy 7 gift A and 20 gift B.

15. Let the number of chickens be x and the number of goats be y.

$x + y = 50$ — (1)

$2x + 4y = 140$ — (2)

From (1), $y = 50 - x$ — (3)

Substitute (3) into (2):

$2x + 4(50 - x) = 140$

$2x + 200 - 4x = 140$

$2x = 60$

$x = 30$

Substitute $x = 30$ into (3):

$y = 50 - 30$

$= 20$

Number of more chickens than goats = 30 – 20

= 10

∴ There are 10 more chickens than goats.

16. Let the amount Ethan has be \$$x$ and the amount Michael has be \$$y$.

$x + y = 80$ — (1)

$\frac{1}{4}x = \frac{1}{6}y$ — (2)

From (1), $y = 80 - x$ — (3)

Substitute (3) into (2):

$\frac{1}{4}x = \frac{1}{6}(80 - x)$

$3x = 160 - 2x$

$5x = 160$

$x = 32$

Substitute $x = 32$ into (1):

$32 + y = 80$

$y = 48$

∴ Ethan received \$32 and Michael received \$48.

17. Let the amount deposited in Bank A be $\$x$ and the amount deposited in Bank B be $\$y$.

$x + y = 25\ 000$ — (1)

$\frac{0.6}{100}x = \frac{0.65}{100}y$ — (2)

From (2), $y = \frac{12}{13}x$ — (3)

Substitute (3) into (1):

$x + \frac{12}{13}x = 25\ 000$

$\frac{25}{13}x = 25\ 000$

$x = 13\ 000$

Substitute $x = 13\ 000$ into (3):

$y = \frac{12}{13}(13\ 000)$

$= 12\ 000$

∴ Rui Feng deposited \$13 000 in Bank A and \$12 000 in Bank B.

18. Let the smaller number be x and the greater number be y.

$\frac{y-2}{x} = 2$ — (1)

$\frac{5x-2}{y} = 2$ — (2)

From (1), $y - 2 = 2x$

$x = \frac{y-2}{x}$ — (3)

Substitute (3) into (2):

$\frac{5\left(\frac{y-2}{2}\right) - 2}{y} = 2$

$5\left(\frac{y-2}{2}\right) - 2 = 2y$

$\frac{5}{2}y - 5 - 2 = 2y$

$\frac{1}{2}y = 7$

$y = 14$

Substitute $y = 14$ into (3):

$x = \frac{14-2}{2}$

$= 6$

∴ The two numbers are 6 and 14.

19. Let the tens digit of the original number be x and its ones digit be y. Then the original number is $10x + y$, the number obtained when the digits of the original number are reversed is $10y + x$.

$x + y = \frac{1}{8}(10x + y)$ — (1)

$(10x + y) - (10y + x) = 45$ — (2)

From (1),

$8(x + y) = 10x + y$

$8x + 8y = 10x + y$

$2x = 7y$

$x = \frac{7}{2}y$ — (3)

From (2),

$10x + y - 10y - x = 45$

$9x - 9y = 45$

$x - y = 5$ — (4)

Substitute (3) into (4):

$\frac{7}{2}y - y = 5$

$\frac{5}{2}y = 5$

$y = 2$

Substitute $y = 2$ into (3):

$x = \frac{7}{2}(2)$

$= 7$

∴ The original number is 72.

20. Let the cost of 1 pear be $\$x$ and the cost of 1 mango be $\$y$.

$8x + 5y = 10 + 1.1$ — (1)

$5x + 4y = 10 - 1.75$ — (2)

$4 \times (1)$: $32x + 20y = 44.4$ — (3)

$5 \times (2)$: $25x + 20y = 41.25$ — (4)

(3) – (4):

$(32x + 20y) - (25x + 20y) = 44.4 - 41.25$

$32x + 20y - 25x - 20y = 3.15$

$7x = 3.15$

$x = 0.45$

Substitute $x = 0.45$ into (2):

$5(0.45) + 4y = 8.25$

$2.25 + 4y = 8.25$

$4y = 6$

$y = 1.5$

∴ 1 pear costs \$0.45 and 1 mango costs \$1.50.

21. **(i)** Let the number of shares of Company A Huixian's mother has be x and the share price of Company B on Day 7 be $\$y$.

$4.6x - 2000y = 7400$ — (1)

$4.8x - 5000(y - 0.5) = -5800$ — (2)

From (1),

$2000y = 4.6x - 7400$

$y = \frac{4.6x - 7400}{2000}$ — (3)

Substitute (3) into (2):

$4.8x - 5000\left(\frac{4.6x - 7400}{2000} - 0.5\right) = -5800$

$4.8x - 11.5x + 18\ 500 + 2500 = -5800$

$6.7x = 26\ 800$

$x = 4000$

∴ Huixian's mother has 4000 shares of Company A.

(ii) From **(i)**,

substitute $x = 4000$ into (3):

$y = \frac{4.6(4000) - 7400}{2000}$

$= 5.5$

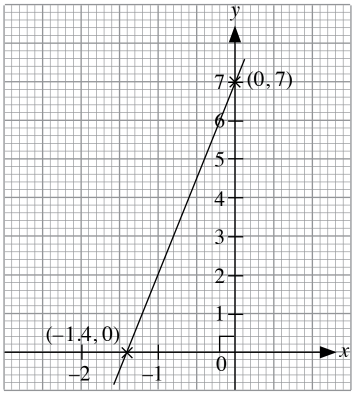
Share price of Company B on Day 12 $= 5.5 - 0.5$

$= 5$

∴ The share price of Company B on Day 12 is \$5.

Review Exercise 2

1. **(a)**

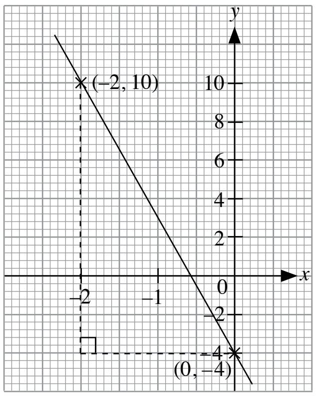


$m = \frac{7}{1.4}$

$= 5$

$c = 7$

(b)

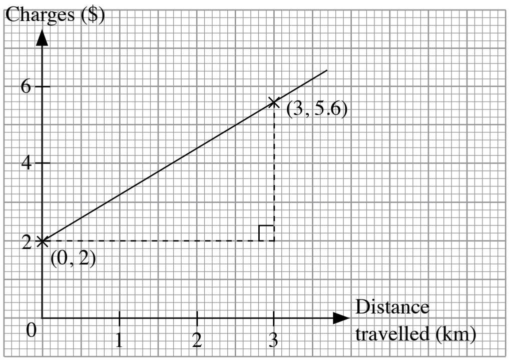


$m = -\frac{14}{2}$

$= -7$

$c = -4$

2.



m = y-intercept

$= 2$

n = gradient of line

$= \frac{3.6}{3}$

$= 1.2$

3. **(i)** Price Company A charges for 20 minutes of talk time = \$0.80

(ii) Price Company B charges for 50 minutes of talk time = \$3.80

(iii) For less than 30 minutes of talk time, Company B charges a lower price than Company A, thus Company B would be able to offer Jun Wei a better price.

(iv) m_A = gradient of A

$= \frac{4}{50}$

$= \frac{2}{25}$

m_B = gradient of B

$= \frac{5}{40}$

$= \frac{1}{8}$

Since $m_B > m_A$, Company B has a greater rate of increase in charges.

(v) At \$4 per month,

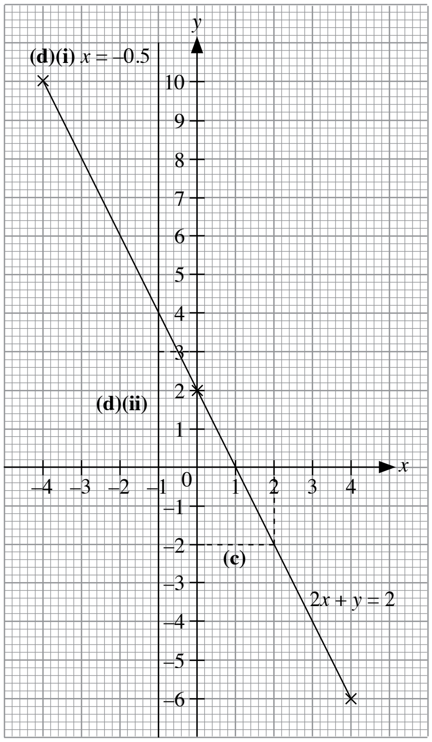
duration of talk time offered by Company A = 60 minutes and duration of talk time offered by Comapny B = 52 minutes.

Since Company A offers more talk time for \$4 per month, Michael should choose Company A.

4. **(a)** $2x + y = 2$

x	–4	0	4
y	10	2	–6

(b)



(c) From the graph in **(b)**,

When $y = -2$,

$p = x = 2$

(d) **(ii)** The coordinates of the point of intersection are (–0.5, 3).

5. **(a)** **(i)** When $x = -5$, $y = p$,

$5(-5) - 3p = 2$

$-25 - 3p = 2$

$3p = -27$

$p = -9$

When $x = 7$, $y = q$,

$5(7) - 3q = 2$

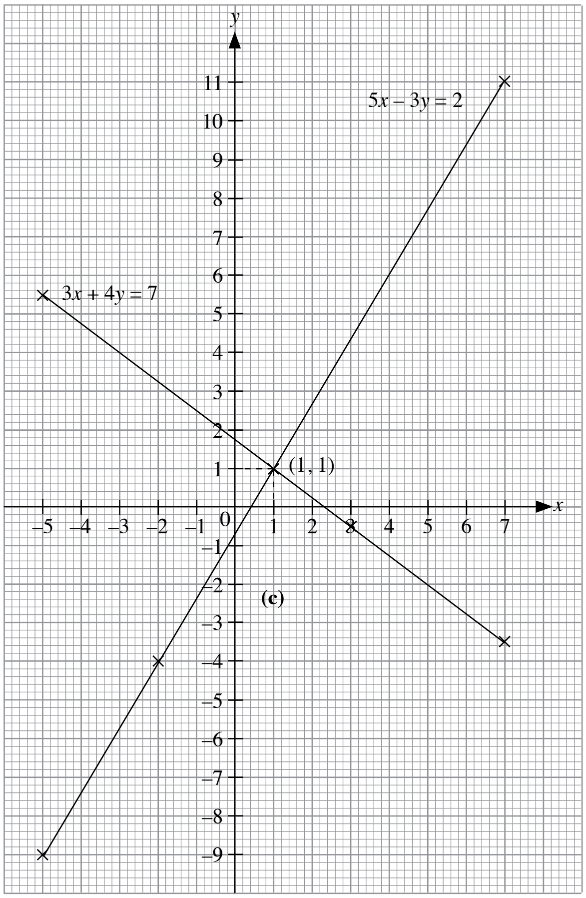
$35 - 3q = 2$

$3q = 33$

$q = 11$

$\therefore p = -9, q = 11$

(ii)



(b) **(i)** $3x + 4y = 7$

x	–5	3	7
y	5.5	–0.5	–3.5

(c) The graphs intersect at the point (1, 1).

$\therefore$ The solution is $x = 1$ and $y = 1$.

6. **(a)** $7x + 2y = 10$ — (1)

$5x + 2y = 6$ — (2)

(1) – (2):

$(7x + 2y) - (5x + 2y) = 10 - 6$

$7x + 2y - 5x - 2y = 4$

$2x = 4$

$x = 2$

Substitute $x = 2$ into (2):

$5(2) + 2y = 6$

$10 + 2y = 6$

$2y = -4$

$y = -2$

$\therefore$ The solution is $x = 2$ and $y = -2$.

(b) $9x + 4y = 28$ — (1)

$4y - 11x = -12$ — (2)

(1) – (2):

$(9x + 4y) - (4y - 11x) = 28 - (-12)$

$9x + 4y - 4y + 11x = 28 + 12$

$20x = 40$

$x = 2$

Substitute $x = 2$ into (1):

$9(2) + 4y = 28$

$18 + 4y = 28$

$4y = 10$

$y = 2\frac{1}{2}$

$\therefore$ The solution is $x = 2$ and $y = 2\frac{1}{2}$.

(c) $2x - 5y = 22$ — (1)

$2x - 3y = 14$ — (2)

(1) – (2):

$(2x - 5y) - (2x - 3y) = 22 - 14$

$2x - 5y - 2x + 3y = 8$

$2y = -8$

$y = -4$

Substitute $y = -4$ into (2):

$2x - 3(-4) = 14$

$2x + 12 = 14$

$2x = 2$

$x = 1$

$\therefore$ The solution is $x = 1$ and $y = -4$.

(d) $6x - y = 16$ — (1)

$3x + 2y = -12$ — (2)

From (1), $y = 6x - 16$ — (3)

Substitute (3) into (2):

$3x + 2(6x - 16) = -12$

$3x + 12x - 32 = -12$

$15x = 20$

$x = 1\frac{1}{3}$

Substitute $x = 1\frac{1}{3}$ into (3):

$y = 6\left(1\frac{1}{3}\right) - 16$

$= -8$

$\therefore$ The solution is $x = 1\frac{1}{3}$ and $y = -8$.

(e) $4x + 3y = 0$ — (1)

$5y + 53 = 11x$ — (2)

From (1), $x = -\frac{3}{4}y$ — (3)

Substitute $x = -\frac{3}{4}y$ into (2):

$5y + 53 = 11\left(-\frac{3}{4}y\right)$

$= -\frac{33}{4}y$

$13\frac{1}{4}y = -53$

$y = -4$

Substitute $y = -4$ into (3):

$x = -\frac{3}{4}(-4)$

$= 3$

∴ The solution is $x = 3$ and $y = -4$.

(f) $5x - 4y = 4$ — (1)

$2x - y = 2.5$ — (2)

From (2), $y = 2x - 2.5$ — (3)

Substitute (3) into (1):

$5x - 4(2x - 2.5) = 4$

$5x - 8x + 10 = 4$

$3x = 6$

$x = 2$

Substitute $x = 2$ into (3):

$y = 2(2) - 2.5$

$= 1.5$

∴ The solution is $x = 2$ and $y = 1.5$.

7. Let the first number be x and the second number be y.

$x + 11 = 2y$ — (1)

$y + 20 = 2x$ — (2)

From (1), $x = 2y - 11$ — (3)

Substitute (3) into (2):

$y + 20 = 2(2y - 11)$

$= 4y - 22$

$3y = 42$

$y = 14$

Substitute $y = 14$ into (3):

$x = 2(14) - 11$

$= 17$

∴ The two numbers are 17 and 14.

8. The parallel sides of a parallelogram are equal.

$x + y + 1 = 3x - 4$ — (1)

$2y - x = x + 2$ — (2)

From (1), $y = 2x - 5$ — (3)

Substitute (3) into (2):

$2(2x - 5) - x = x + 2$

$4x - 10 - x = x + 2$

$2x = 12$

$x = 6$

Substitute $x = 6$ into (3):

$y = 2(6) - 5$

$= 7$

Perimeter of parallelogram $= 2\{[2(7) - 6] + (6 + 7 + 1)\}$

$= 44$ cm

∴ The perimeter of the parallelogram is 44 cm.

9. Let the numerator of the fraction be x and its denominator be y, i.e. let the fraction be $\frac{x}{y}$.

$\frac{x - 1}{y + 2} = \frac{1}{2}$ — (1)

$\frac{x + 3}{y - 2} = 1\frac{1}{4}$ — (2)

From (1),

$2(x - 1) = y + 2$

$2x - 2 = y + 2$

$y = 2x - 4$ — (3)

From (2),

$4(x + 3) = 5(y - 2)$

$4x + 12 = 5y - 10$

$4x - 5y = -22$ — (4)

Substitute (3) into (4):

$4x - 5(2x - 4) = -22$

$4x - 10x + 20 = -22$

$6x = 42$

$x = 7$

Substitute $x = 7$ into (3):

$y = 2(7) - 4$

$= 10$

∴ The fraction is $\frac{7}{10}$.

10. Let the tens digit of the number be x and its ones digit be y.

$x + y = 12$ — (1)

$y = 2x$ — (2)

Substitute (2) into (1):

$x + 2x = 12$

$3x = 12$

$x = 4$

Substitute $x = 4$ into (2):

$y = 2(4)$

$= 8$

∴ The number is 48.

11. (i) Let Khairul's present age be x years old and Khairul's monther's present age be y years old.

$y + 4 = 3(x + 4)$ — (1)

$y - 6 = 7(x - 6)$ — (2)

From (1), $y + 4 = 3x + 12$

$y = 3x + 8$ — (3)

Substitute (3) into (2):

$3x + 8 - 6 = 7(x - 6)$

$3x + 2 = 7x - 42$

$4x = 44$

$x = 11$

∴ Khairul's present age is 11 years.

(ii) From **(i)**,

Substitute $x = 11$ into (3):

$y = 3(11) + 8$

$= 41$

Age of Khairul's mother when he was born $= 41 - 11$

$= 30$

$\therefore$ The age of Khairul's mother was 30 years.

12. Let the amount Shirley has be $\$x$ and the amount Priya has be $\$y$.

$2(x - 3) = y + 3$ — (1)

$x + 5 = 2(y - 5)$ — (2)

From (1), $2x - 6 = y + 3$

$y = 2x - 9$ — (3)

Substitute (3) into (2):

$x + 5 = 2(2x - 9 - 5)$

$= 4x - 28$

$3x = 33$

$x = 11$

Substitute $x = 11$ into (3):

$y = 2(11) - 9$

$= 13$

$\therefore$ Shirley has \$13 and Priya has \$11.

13. Let the number of smartphones be x and the number of tablet computers be y.

$x + y = 36$ — (1)

$895x + 618y = 28\,065$ — (2)

From (1), $y = 36 - x$ — (3)

Substitute (3) into (2):

$895x + 618(36 - x) = 28\,065$

$895x + 22\,248 - 618x = 28\,065$

$277x = 5817$

$x = 21$

Substitute $x = 21$ into (3):

$y = 36 - 21$

$= 15$

$\therefore$ The vendor buys 21 smartphones and 15 tablet computers.

14. Let the cost of 1 cup of ice-cream milk tea be $\$x$ and the cost of 1 cup of citron tea be $\$y$.

$5x + 4y = 26.8$ — (1)

$7x + 6y = 38.6$ — (2)

$3 \times (1)$: $15x + 12y = 80.4$ — (3)

$2 \times (2)$: $14x + 12y = 77.2$ — (4)

(3) – (4):

$(15x + 12y) - (14x + 12y) = 80.4 - 77.2$

$15x + 12y - 14x - 12y = 3.2$

$x = 3.2$

Substitute $x = 3.2$ into (1):

$5(3.2) + 4y = 26.8$

$16 + 4y = 26.8$

$6y = 10.8$

$y = 2.7$

Difference in cost $= \$3.20 - \$2.70$

$= \$0.50$

$\therefore$ The difference in cost is \$0.50.

15. Let the mass of \$2.50 per kg coffee powder be x kg and the mass of \$3.50 per kg coffee powder be y kg.

$x + y = 20$ — (1)

$2.5x + 3.5y = 20 \times 2.8$

$= 56$ — (2)

From (1), $y = 20 - x$ — (3)

Substitute (3) into (2):

$2.5x + 3.5(20 - x) = 56$

$2.5x + 70 - 3.5x = 56$

$x = 14$

Substitute $x = 14$ into (3):

$y = 20 - 14$

$= 6$

$\therefore$ Vishal uses 14 kg of the \$2.50 per kg coffee powder and 6 kg of the \$3.50 per kg coffee powder.

16. $120x + (175 - 120)y = 2680$ — (1)

$120x + (210 - 120)y = 3240$ — (2)

From (1), $120x + 55y = 2680$ — (3)

From (2), $120x + 90y = 3240$ — (4)

(4) – (3):

$(120x + 90y) - (120x + 55y) = 3240 - 2680$

$120x + 90y - 120x - 55y = 560$

$35y = 560$

$y = 16$

Substitute $y = 16$ into (3):

$120x + 55(16) = 2680$

$120x + 880 = 2680$

$120x = 1800$

$x = 15$

Amount to pay for 140 minutes of talk time

$= 120 \times 15 + (140 - 120) \times 16$

$= 2120$ cents

$= \$21.20$

$\therefore$ The amount Ethan has to pay is \$21.20.

17. Let the number of students in class 2A be x. and the number of students in class 2B be y.

$72x + 75y = 75(73.48)$

$= 5511$ — (1)

$x + y = 75$ — (2)

From (2), $y = 75 - x$ — (3)

Substitute (3) into (1):

$72x + 75(75 - x) = 5511$

$72x + 5625 - 75x = 5511$

$3x = 114$

$x = 38$

Substitute $x = 38$ into (3):

$y = 75 - 38$

$= 37$

$\therefore$ Class 2A has 38 students and class 2B has 37 students.

Challenge Yourself

1. **(i)** $px - y = 6$ — (1)

$8x - 2y = q$ — (2)

From (1), $2px - 2y = 12$

For the simultaneous equations to have an infinite number of solutions, the two equations should be identical.

$2p = 8$

$p = 4$

$q = 12$

(ii) For the simultaneous equations to have no solution, the two equations should have no point of intersection.

$\therefore p = 4, q \neq 12$

(iii) For the simultaneous equations to have a unique solution, the two equations should have one and only one point of intersection.

$\therefore p \neq 4$ and q is any real number.

2. $\frac{4}{x} + \frac{15}{y} = 15$ — (1)

$\frac{7}{5x} - \frac{6}{y} = 3$ — (2)

From (1), $4y + 15x = 15xy$ — (3)

From (2), $7y - 30x = 15xy$ — (4)

(3) = (4):

$4y + 15x = 7y - 30x$

$3y = 45x$

$y = 15x$ — (5)

Substitute (5) into (1):

$\frac{4}{x} + \frac{15}{15x} = 15$

$\frac{4}{x} + \frac{1}{x} = 15$

$\frac{5}{x} = 15$

$x = \frac{1}{3}$

Substitute $x = \frac{1}{3}$ into (5):

$y = 15\left(\frac{1}{3}\right)$

$= 5$

$\therefore$ The solution is $x = \frac{1}{3}$ and $y = 5$.

3. Let the first number be x and the second number of y.

$11x^2 + 13y^3 = 395$ — (1)

$26y^3 - 218 = 121x^2$ — (2)

$2 \times (1)$: $22x^2 + 26y^3 = 790$ — (3)

(3) – (2):

$(22x^2 + 26y^3) - (26y^2 - 218) = 790 - 121x^2$

$22x^2 + 26y^3 - 26y^3 + 218 = 790 - 121x^2$

$143x^2 = 572$

$x^2 = 4$

$x = 2\ (x > 0)$

Substitute $x = 2$ into (1):

$11(2)^2 + 13y^3 = 395$

$44 + 13y^3 = 395$

$13y^3 = 351$

$y^3 = 27$

$y = 3$

$\therefore$ The solution is $x = 2$ and $y = 3$.

4. **(i)** Let the number of spiders be x, the number of dragonflies be y and the number of houseflies be z.

$x + y + z = 20$ — (1)

$8x + 6y + 6z = 136$ — (2)

$2y + z = 19$ — (3)

From (3), $z = 19 - 2y$ — (4)

Substitute (4) into (1):

$x + y + 19 - 2y = 20$

$y = x - 1$ — (5)

Substitute (4) into (2):

$8x + 6y + 6(19 - 2y) = 136$

$8x + 6y + 114 - 12y = 136$

$8x - 6y = 22$

$4x - 3y = 11$ — (6)

Substitute (5) into (6):

$4x - 3(x - 1) = 11$

$4x - 3x + 3 = 11$

$x = 8$

$\therefore$ The number of spiders is 8.

(ii) From **(i)**,

Substitute $x = 8$ into (5):

$y = 8 - 1$

$= 7$

$\therefore$ The number of dragonflies is 7.

(iii) From **(i)** and **(ii)**,

Substitute $y = 7$ into (4):

$z = 19 - 2(7)$

$= 5$

$\therefore$ The number of houseflies is 5.

5. Let the number of roosters be r, the number of hens be h and the number of chicks be c.

$r + h + c = 100$ — (1)

$5r + 3h + \frac{c}{3} = 100$ — (2)

$3 \times (2)$: $15r + 9h + c = 300$ — (3)

(3) – (1):

$(15r + 9h + c) - (r + h + c) = 300 - 100$

$15r + 9h + c - r - h - c = 200$

$14r + 8h = 200$

$7r + 4h = 100$

$\therefore h = \frac{100 - 7r}{4}$

$= 25 - \frac{7r}{4}$

Since h is a positive integer, by divisibility, r must be a multiple of 4.

$$25 - \frac{7r}{4} > 0$$

$$\frac{7r}{4} < 25$$

$$7r < 100$$

$$r < 14\frac{2}{7}$$

$\therefore$ Possible values of $r = 4$, 8 and 12

r	h	c	$r + h + c$
4	18	78	100
8	11	81	100
12	4	84	100

$\therefore$ The farmer can buy 4 roosters, 18 hens and 78 chicks, 8 roosters, 11 hens and 81 chicks or 12 roosters, 4 hens and 84 chicks.

Chapter 3 Expansion and Factorisation of Quadratic Expressions

TEACHING NOTES

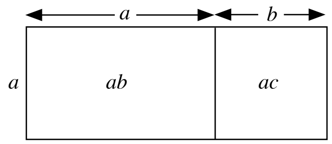
Suggested Approach

The teaching of the expansion and factorisation of algebraic expressions should focus primarily on the Concrete-Pictorial-Approach. In Secondary One, students have learnt how to expand simple linear expressions using the Distributive Law of Multiplication. Teachers may want to show the expansion of algebraic expressions using the area of rectangles.

E.g. Expand $a(b + c)$.

Area of rectangle $= a(b + c)$

$= ab + ac$



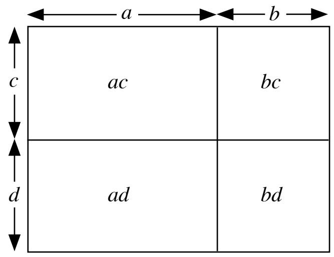
Teachers can further reinforce the concept of expanding quadratic expressions using the area of rectangles.

E.g. Expand $(a + b)(c + d)$.

Area of rectangle

$= (a + b)(c + d)$

$= ac + ad + bc + bd$



Section 3.1: Quadratic Expressions

Students have learnt how to simplify simple linear algebraic expressions in Secondary One using algebra discs (E.g. 'x' disc, '$-x$' disc, '1' disc, '–1' disc). Teachers should further introduce another two more digital algebra discs (E.g. 'x^2' disc, '$-x^2$' disc) to help students to visualise and learn how to form and simplify simple quadratic expressions. Use the Practise Now examples in the textbook.

Section 3.2: Expansion and Simplification of Quadratic Expressions

Teachers can build upon prerequisites and move from expanding linear expressions to expanding simple quadratic expressions, of the form $a(b + c)$ using the algebra discs (see Practise Now on page 98 of the textbook).

To expand quadratic expressions of the form $(a + b)(c + d)$, teachers may use the algebra discs to illustrate how the 'expanded terms' can be arranged in the rectangular array. Teachers should also highlight to students how to 'fill up' the 'terms' in the multiplication frame after the expansion process (see Class Discussion: Expansion of Quadratic Expressions of the Form $(a + b)(c + d)$).

Section 3.3: Factorisation of Quadratic Expressions

Most students would find factorising quadratic expressions of the form $ax^2 + bx + c$ difficult. Students should be provided with ample practice questions and the factorisation process may need to be reiterated multiple times. Teachers should begin with simple quadratic expressions (E.g. those of the form $x^2 + bx + c$) to allow students to gain confidence in obtaining the 2 linear factors of the quadratic expressions.

Teachers should instruct students to explore the factorisation process of simple quadratic expressions using the algebra discs (see Practise Now on page 107 of the textbook).

Next, without using algebra discs, teacher should illustrate to students the steps to factorising quadratic expressions using a 'Multiplication Frame' (see Page 108).

Once students have acquired the technique in factorising simple quadratic expressions, teachers can then challenge the students with more difficult quadratic expressions.

WORKED SOLUTIONS

Class Discussion (Expansion of Quadratic Expressions of the Form (*a* + *b*)(*c* + *d*))

1. (a) $(x + 3)(x + 6) = x(x + 6) + 3(x + 6)$
$= x^2 + 6x + 3x + 18$
$= x^2 + 9x + 18$

(b) $(x + 3)(x - 6) = x(x - 6) + 3(x - 6)$
$= x^2 - 6x + 3x - 18$
$= x^2 - 3x - 18$

(c) $(x - 3)(x + 6) = x(x + 6) - 3(x + 6)$
$= x^2 + 6x - 3x - 18$
$= x^2 + 3x - 18$

(d) $(x - 3)(x - 6) = x(x - 6) - 3(x - 6)$
$= x^2 - 6x - 3x + 18$
$= x^2 - 9x + 18$

(e) $(3x + 1)(2x + 3) = 3x(2x + 3) + 1(2x + 3)$
$= 6x^2 + 9x + 2x + 3$
$= 6x^2 + 11x + 3$

(f) $(3x + 1)(2x - 3) = 3x(2x - 3) + 1(2x - 3)$
$= 6x^2 - 9x + 2x - 3$
$= 6x^2 - 7x - 3$

(g) $(3x - 1)(2x + 3) = 3x(2x + 3) - 1(2x + 3)$
$= 6x^2 + 9x - 2x - 3$
$= 6x^2 + 7x - 3$

(h) $(3x - 1)(2x - 3) = 3x(2x - 3) - 1(2x - 3)$
$= 6x^2 - 9x - 2x + 3$
$= 6x^2 - 11x + 3$

2. $(a + b)(c + d) = a(c + d) + b(c + d)$
$= ac + ad + bc + bd$

Practise Now (Page 93)

(a) $-4x^2 + 2x^2 = -2x^2$

(b) $-4x^2 + (-2x^2) = -4x^2 - 2x^2$
$= -6x^2$

(c) $4x^2 - 2x^2 = 2x^2$

(d) $4x^2 - (-2x^2) = 4x^2 + 2x^2$
$= 6x^2$

(e) $2x^2 - 3 - x^2 + 1 = 2x^2 - x^2 - 3 + 1$
$= x^2 - 2$

(f) $5x^2 + (-x) + 2 - (-2x^2) - 3x - 4$
$= 5x^2 - x + 2 + 2x^2 - 3x - 4$
$= 5x^2 + 2x^2 - x - 3x + 2 - 4$
$= 7x^2 - 4x - 2$

Practise Now (Page 94)

(a) $-(2x^2 + x + 1) = -2x^2 - x - 1$

(b) $-(-2x^2 - x + 1) = 2x^2 + x - 1$

(c) $x^2 + 2x + 1 - (3x^2 + 5x - 2)$
$= x^2 + 2x + 1 - 3x^2 - 5x + 2$
$= x^2 - 3x^2 + 2x - 5x + 1 + 2$
$= -2x^2 - 3x + 3$

(d) $-(-x^2 - 4) + 2x^2 - 7x + 3$
$= x^2 + 4 + 2x^2 - 7x + 3$
$= x^2 + 2x^2 - 7x + 4 + 3$
$= 3x^2 - 7x + 7$

Practise Now (Page 96)

(a) $2(-2x^2 + x - 1) = -4x^2 + 2x - 2$

(b) $3(x^2 - 2x + 3) = 3x^2 - 6x + 9$

(c) $4x^2 + (-3x) + (-1) + 3(x^2 - 4)$
$= 4x^2 - 3x - 1 + 3x^2 - 12$
$= 4x^2 + 3x^2 - 3x - 1 - 12$
$= 7x^2 - 3x - 13$

(d) $2(x^2 + 4x - 5) - (6 + x^2)$
$= 2x^2 + 8x - 10 - 6 - x^2$
$= 2x^2 - x^2 + 8x - 10 - 6$
$= x^2 + 8x - 16$

Practise Now 1

(a) $7x^2 - 4x + 6x^2 - x = 7x^2 + 6x^2 - 4x - x$
$= 13x^2 - 5x$

(b) $-(-5x^2) + 3x + (-6) + 2(3x^2 - 8x + 4)$
$= 5x^2 + 3x - 6 + 6x^2 - 16x + 8$
$= 5x^2 + 6x^2 + 3x - 16x - 6 + 8$
$= 11x^2 - 13x + 2$

(c) $4x^2 - 1 - (7x^2 + 13x - 2)$
$= 4x^2 - 1 - 7x^2 - 13x + 2$
$= 4x^2 - 7x^2 - 13x - 1 + 2$
$= -3x^2 - 13x + 1$

(d) $-(3x^2 + 5x - 8) + x^2 + 6x + 5$
$= -3x^2 - 5x + 8 + x^2 + 6x + 5$
$= -3x^2 + x^2 - 5x + 6x + 8 + 5$
$= -2x^2 + x + 13$

Practise Now (Page 98)

(a) $3(2x + 1) = 3(2x) + 3(1)$
$= 6x + 3$

(b) $-3(2x - 1) = -3(2x) + (-3)(-1)$
$= -6x + 3$

(c) $x(-2x + 3) = x(-2x) + x(3)$
$= -2x^2 + 3x$

(d) $-2x(x - 3) = -2x(x) + (-2x)(-3)$
$= -2x^2 + 6x$

Practise Now 2

(a) $3(4x + 1) = 12x + 3$

(b) $7(5x - 2) = 35x - 14$

(c) $5x(2x - 3) = 10x^2 - 15x$

(d) $-2x(8x - 3) = -16x^2 + 6x$

Practise Now 3

(a) $5(x - 4) - 3(2x + 4)$
$= 5x - 20 - 6x - 12$
$= 5x - 6x - 20 - 12$
$= -x - 32$

(b) $2x(2x + 3) - x(2 - 5x)$
$= 4x^2 + 6x - 2x + 5x^2$
$= 4x^2 + 5x^2 + 6x - 2x$
$= 9x^2 + 4x$

Practise Now 4

(a) $(x + 2)(x + 4)$
$= x(x + 4) + 2(x + 4)$
$= x^2 + 4x + 2x + 8$
$= x^2 + 6x + 8$

(b) $(3x - 4)(5x - 6)$
$= 3x(5x - 6) - 4(5x - 6)$
$= 15x^2 - 18x - 20x + 24$
$= 15x^2 - 38x + 24$

(c) $(5 + x)(2 - 3x)$
$= 5(2 - 3x) + x(2 - 3x)$
$= 10 - 15x + 2x - 3x^2$
$= 10 - 13x - 3x^2$

(d) $(1 - 7x)(11x - 4)$
$= (11x - 4) - 7x(11x - 4)$
$= 11x - 4 - 77x^2 + 28x$
$= -77x^2 + 11x + 28x - 4$
$= -77x^2 + 39x - 4$

Practise Now 5

$(2x - 1)(x + 5) - 5x(x - 4)$
$= 2x(x + 5) - (x + 5) - 5x(x - 4)$
$= 2x^2 + 10x - x - 5 - 5x^2 + 20x$
$= 2x^2 - 5x^2 + 10x - x + 20x - 5$
$= -3x^2 + 29x - 5$

Practise Now (Page 107)

(a)

×	x	5
x	x^2	$5x$
1	x	5

$\therefore x^2 + 6x + 5 = (x + 1)(x + 5)$

(b)

×	x	–5
x	x^2	$-5x$
–1	$-x$	5

$\therefore x^2 - 6x + 5 = (x - 1)(x - 5)$

(c)

×	x	6
x	x^2	$6x$
2	$2x$	12

$\therefore x^2 + 8x + 12 = (x + 2)(x + 6)$

(d)

×	x	–6
x	x^2	$-6x$
–2	$-2x$	12

$\therefore x^2 - 8x + 12 = (x - 2)(x - 6)$

Practise Now 6

(a) $x^2 = x \times x$
$7 = 1 \times 7$ or $(-1) \times (-7)$

×	x	7
x	x^2	$7x$
1	x	7

$x + 7x = 8x$
$\therefore x^2 + 8x + 7 = (x + 1)(x + 7)$

(b) $x^2 = x \times x$
$28 = 1 \times 28$ or $(-1) \times (-28)$
$= 2 \times 14$ or $(-2) \times (-14)$
$= 4 \times 7$ or $(-4) \times (-7)$

×	x	–7
x	x^2	$-7x$
–4	$-4x$	28

$(-4x) + (-7x) = -11x$
$\therefore x^2 - 11x + 28 = (x - 4)(x - 7)$

(c) $x^2 = x \times x$
$-2 = 1 \times (-2)$ or $(-1) \times 2$

×	x	2
x	x^2	$2x$
–1	$-x$	–2

$(-x) + 2x = x$
$\therefore x^2 + x - 2 = (x - 1)(x + 2)$

(d) $x^2 = x \times x$
$-8 = 1 \times (-8)$ or $(-1) \times 8$
$= 2 \times (-4)$ or $(-2) \times 4$

×	x	–8
x	x^2	$-8x$
1	x	–8

$x + (-8x) = -7x$
$\therefore x^2 - 7x - 8 = (x + 1)(x - 8)$

Practise Now 7

(a) $2x^2 = 2x \times x$

$12 = 1 \times 12$ or $(-1) \times (-12)$

$= 2 \times 6$ or $(-2) \times (-6)$

$= 3 \times 4$ or $(-3) \times (-4)$

$\times$	x	4
$2x$	$2x^2$	$8x$
3	$3x$	12

$3x + 8x = 11x$

$\therefore 2x^2 + 11x + 12 = (2x + 3)(x + 4)$

(b) $5x^2 = 5x \times x$

$6 = 1 \times 6$ or $(-1) \times (-6)$

$= 2 \times 3$ or $(-2) \times (-3)$

$\times$	x	-2
$5x$	$5x^2$	$-10x$
-3	$-3x$	6

$(-3x) + (-10x) = -13x$

$\therefore 5x^2 - 13x + 6 = (5x - 3)(x - 2)$

(c) $-2x^2 = -2x \times x$

$-9 = 1 \times (-9)$ or $(-1) \times 9$

$= 3 \times (-3)$ or $(-3) \times 3$

$\times$	x	-3
$-2x$	$-2x^2$	$6x$
3	$3x$	-9

$3x + 6x = 9x$

$\therefore -2x^2 + 9x - 9 = (-2x + 3)(x - 3)$

(d) $9x^2 - 33x + 24 = 3(3x^2 - 11x + 8)$

$3x^2 = 3x \times x$

$8 = 1 \times 8$ or $(-1) \times (-8)$

$= 2 \times 4$ or $(-2) \times (-4)$

$\times$	x	-1
$3x$	$3x^2$	$-3x$
-8	$-8x$	8

$(-8x) + (-3x) = -11x$

$\therefore 9x^2 - 33x + 24 = 3(3x - 8)(x - 1)$

Exercise 3A

1. (a) $6x^2 + 19 + 9x^2 - 8$
$= 6x^2 + 9x^2 + 19 - 8$
$= 15x^2 + 11$

(b) $x^2 + 2x - 7 - (-11x^2) - 5x - 1$
$= x^2 + 2x - 7 + 11x^2 - 5x - 1$
$= x^2 + 11x^2 + 2x - 5x - 7 - 1$
$= 12x^2 - 3x - 8$

(c) $y + (-3y^2) + 2(y^2 - 6y)$
$= y - 3y^2 + 2y^2 - 12y$
$= -3y^2 + 2y^2 + y - 12y$
$= -y^2 - 11y$

(d) $5x^2 - x - (x^2 - 10x)$
$= 5x^2 - x - x^2 + 10x$
$= 5x^2 - x^2 - x + 10x$
$= 4x^2 + 9x$

(e) $-(4x^2 + 9x + 2) + 3x^2 - 7x + 2$
$= -4x^2 - 9x - 2 + 3x^2 - 7x + 2$
$= -4x^2 + 3x^2 - 9x - 7x - 2 + 2$
$= -x^2 - 16x$

(f) $-(1 - 7y - 8y^2) + 2(y^2 - 3y - 1)$
$= -1 + 7y + 8y^2 + 2y^2 - 6y - 2$
$= 8y^2 + 2y^2 + 7y - 6y - 1 - 2$
$= 10y^2 + y - 3$

2. (a) $12 \times 5x = 12 \times 5 \times x$
$= 60x$

(b) $x \times 6x = x \times 6 \times x$
$= 6 \times x \times x$
$= 6x^2$

(c) $(-2x) \times 8x = (-2) \times x \times 8 \times x$
$= (-2) \times 8 \times x \times x$
$= -16x^2$

(d) $(-3x) \times (-10x)$
$= (-3) \times x \times (-10) \times x$
$= (-3) \times (-10) \times x \times x$
$= 30x^2$

3. (a) $4(3x + 4) = 12x + 16$

(b) $-6(-7x - 3) = 42x + 18$

(c) $8(-x - 3) = -8x - 24$

(d) $-2(5x - 1) = -10x + 2$

(e) $5x(3x - 4) = 15x^2 - 20x$

(f) $-8x(3x + 5) = -24x^2 - 40x$

(g) $-5x(2 - 3x) = -10x + 15x^2$

(h) $-x(-x - 1) = x^2 + x$

4. (a) $4(2a + 3) + 5(a + 3) = 8a + 12 + 5a + 15$
$= 8a + 5a + 12 + 15$
$= 13a + 27$

(b) $9(5 - 2b) + 3(6 - 5b) = 45 - 18b + 18 - 15b$
$= 45 + 18 - 18b - 15b$
$= 63 - 33b$

(c) $c(3c + 1) + 2c(c + 3) = 3c^2 + c + 2c^2 + 6c$
$= 3c^2 + 2c^2 + c + 6c$
$= 5c^2 + 7c$

(d) $6d(5d - 4) + 2d(3d - 2) = 30d^2 - 24d + 6d^2 - 4d$
$= 30d^2 + 6d^2 - 24d - 4d$
$= 36d^2 - 28d$

5. (a) $(x + 3)(x + 7) = x(x + 7) + 3(x + 7)$
$= x^2 + 7x + 3x + 21$
$= x^2 + 10x + 21$

(b) $(4x + 1)(3x + 5) = 4x(3x + 5) + (3x + 5)$
$= 12x^2 + 20x + 3x + 5$
$= 12x^2 + 23x + 5$

6. **(a)** $7(2a + 1) - 4(8a + 3)$
$= 14a + 7 - 32a - 12$
$= 14a - 32a + 7 - 12$
$= -18a - 5$

(b) $3(2b - 1) - 2(5b - 3)$
$= 6b - 3 - 10b + 6$
$= 6b - 10b - 3 + 6$
$= -4b + 3$

(c) $3c(5 + c) - 2c(3c - 7)$
$= 15c + 3c^2 - 6c^2 + 14c$
$= 15c + 14c + 3c^2 - 6c^2$
$= 29c - 3c^2$

(d) $2d(3d - 5) - d(2 - d)$
$= 6d^2 - 10d - 2d + d^2$
$= 6d^2 + d^2 - 10d - 2d$
$= 7d^2 - 12d$

(e) $-f(9 - 2f) + 4f(f - 8)$
$= -9f + 2f^2 + 4f^2 - 32f$
$= 2f^2 + 4f^2 - 9f - 32f$
$= 6f^2 - 41f$

(f) $-2h(3 + 4h) - 5h(h - 1)$
$= -6h - 8h^2 - 5h^2 + 5h$
$= -8h^2 - 5h^2 - 6h + 5h$
$= -13h^2 - h$

7. **(a)** $(a + 1)(a - 9) = a(a - 9) + (a - 9)$
$= a^2 - 9a + a - 9$
$= a^2 - 8a - 9$

(b) $(b - 2)(b + 7) = b(b + 7) - 2(b + 7)$
$= b^2 + 7b - 2b - 14$
$= b^2 + 5b - 14$

(c) $(c - 5)(c - 6) = c(c - 6) - 5(c - 6)$
$= c^2 - 6c - 5c + 30$
$= c^2 - 11c + 30$

(d) $(3d + 1)(5 - 2d) = 3d(5 - 2d) + (5 - 2d)$
$= 15d - 6d^2 + 5 - 2d$
$= -6d^2 + 15d - 2d + 5$
$= -6d^2 + 13d + 5$

(e) $(1 - f)(7f + 6) = (7f + 6) - f(7f + 6)$
$= 7f + 6 - 7f^2 - 6f$
$= -7f^2 + 7f - 6f + 6$
$= -7f^2 + f + 6$

(f) $(4 - 3h)(10 - 9h) = 4(10 - 9h) - 3h(10 - 9h)$
$= 40 - 36h - 30h + 27h^2$
$= 40 - 66h + 27h^2$

8. **(a)** $5 + (x + 1)(x + 3) = 5 + x(x + 3) + (x + 3)$
$= 5 + x^2 + 3x + x + 3$
$= x^2 + 3x + x + 5 + 3$
$= x^2 + 4x + 8$

(b) $3x + (x + 7)(2x - 1) = 3x + x(2x - 1) + 7(2x - 1)$
$= 3x + 2x^2 - x + 14x - 7$
$= 2x^2 + 3x - x + 14x - 7$
$= 2x^2 + 16x - 7$

(c) $(3x + 2)(x - 9) + 2x(4x + 1)$
$= 3x(x - 9) + 2(x - 9) + 8x^2 + 2x$
$= 3x^2 - 27x + 2x - 18 + 8x^2 + 2x$
$= 3x^2 + 8x^2 - 27x + 2x + 2x - 18$
$= 11x^2 - 23x - 18$

(d) $(x - 3)(x - 8) + (x - 4)(2x + 9)$
$= x(x - 8) - 3(x - 8) + x(2x + 9) - 4(2x + 9)$
$= x^2 - 8x - 3x + 24 + 2x^2 + 9x - 8x - 36$
$= x^2 + 2x^2 - 8x - 3x + 9x - 8x + 24 - 36$
$= 3x^2 - 10x - 12$

9. **(a)** $4x^2 - (3x - 4)(2x + 1) = 4x^2 - [3x(2x + 1) - 4(2x + 1)]$
$= 4x^2 - (6x^2 + 3x - 8x - 4)$
$= 4x^2 - 6x^2 - 3x + 8x + 4$
$= -2x^2 + 5x + 4$

(b) $2x(x - 6) - (2x + 5)(7 - x) = 2x^2 - 12x - [2x(7 - x) + 5(7 - x)]$
$= 2x^2 - 12x - (14x - 2x^2 + 35 - 5x)$
$= 2x^2 - 12x - 14x + 2x^2 - 35 + 5x$
$= 2x^2 + 2x^2 - 12x - 14x + 5x - 35$
$= 4x^2 - 23x - 35$

(c) $(4x - 3)(x + 2) - (3x - 5)(-x - 9)$
$= [4x(x + 2) - 3(x + 2)] - [3x(-x - 9) - 5(-x - 9)]$
$= (4x^2 + 8x - 3x - 6) - (-3x^2 - 27x + 5x + 45)$
$= 4x^2 + 5x - 6 + 3x^2 + 22x - 45$
$= 4x^2 + 3x^2 + 5x + 22x - 6 - 45$
$= 7x^2 + 27x - 51$

(d) $(2x + 3)(5x - 2) - 2(5x - 3)(x + 1)$
$= [2x(5x - 2) + 3(5x - 2)] - 2[5x(x + 1) - 3(x + 1)]$
$= (10x^2 - 4x + 15x - 6) - 2(5x^2 + 5x - 3x - 3)$
$= 10x^2 + 11x - 6 - 2(5x^2 + 2x - 3)$
$= 10x^2 + 11x - 6 - 10x^2 - 4x + 6$
$= 10x^2 - 10x^2 + 11x - 4x - 6 + 6$
$= 7x$

Exercise 3B

1. **(a)** $a^2 = a \times a$
$8 = 1 \times 8$ or $(-1) \times (-8)$
$= 2 \times 4$ or $(-2) \times (-4)$

$\times$	a	8
a	a^2	$8a$
1	a	8

$a + 8a = 9a$
$\therefore a^2 + 9a + 8 = (a + 1)(a + 8)$

(b) $b^2 = b \times b$
$15 = 1 \times 15$ or $(-1) \times (-15)$
$= 3 \times 5$ or $(-3) \times (-5)$

$\times$	b	5
b	b^2	$5b$
3	$3b$	15

$3b + 5b = 8b$
$\therefore b^2 + 8b + 15 = (b + 3)(b + 5)$

(c) $c^2 = c \times c$

$20 = 1 \times 20$ or $(-1) \times (-20)$
$= 2 \times 10$ or $(-2) \times (-10)$
$= 4 \times 5$ or $(-4) \times (-5)$

$\times$	c	-5
c	c^2	$-5c$
-4	$-4c$	20

$(-4c) + (-5c) = -9c$

$\therefore c^2 - 9c + 20 = (c - 4)(c - 5)$

(d) $d^2 = d \times d$

$28 = 1 \times 28$ or $(-1) \times (-28)$
$= 2 \times 14$ or $(-2) \times (-14)$
$= 4 \times 7$ or $(-4) \times (-7)$

$\times$	d	-14
d	d^2	$-14d$
-2	$-2d$	28

$(-2d) + (-14d) = -16d$

$\therefore d^2 - 16d + 28 = (d - 2)(d - 14)$

(e) $f^2 = f \times f$

$-16 = 1 \times (-16)$ or $(-1) \times 16$
$= 2 \times (-8)$ or $(-2) \times 8$
$= 4 \times (-4)$ or $(-4) \times 4$

$\times$	f	8
f	f^2	$8f$
-2	$-2f$	-16

$(-2f) + 8f = 6f$

$\therefore f^2 + 6f - 16 = (f - 2)(f + 8)$

(f) $h^2 = h \times h$

$-120 = 1 \times (-120)$ or $(-1) \times 120$
$= 2 \times (-60)$ or $(-2) \times 60$
$= 3 \times (-40)$ or $(-3) \times 40$
$= 4 \times (-30)$ or $(-4) \times 30$
$= 5 \times (-24)$ or $(-5) \times 24$
$= 6 \times (-20)$ or $(-6) \times 20$
$= 8 \times (-15)$ or $(-8) \times 15$
$= 10 \times (-12)$ or $(-10) \times 12$

$\times$	h	12
h	h^2	$12h$
-10	$-10h$	-120

$(-10h) + 12h = 2h$

$\therefore h^2 + 2h - 120 = (h - 10)(h + 12)$

(g) $k^2 = k \times k$

$-12 = 1 \times (-12)$ or $(-1) \times 12$
$= 2 \times (-6)$ or $(-2) \times 6$
$= 3 \times (-4)$ or $(-3) \times 4$

$\times$	k	-6
k	k^2	$-6k$
2	$-2k$	-12

$2k + (-6k) = -4k$

$\therefore k^2 - 4k - 12 = (k + 2)(k - 6)$

(h) $m^2 = m \times m$

$-21 = 1 \times (-21)$ or $(-1) \times 21$
$= 3 \times (-7)$ or $(-3) \times 7$

$\times$	m	-21
m	m^2	$-21m$
1	m	21

$m + (-21m) = -20m$

$\therefore m^2 - 20m - 21 = (m + 1)(m - 21)$

2. **(a)** $3n^2 = 3n \times n$

$7 = 1 \times 7$ or $(-1) \times (-7)$

$\times$	n	1
$3n$	$3n^2$	$3n$
7	$7n$	7

$7n + 3n = 10n$

$\therefore 3n^2 + 10n + 7 = (3n + 7)(n + 1)$

(b) $4p^2 = 4p \times p$ or $2p \times 2p$

$3 = 1 \times 3$ or $(-1) \times (-3)$

$\times$	$2p$	3
$2p$	$4p^2$	$6p$
1	$2p$	3

$2p + 6p = 8p$

$\therefore 4p^2 + 8p + 3 = (2p + 1)(2p + 3)$

(c) $6q^2 = 6q \times q$ or $3q \times 2q$

$12 = 1 \times 12$ or $(-1) \times (-12)$
$= 2 \times 6$ or $(-2) \times (-6)$
$= 3 \times 4$ or $(-3) \times (-4)$

$\times$	$2q$	-3
$3p$	$6p^2$	$-9q$
-4	$-8q$	12

$(-8q) + (-9q) = -17q$

$\therefore 6q^2 - 17q + 12 = (3q - 4)(2q - 3)$

(d) $4r^2 = 4r \times r$ or $2r \times 2r$

$3 = 1 \times 3$ or $(-1) \times (-3)$

$\times$	r	-1
$4r$	$4r^2$	$-4r$
-3	$-3r$	3

$(-3r) + (-4r) = -7r$

$\therefore 4r^2 - 7r + 3 = (4r - 3)(r - 1)$

(e) $8s^2 = 8s \times s$ or $4s \times 2s$

$-15 = 1 \times (-15)$ or $(-1) \times 15$

$= 3 \times (-5)$ or $(-3) \times 5$

$\times$	$2s$	3
$4s$	$8s^2$	$-12s$
-5	$-10s$	-15

$(-10s) + 12s = 2s$

$\therefore 8s^2 + 2s - 15 = (4s - 5)(2s + 3)$

(f) $6t^2 = 6t \times t$ or $3t \times 2t$

$-20 = 1 \times (-20)$ or $(-1) \times 20$

$= 2 \times (-10)$ or $(-2) \times 10$

$= 4 \times (-5)$ or $(-4) \times 5$

$\times$	t	4
$6t$	$6t^2$	$24c$
-5	$-5t$	-20

$(-5t) + 24t = 19t$

$\therefore 6t^2 + 19t - 20 = (6t - 5)(t + 4)$

(g) $4u^2 = 4u \times u$ or $2u \times 2u$

$-21 = 1 \times (-21)$ or $(-1) \times 21$

$= 3 \times (-7)$ or $(-3) \times 7$

$\times$	$2u$	-7
$2u$	$4u^2$	$-14u$
3	$6u$	-21

$6u + (-14u) = -8u$

$\therefore 4u^2 - 8u - 21 = (2u + 3)(2u - 7)$

(h) $18w^2 = 18w \times w$ or $9w \times 2w$ or $6w \times 3w$

$-39 = 1 \times (-39)$ or $(-1) \times 39$

$= 3 \times (-13)$ or $(-3) \times 13$

$\times$	$2w$	-3
$9w$	$18w^2$	$-27w$
13	$26w$	-39

$26w + (-27w) = -w$

$\therefore 18w^2 - w - 39 = (9w + 13)(2w - 3)$

3. (a) $-a^2 = -a \times a$

$35 = 1 \times 35$ or $(-1) \times (-35)$

$= 5 \times 7$ or $(-5) \times (-7)$

$\times$	a	5
$-a$	$-a^2$	$-5a$
7	$7a$	35

$7a + (-5a) = 2a$

$\therefore -a^2 + 2a + 35 = (-a + 7)(a + 5)$

(b) $-3b^2 = -3b \times b$

$-25 = 1 \times (-25)$ or $(-1) \times 25$

$= 5 \times (-5)$ or $(-5) \times 5$

$\times$	b	-25
$-3b$	$-3b^2$	$75b$
1	b	-25

$b + 75b = 76b$

$\therefore -3b^2 + 76b - 25 = (-3b + 1)(b - 25)$

(c) $4c^2 + 10c + 4 = 2(2c^2 + 5c + 2)$

$2c^2 = 2c \times c$

$2 = 1 \times 2$ or $(-1) \times (-2)$

$\times$	c	2
$2c$	$2c^2$	$4c$
1	c	2

$c + 4c = 5c$

$\therefore 4c^2 + 10c + 4 = 2(2c + 1)(c + 2)$

(d) $5d^2 - 145d + 600 = 5(d^2 - 29d + 120)$

$d^2 = d \times d$

$120 = 1 \times 120$ or $(-1) \times (-120)$

$= 2 \times 60$ or $(-2) \times (-60)$

$= 3 \times 40$ or $(-3) \times (-40)$

$= 4 \times 30$ or $(-4) \times (-30)$

$= 5 \times 24$ or $(-5) \times (-24)$

$= 6 \times 20$ or $(-6) \times (-20)$

$= 8 \times 15$ or $(-8) \times (-15)$

$= 10 \times 12$ or $(-10) \times (-12)$

$\times$	d	-24
d	d^2	$-24d$
-5	$-5h$	120

$(-5h) + (-24h) = -29d$

$\therefore 5d^2 - 145d + 600 = 5(d - 5)(d - 24)$

(e) $8f^2 + 4f - 60 = 4(2f^2 + f - 15)$

$2f^2 = 2f \times f$

$-15 = 1 \times (-15)$ or $(-1) \times 15$

$= 3 \times (-5)$ or $(-3) \times 5$

$\times$	f	3
$2f$	$2f^2$	$6f$
-5	$-5f$	-15

$(-5f) + 6f = f$

$\therefore 8f^2 + 4f - 60 = 4(2f - 5)(f + 3)$

(f) $24h^2 - 15h - 9 = 3(8h^2 - 5h - 3)$

$8h^2 = 8h \times h$ or $4h \times 2h$

$-3 = 1 \times (-3)$ or $(-1) \times 3$

$\times$	h	-1
$8h$	$8h^2$	$-8h$
3	$3h$	-3

$3h + (-8h) = -5h$

$\therefore 24h^2 - 15h - 9 = 3(8h + 3)(h - 1)$

(g) $30 + 14k - 4k^2 = 2(15 + 7k - 2k^2)$

$-2k = -2k \times k$

$15 = 1 \times 15$ or $(-1) \times (-15)$

$= 3 \times 5$ or $(-3) \times (-5)$

$\times$	$2k$	3
$-k$	$-2k^2$	$-3k$
5	$10k$	15

$10k + (-3k) = 7k$

$\therefore 30 + 14k - 4k^2 = 2(-k + 5)(2k + 3)$

(h) $35m^2n + 5mn - 30n = 5n(7m^2 + m - 6)$

$7m^2 = 7m \times m$

$-6 = 1 \times (-6)$ or $(-1) \times 6$

$= 2 \times (-3)$ or $(-2) \times 3$

$\times$	m	1
$7m$	$7m^2$	$7m$
-6	$-6m$	-6

$(-6m) + 7m = m$

$\therefore 35m^2n + 5mn - 30n = 5n(7m - 6)(m + 1)$

4. $x^2 = x \times x$

$12 = 1 \times 12$ or $(-1) \times (-12)$

$= 2 \times 6$ or $(-2) \times (-6)$

$= 3 \times 4$ or $(-3) \times (-4)$

$\times$	x	6
x	x^2	$6x$
2	$2x$	12

$2x + 6x = 8x$

$\therefore x^2 + 8x + 12 = (x + 2)(x + 6)$

Since the area is $(x^2 + 8x + 12)$ cm^2 and the length is $(x + 6)$ cm,

$\therefore$ the breadth is $(x + 2)$ cm.

5. **(a)** $\frac{4}{9}p^2 + p - 1 = \frac{1}{9}(4p^2 + 9p - 9)$

$4p^2 = 4p \times p$ or $2p \times 2p$

$-9 = 1 \times (-9)$ or $(-1) \times 9$

$= 3 \times (-3)$ or $(-3) \times 3$

$\times$	p	3
$4p$	$4p^2$	$12p$
-3	$-3p$	-9

$(-3p) + 12p = 9p$

$\therefore \frac{4}{9}p^2 + p - 1 = \frac{1}{9}(4p - 3)(p + 3)$

(b) $0.6r - 0.8qr - 12.8q^2r = -0.2r(64q^2 + 4q - 3)$

$64q^2 = 64q \times q$ or $32q \times 2q$ or $16q \times 4q$ or $8q \times 8q$

$-3 = 1 \times (-3)$ or $(-1) \times 3$

$\times$	$4q$	1
$16q$	$64q^2$	$16q$
-3	$-12q$	-3

$(-12q) + 16q = 4q$

$\therefore 0.6r - 0.8qr - 12.8q^2r = -0.2r(16q - 3)(4q + 1)$

Review Exercise 3

1. **(a)** $10a(2a - 7) = 20a^2 - 70a$

(b) $-3b(7 - 4b) = -21b + 12b^2$

(c) $(c - 4)(c - 11) = c(c - 11) - 4(c - 11)$

$= c^2 - 11c - 4c + 44$

$= c^2 - 15c + 44$

(d) $(3d - 5)(4 - d) = 3d(4 - d) - 5(4 - d)$

$= 12d - 3d^2 - 20 + 5d$

$= -3d^2 + 12d + 5d - 20$

$= -3d^2 + 17d - 20$

2. **(a)** $7f(3f - 4) + 4f(3 - 2f) = 21f^2 - 28f + 12f - 8f^2$

$= 21f^2 - 8f^2 - 28f + 12f$

$= 13f^2 - 16f$

(b) $6h^2 + (2h + 3)(h - 1) = 6h^2 + 2h(h - 1) + 3(h - 1)$

$= 6h^2 + 2h^2 - 2h + 3h - 3$

$= 8h^2 + h - 3$

(c) $(2k - 1)(k - 4) - 3k(k - 7)$

$= 2k(k - 4) - (k - 4) - 3k^2 + 21k$

$= 2k^2 - 8k - k + 4 - 3k^2 + 21k$

$= 2k^2 - 3k^2 - 8k - k + 21k + 4$

$= -k^2 + 12k + 4$

(d) $(m + 2)(m + 1) - (3m + 5)(9 - 5m)$

$= m(m + 1) + 2(m + 1) - [3m(9 - 5m) + 5(9 - 5m)]$

$= m^2 + m + 2m + 2 - (27m - 15m^2 + 45 - 25m)$

$= m^2 + m + 2m + 2 - 27m + 15m^2 - 45 + 25m$

$= m^2 + 15m^2 + m + 2m - 27m + 25m + 2 - 45$

$= 16m^2 + m - 43$

3. **(a)** $a^2 = a \times a$

$36 = 1 \times 36$ or $(-1) \times (-36)$

$= 2 \times 18$ or $(-2) \times (-18)$

$= 3 \times 12$ or $(-3) \times (-12)$

$= 4 \times 9$ or $(-4) \times (-9)$

$= 6 \times 6$ or $(-6) \times (-6)$

$\times$	a	9
a	a^2	$9a$
4	$4a$	36

$4a + 9 = 13a$

$\therefore a^2 + 13a + 36 = (a + 4)(a + 9)$

(b) $b^2 = b \times b$

$56 = 1 \times 56$ or $(-1) \times (-56)$

$= 2 \times 28$ or $(-2) \times (-28)$

$= 4 \times 14$ or $(-4) \times (-14)$

$= 7 \times 8$ or $(-7) \times (-8)$

$\times$	b	-8
b	b^2	$-8b$
-7	$-7b$	56

$(-7b) + (-8b) = -15b$

$\therefore b^2 - 15b + 56 = (b - 7)(b - 8)$

(c) $c^2 = c \times c$

$-51 = 1 \times (-51)$ or $(-1) \times 51$

$= 3 \times (-17)$ or $(-3) \times 17$

$\times$	c	17
c	c^2	$17c$
-3	$-3c$	-51

$(-3c) + 17c = 14c$

$\therefore c^2 + 14c - 51 = (c - 3)(c + 17)$

(d) $d^2 = d \times d$

$-45 = 1 \times (-45)$ or $(-1) \times 45$

$= 3 \times (-15)$ or $(-3) \times 15$

$= 5 \times (-9)$ or $(-5) \times 9$

$\times$	d	-15
d	d^2	$-15d$
3	$3d$	-45

$3d + (-15d) = -15d$

$\therefore d^2 - 12d - 45 = (d + 3)(d - 15)$

4. (a) $9f^2 = 9f \times f$ or $3f \times 3f$

$-16 = 1 \times (-16)$ or $(-1) \times 16$

$= 2 \times (-8)$ or $(-2) \times 8$

$= 4 \times (-4)$ or $(-4) \times 4$

$\times$	$3f$	8
$3f$	$9f^2$	$24f$
-2	$-6f$	-16

$(-6f) + 24f = 18f$

$\therefore 9f^2 + 18f - 16 = (3f - 2)(3f + 8)$

(b) $3h^2 = 3h \times h$

$-14 = 1 \times (-14)$ or $(-1) \times 14$

$= 2 \times (-7)$ or $(-2) \times 7$

$\times$	h	-7
$3h$	$3h^2$	$-21h$
2	$2h$	-14

$2h + (-21h) = -19h$

$\therefore 3h^2 - 19h - 14 = (3h + 2)(h - 7)$

(c) $14k^2 + 49k + 21 = 7(2k^2 + 7k + 3)$

$2k^2 = 2k \times k$

$3 = 1 \times 3$ or $(-1) \times (-3)$

$\times$	k	3
$2k$	$2k^2$	$6k$
1	k	3

$k + 6k = 7k$

$\therefore 14k^2 + 49k + 21 = 7(2k + 1)(k + 3)$

(d) $18m^2 - 39m + 18 = 3(6m^2 - 13m + 6)$

$6m^2 = 6m \times m$ or $3m \times 2m$

$6 = 1 \times 6$ or $(-1) \times (-6)$

$= 2 \times 3$ or $(-2) \times (-3)$

$\times$	$2m$	-3
$3m$	$6m^2$	$-9m$
-2	$-4m$	6

$(-4m) + (-9m) = -13m$

$\therefore 18m^2 - 39m + 18 = 3(3m - 2)(2m - 3)$

5. $3x^2 - \frac{11}{2}x - 5 = \frac{1}{2}(6x^2 - 11x - 10)$

$6x^2 = 6x \times x$ or $3x \times 2x$

$-10 = 1 \times (-10)$ or $(-1) \times 10$

$= 2 \times (-5)$ or $(-2) \times 5$

$\times$	$2x$	-5
$3x$	$6x^2$	$-15x$
2	$4x$	-10

$4x + (-15x) = -11x$

$\therefore 3x^2 - \frac{11}{2}x - 5 = \frac{1}{2}(3x + 2)(2x - 5)$

Challenge Yourself

$n^2 = n \times n$

$45 = 1 \times 45$ or $(-1) \times (-45)$

$= 3 \times 15$ or $(-3) \times (-15)$

$= 5 \times 9$ or $(-5) \times (-9)$

$\times$	n	-15
n	n^2	$-15n$
-3	$-3n$	45

$(-3n) + (-15n) = -18n$

$\therefore n^2 - 18n + 45 = (n - 3)(n - 15)$ or $(3 - n)(15 - n)$

For $n^2 - 18n + 45$ to be a prime number,

$(n - 3)(n - 15)$ or $(3 - n)(15 - n)$ must be a prime number.

The factors of a prime number are 1 and itself.

$\therefore$ $n - 3 = 1$ or $n - 15 = 1$ or $3 - n = 1$ or $15 - n = 1$

$n = 4$ $\quad n = 16$ $\quad n = 2$ $\quad n = 14$

When $n = 4$,

$4^2 - 18(4) + 45 = -11$

When $n = 16$,

$16^2 - 18(16) + 45 = 13$

When $n = 2$,

$2^2 - 18(2) + 45 = 13$

When $n = 14$,

$14^2 - 18(14) + 45 = -11$

$\therefore n = 2$ or 16

Chapter 4 Further Expansion and Factorisation of Algebraic Expressions

TEACHING NOTES

Suggested Approach

The general form of a quadratic expression in one variable is $ax^2 + bx + c$, where x is the variable and a, b and c are given numbers. In the expression, c is known as the constant term as it does not involve the variable x. When we expand the product of two linear expressions in x, we obtain a quadratic expression in x.

Factorisation is the reverse of expansion. When we expand the product of two linear expressions, we obtain a quadratic expression. By reversing the process, we factorise the quadratic expression into a product of two linear factors.

Teachers can use the Concrete-Pictorial-Approach using the algebra discs to illustrate the process of expansion and factorisation of quadratic expressions. However, the emphasis should be for the students to use a Multiplication Frame when factorising any quadratic expressions.

Section 4.1: Expansion and Factorisation of Algebraic Expressions

In this section, students should have ample practice to expand and factorise slightly more difficult and complicated algebraic expressions. The focus for expansion of algebraic expressions should be on applying the Distributive Law while for factorisation of algebraic expressions, students should be using the Multiplication Frame.

Section 4.2: Expansion Using Special Algebraic Identities

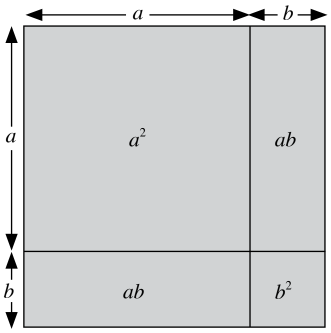
The area of squares and rectangles can be used to show the expansion of the three special algebraic identities. Teachers can also guide students to complete the Class Discussion on page 121 (see Class Discussion: Special Algebraic Identities).

From the Class Discussion activity, students should conclude that these algebraic identities known as **perfect squares**, $(a + b)^2$ and $(a - b)^2$ and the **difference of two squares** $(a + b)(a - b)$, are useful for expanding algebraic expressions.

Special Algebraic Identity 1

Expand $(a + b)^2$.

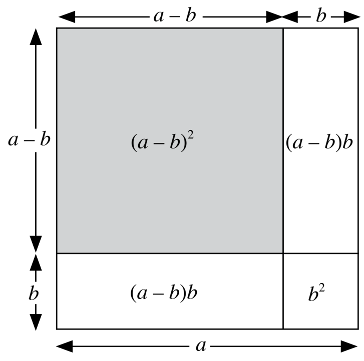
Area of square $= (a + b)^2$
$= a^2 + ab + ab + b^2$
$= a^2 + 2ab + b^2$



Special Algebraic Identity 2

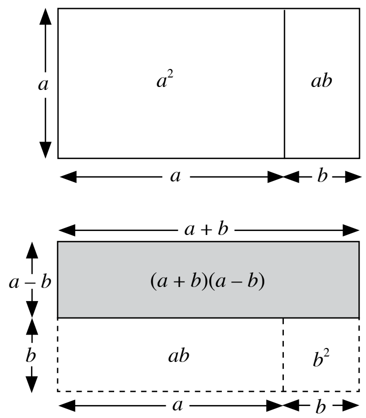
Expand $(a - b)^2$.

Area of small square
$= (a - b)^2$
$= a^2 - b^2 - 2(a - b)b$
$= a^2 - b^2 - 2ab + 2b^2$
$= a^2 - 2ab + b^2$



Special Algebraic Identity 3

Area of rectangle
$= (a + b)(a - b)$
$= (a^2 - ab) + (ab - b^2)$
$= a^2 - ab + ab - b^2$
$= a^2 - b^2$



As an additional activity, teachers may want to ask students the following:

Is $(a + b)^2 = a^2 + b^2$ and $(a - b)^2 = a^2 - b^2$? Explain your answer.

Below are some common misconceptions regarding expansion that teachers may want to remind students of.

- $(x + 2)^2 = x^2 + 4$ instead of $(x + 2)^2 = x^2 + 4x + 4$
- $(2x - 1)^2 = 4x^2 - 1$ instead of $(2x - 1)^2 = 4x^2 - 4x + 1$

Section 4.3 Factorisation Using Special Algebraic Identities

Since factorisation is the reverse of expansion, when we factorise the quadratic expression using the special algebraic identities, we have

- $a^2 + 2ab + b^2 = (a + b)^2$
- $a^2 - 2ab + b^2 = (a - b)^2$
- $a^2 - b^2 = (a + b)(a - b)$

Teachers should provide ample practice for students to check if the given quadratic expression can be factorised using the special algebraic identities. Get students to learn to identify the 'a' and 'b' terms in any given expression.

Section 4.4: Factorisation by Grouping

Students have learnt how to factorise algebraic expressions of the form $ax + ay$ by identifying the common factors (either common numbers or common variables of the terms) in Secondary One.

To factorise algebraic expressions of the form $ax + bx + kay + kby$, it may be necessary to regroup the terms of the algebraic expression before being able to identify the common factors. The idea is to identify the common factor(s) in the first two terms and another common factor(s) in the last 2 terms.

For example, to factorise by grouping, we have

$ax + bx + kay + kby$
$= x(a + b) + ky(a + b)$
$= (a + b)(x + ky)$

WORKED SOLUTIONS

Thinking Time (Page 118)

$(a + b)(c + d + e) = a(c + d + e) + b(c + d + e)$
$= ac + ad + ae + bc + bd + be$

Class Discussion (Special Algebraic Identities)

1. $(a + b)^2 = (a + b)(a + b)$
$= a(a + b) + b(a + b)$
$= a^2 + ab + ab + b^2$
$= a^2 + 2ab + b^2$

2. $(a - b)^2 = (a - b)(a - b)$
$= a(a - b) - b(a - b)$
$= a^2 - ab - ab + b^2$
$= a^2 - 2ab + b^2$

3. $(a + b)(a - b) = a(a - b) + b(a - b)$
$= a^2 - ab + ab - b^2$
$= a^2 - b^2$

Thinking Time (Page 130)

$5x^2 - 12x - 9 = 5x^2 - 15x + 3x - 9$
$= 5x(x - 3) + 3(x - 3)$
$= (5x + 3)(x - 3)$

Class Discussion (Equivalent Expressions)

$A = (x - y)^2 = (x - y)(x - y) = I$
$A = (x - y)^2 = x^2 - 2xy + y^2 = M$
$B = (x + y)(x + y) = (x + y)^2 = G$
$B = (x + y)(x + y) = x^2 + 2xy + y^2 = O$
$D = (2w - x)(z - 3y) = 2wz - 6wy + 3xy - xz = F$
$E = -5x^2 + 28x - 24 = 2x - (x - 4)(5x - 6) = L$
$J = x^2 - y^2 = (x + y)(x - y) = K$

Practise Now 1

1. (a) $5x \times 6y = (5 \times x) \times (6 \times y)$
$= (5 \times 6) \times x \times y$
$= 30xy$

(b) $(-8x) \times 2y = (-8 \times x) \times (2 \times y)$
$= (-8 \times 2) \times x \times y$
$= -16xy$

(c) $x^2yz \times y^2z = (x \times x \times y \times z) \times (y \times y \times z)$
$= (x \times x) \times (y \times y \times y) \times (z \times z)$
$= x^2y^3z^2$

(d) $(-xy) \times (-11x^3y^2)$
$= (-1 \times x \times y) \times (-11 \times x \times x \times x \times y \times y)$
$= [-1 \times (-11)] \times (x \times x \times x \times x) \times (y \times y \times y)$
$= 11x^4y^3$

2. $\frac{1}{2}a \times \left(-\frac{8}{3}b\right) = \left(\frac{1}{2} \times a\right) \times \left(-\frac{8}{3} \times b\right)$
$= \left[\frac{1}{2} \times \left(-\frac{8}{3}\right)\right] \times a \times b$
$= -\frac{4}{3}ab$

Practise Now 2

(a) $-y(5 - 2x) = -5y + 2xy$
(b) $2x(7x + 3y) = 14x^2 + 6xy$

Practise Now 3

(a) $4x(3y - 5z) - 5x(2y - 3z) = 12xy - 20xz - 10xy + 15xz$
$= 12xy - 10xy - 20xz + 15xz$
$= 2xy - 5xz$

(b) $x(2x - y) + 3x(y - 3x) = 2x^2 - xy + 3xy - 9x^2$
$= 2x^2 - 9x^2 - xy + 3xy$
$= -7x^2 + 2xy$

Practise Now 4

(a) $(x + 9y)(2x - y) = x(2x - y) + 9y(2x - y)$
$= 2x^2 - xy + 18xy - 9y^2$
$= 2x^2 + 17xy - 9y^2$

(b) $(x^2 - 3)(6x + 7) = x^2(6x + 7) - 3(6x + 7)$
$= 6x^3 + 7x^2 - 18x - 21$

Practise Now 5

$2x(3x - 4y) - (x - y)(x + 3y)$
$= 6x^2 - 8xy - [x(x + 3y) - y(x + 3y)]$
$= 6x^2 - 8xy - (x^2 + 3xy - xy - 3y^2)$
$= 6x^2 - 8xy - (x^2 + 2xy - 3y^2)$
$= 6x^2 - 8xy - x^2 - 2xy + 3y^2$
$= 6x^2 - x^2 - 8xy - 2xy + 3y^2$
$= 5x^2 - 10xy + 3y^2$

Practise Now 6

(a) $(x - 5y)(x + 4y - 1)$
$= x(x + 4y - 1) - 5y(x + 4y - 1)$
$= x^2 + 4xy - x - 5xy - 20y^2 + 5y$
$= x^2 + 4xy - 5xy - x - 20y^2 + 5y$
$= x^2 - xy - x - 20y^2 + 5y$

(b) $(x + 3)(x^2 - 7x - 2)$
$= x(x^2 - 7x - 2) + 3(x^2 - 7x - 2)$
$= x^3 - 7x^2 - 2x + 3x^2 - 21x - 6$
$= x^3 - 7x^2 + 3x^2 - 2x - 21x - 6$
$= x^3 - 4x^2 - 23x - 6$

Practise Now 7

1. **(a)** $x^2 = x \times x$
$-15y^2 = y \times (-15y)$ or $(-y) \times 15y$
$= 3y \times (-5y)$ or $(-3y) \times 5y$

$\times$	x	$-5y$
x	x^2	$-5xy$
$3y$	$3xy$	$-15y^2$

$3xy + (-5xy) = -2xy$
$\therefore x^2 - 2xy - 15y^2 = (x + 3y)(x - 5y)$

(b) $6x^2 = 6x \times x$ or $3x \times 2x$
$5y^2 = y \times 5y$ or $(-y) \times (-5y)$

$\times$	x	y
$6x$	$6x^2$	$6xy$
$5y$	$5xy$	$5y^2$

$5xy + 6xy = 11xy$
$\therefore 6x^2 + 11xy + 5y^2 = (6x + 5y)(x + y)$

2. $3x^2y^2 = 3xy \times xy$
$16 = 1 \times 16$ or $(-1) \times (-16)$
$= 2 \times 8$ or $(-2) \times (-8)$
$= 4 \times 4$ or $(-4) \times (-4)$

$\times$	xy	-2
$3xy$	$3x^2y^2$	$-6xy$
-8	$-8xy$	16

$(-8xy) + (-6xy) = -14xy$
$\therefore 3x^2y^2 - 14xy + 16 = (3xy - 8)(xy - 2)$

Practise Now 8

1. **(a)** $(x + 2)^2 = x^2 + 2(x)(2) + 2^2$
$= x^2 + 4x + 4$

(b) $(5x + 3)^2 = (5x)^2 + 2(5x)(3) + 3^2$
$= 25x^2 + 30x + 9$

2. $\left(\frac{1}{2}x + 3\right)^2 = \left(\frac{1}{2}x\right)^2 + 2\left(\frac{1}{2}x\right)(3) + 3^2$
$= \frac{1}{4}x^2 + 3x + 9$

Practise Now 9

1. **(a)** $(1 - 3x)^2 = 1^2 - 2(1)(3x) + (3x)^2$
$= 1 - 6x + 9x^2$

(b) $(2x - 3y)^2 = (2x)^2 - 2(2x)(3y) + (3y)^2$
$= 4x^2 - 12xy + 9y^2$

2. $\left(x - \frac{1}{3}y\right)^2 = x^2 - 2(x)\left(\frac{1}{3}y\right) + \left(\frac{1}{3}y\right)^2$
$= x^2 - \frac{2}{3}xy + \frac{1}{9}y^2$

Practise Now 10

1. **(a)** $(5x + 8)(5x - 8) = (5x)^2 - 8^2$
$= 25x^2 - 64$

(b) $(-2x + 7y)(-2x - 7y) = (-2x)^2 - (7y)^2$
$= 4x^2 - 49y^2$

2. $\left(\frac{x}{4} + y\right)\left(\frac{x}{4} - y\right) = \left(\frac{x}{4}\right)^2 - y^2$
$= \frac{1}{16}x^2 - y$

Practise Now 11

1. **(a)** $1001^2 = (1000 + 1)^2$
$= 1000^2 + 2(1000)(1) + 1^2$
$= 1\,000\,000 + 2000 + 1$
$= 1\,002\,001$

(b) $797^2 = (800 - 3)^2$
$= 800^2 - 2(800)(3) + 3^2$
$= 640\,000 - 4800 + 9$
$= 635\,209$

(c) $305 \times 295 = (305 + 5)(300 - 5)$
$= 300^2 - 5^2$
$= 90\,000 - 25$
$= 89\,975$

Practise Now 12

$(x - y)^2 = 441$
$x^2 - 2xy + y^2 = 441$
Since $xy = 46$,
$\therefore x^2 - 2(46) + y^2 = 441$
$x^2 - 92 + y^2 = 441$
$\therefore x^2 + y^2 = 533$

Practise Now 13

1. **(a)** $x^2 + 12x + 36 = x^2 + 2(x)(6) + 6^2$
$= (x + 6)^2$

(b) $4x^2 + 20x + 25 = (2x)^2 + 2(2x)(5) + 5^2$
$= (2x + 5)^2$

2. $4x^2 + 2x + \frac{1}{4} = (2x)^2 + 2(2x)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2$
$= \left(2x + \frac{1}{2}\right)^2$

Practise Now 14

1. **(a)** $4 - 36x + 81x^2 = 2^2 - 2(2)(9x) + (9x)^2$
$= (2 - 9x)^2$

(b) $25x^2 - 10xy + y^2 = (5x^2) - 2(5x)(1) + y^2$
$= (5x - y)^2$

2. $36x^2 - 4xy + \frac{1}{9}y^2 = (6x)^2 - 2(6x)\left(\frac{1}{3}y\right) + \left(\frac{1}{3}y\right)^2$
$= \left(6x - \frac{1}{3}y\right)^2$

Practise Now 15

1. **(a)** $36x^2 - 121y^2 = (6x)^2 - (11y)^2$
$= (6x + 11y)(6x - 11y)$

(b) $-4x^2 + 81 = 81 - 4x^2$
$= 9^2 - (2x)^2$
$= (9 + 2x)(9 - 2x)$

2. $4x^2 - \frac{9}{25}y^2 = (2x)^2 - \left(\frac{3}{5}y\right)^2$
$= \left(2x + \frac{3}{5}y\right)\left(2x - \frac{3}{5}y\right)$

3. $4(x + 1)^2 - 49 = [2(x + 1)]^2 - 7^2$
$= [2(x + 1) + 7][2(x + 1) - 7]$
$= (2x + 9)(2x - 5)$

Practise Now 16

$256^2 - 156^2 = (256 + 156)(256 - 156)$
$= 412 \times 100$
$= 41\ 200$

Practise Now 17

(a) $8x^2y + 4x = 4x(2xy + 1)$
(b) $\pi r^2 + \pi rl = \pi r(r + l)$
(c) $-a^3by + a^2y = a^2y(-ab + 1)$
(d) $3c^2d + 6c^2d^2 + 3c^3 = 3c^2(d + 2d^2 + c)$

Practise Now 18

(a) $2(x + 1) + a(1 + x) = (x + 1)(2 + a)$
(b) $9(x + 2) - b(x + 2) = (x + 2)(9 - b)$
(c) $3c(2x - 3) - 6d(2x - 3) = 3[c(2x - 3) - 2d(2x - 3)]$
$= 3(2x - 3)(c - 2d)$
(d) $7h(4 - x) - (x - 4) = 7h(4 - x) + (4 - x)$
$= (4 - x)(7h + 1)$

Practise Now 19

(a) $xy + 4x + 3y + 12 = x(y + 4) + 3(y + 4)$
$= (y + 4)(x + 3)$
(b) $3by + 4ax + 12ay + bx = 4ax + 12ay + bx + 3by$
$= 4a(x + 3y) + b(x + 3y)$
$= (x + 3y)(4a + b)$
(c) $x^3 - x^2 - 1 + x = x^3 - x^2 + x - 1$
$= x^2(x - 1) + (x - 1)$
$= (x - 1)(x^2 + 1)$
(d) $6xy - 4x - 2z + 3yz = 6xy - 4x + 3yz - 2z$
$= 2x(3y - 2) + z(3y - 2)$
$= (3y - 2)(2x + z)$

Exercise 4A

1. **(a)** $6x \times (-2y) = (6 \times x) \times (-2 \times y)$
$= [6 \times (-2)] \times x \times y$
$= -12xy$

(b) $14x \times \frac{1}{2}y = (14 \times x) \times \left(\frac{1}{2} \times y\right)$
$= \left(14 \times \frac{1}{2}\right) \times x \times y$
$= 7xy$

2. **(a)** $8x(y - 1) = 8xy - 8x$
(b) $-9x(3y - 2z) = -27xy + 18xz$
(c) $3x(2x + 7y) = 6x^2 + 21xy$
(d) $3y(x - 11y) = 3xy - 33y^2$
(e) $-3a(2a + 3b) = -6a^2 - 9ab$
(f) $-4c(2c - 5d) = -8c^2 + 20cd$
(g) $-6h(7k - 3h) = -42hk + 18h^2$
(h) $-8m(-12m - 7n) = 96m^2 + 56mn$
(i) $2p(3p + q + 7r) = 6p^2 + 2pq + 14pr$
(j) $-7s(s - 4t - 3u) = -7s^2 + 28st + 21su$

3. **(a)** $7a(3b - 4c) + 4a(3c - 2b) = 21ab - 28ac + 12ac - 8ab$
$= 21ab - 8ab - 28ac + 12ac$
$= 13ab - 16ac$
(b) $4d(d - 5f) + 2f(3d + 7f) = 4d^2 - 20df + 6df + 14f^2$
$= 4d^2 - 14df + 14f^2$

4. **(a)** $(x + y)(x + 6y) = x(x + 6y) + y(x + 6y)$
$= x^2 + 6xy + xy + 6y^2$
$= x^2 + 7xy + 6y^2$
(b) $(x^2 + 2)(x + 5) = x^2(x + 5) + 2(x + 5)$
$= x^3 + 5x^2 + 2x + 10$

5. **(a)** $\left(-\frac{3}{7}x\right) \times \frac{14}{9}y = \left(-\frac{3}{7} \times \frac{14}{9}\right) \times x \times y$
$= -\frac{2}{3}xy$
(b) $9x^3y \times 3x^2y^2 = (9 \times x \times x \times x \times y) \times (3 \times x \times x \times y \times y)$
$= (9 \times 3) \times (x \times x \times x \times x \times x) \times (y \times y \times y)$
$= 27x^5y^3$
(c) $2x^3y \times (-13y^2) = (2 \times x \times x \times x \times y) \times (-13 \times y \times y)$
$= [2 \times (-13)] \times (x \times x \times x) \times (y \times y \times y)$
$= -26x^3y^3$
(d) $(-4xyz) \times (-2x^2y^3z^4)$
$= (-4 \times x \times y \times z) \times (-2 \times x \times x \times y \times y \times y \times z \times z \times z \times z)$
$= [(-4) \times (-2)] \times (x \times x \times x) \times (y \times y \times y \times y) \times (z \times z \times z \times z \times z)$
$= 8x^3y^4z^5$

6. **(a)** $-3xy(x - 2y) = -3x^2y + 6xy^2$
(b) $9x(-3x^2y - 7xz) = -27x^3y - 63x^2z$
(c) $-13x^2y(3x - y) = -39x^3y + 13x^2y^2$
(d) $-5x(-6x - 4x^3y - 3y) = 30x^2 + 20x^4y + 15xy$

7. **(a)** $a(5b + c) - 2a(3c - b) = 5ab + ac - 6ac + 2ab$
$= 5ab + 2ab + ac - 6ac$
$= 7ab - 5ac$

(b) $-2d(4f - 5h) - f(3d + 7h)$
$= -8df + 10dh - 3df - 7fh$
$= -8df - 3df + 10dh - 7fh$
$= -11df + 10dh - 7fh$

(c) $4k(3k + m) - 3k(2k - 5m)$
$= 12k^2 + 4km - 6k^2 + 15km$
$= 12k^2 - 6k^2 + 4km + 15km$
$= 6k^2 + 19km$

(d) $2n(p - 2n) - 4n(n - 2p)$
$= 2np - 4n^2 - 4n^2 + 8np$
$= -4n^2 - 4n^2 + 2np + 8np$
$= -8n^2 + 10np$

8. **(a)** $(a + 3b)(a - b) = a(a - b) + 3b(a - b)$
$= a^2 - ab + 3ab - 3b^2$
$= a^2 + 2ab - 3b^2$

(b) $(3c + 7d)(c - 2d) = 3c(c - 2d) + 7d(c - 2d)$
$= 3c^2 - 6cd + 7cd - 14d^2$
$= 3c^2 + cd - 14d^2$

(c) $(3k - 5h)(-h - 7k) = 3k(-h - 7k) - 5h(-h - 7k)$
$= -3hk - 21k^2 + 5h^2 + 35hk$
$= - 21k^2 - 3hk + 35hk + 5h^2$
$= -21k^2 + 32hk + 5h^2$

(d) $(7m^2 + 2)(m - 4) = 7m^2(m - 4) + 2(m - 4)$
$= 7m^3 - 28m^2 + 2m - 8$

9. **(a)** $5x(x - 6y) + (x + 3y)(3x - 4y)$
$= 5x^2 - 30xy + x(3x - 4y) + 3y(3x - 4y)$
$= 5x^2 - 30xy + 3x^2 - 4xy + 9xy - 12y^2$
$= 5x^2 + 3x^2 - 30xy - 4xy + 9xy - 12y^2$
$= 8x^2 - 25xy - 12y^2$

(b) $(7x - 3y)(x - 4y) + (5x - 9y)(y - 2x)$
$= 7x(x - 4y) - 3y(x - 4y) + 5x(y - 2x) - 9y(y - 2x)$
$= 7x^2 - 28xy - 3xy + 12y^2 + 5xy - 10x^2 - 9y^2 + 18xy$
$= 7x^2 - 10x^2 - 28xy - 3xy + 5xy + 18xy + 12y^2 - 9y^2$
$= -3x^2 - 8xy + 3y^2$

10. **(a)** $(x + 9y)(x + 3y + 1)$
$= x(x + 3y + 1) + 9y(x + 3y + 1)$
$= x^2 + 3xy + x + 9xy + 27y^2 + 9y$
$= x^2 + x + 3xy + 9xy + 9y + 27y^2$
$= x^2 + x + 12xy + 9y + 27y^2$

(b) $(x + 2)(x^2 + x + 1)$
$= x(x^2 + x + 1) + 2(x^2 + x + 1)$
$= x^3 + x^2 + x + 2x^2 + 2x + 2$
$= x^3 + x^2 + 2x^2 + x + 2x + 2$
$= x^3 + 3x^2 + 3x + 2$

11. **(a)** $a^2 = a \times a$
$-4b^2 = b \times (-4b)$ or $(-b) \times 4b$
$= 2b \times (-2b)$ or $(-2b) \times 2b$

$\times$	a	$4b$
a	a^2	$4ab$
$-b$	$-ab$	$-4b^2$

$(-ab) + 4ab = 3ab$
$\therefore a^2 + 3ab - 4b^2 = (a - b)(a + 4b)$

(b) $c^2 = c \times c$
$-21d^2 = d \times (-21b)$ or $(-d) \times 21d$
$= 3d \times (-7d)$ or $(-3d) \times 7d$

$\times$	c	$-7d$
c	c^2	$-7cd$
$3d$	$3cd$	$-21d^2$

$3cd + (-7cd) = -4cd$
$\therefore c^2 - 4cd - 21d^2 = (c + 3d)(c - 7d)$

(c) $2h^2 = 2h \times h$
$-15k^2 = k \times (-15k)$ or $(-k) \times 15k$
$= 3k \times (-5k)$ or $(-3k) \times 5k$

$\times$	h	$5k$
$2h$	$2h^2$	$10hk$
$-3k$	$-3hk$	$-15k^2$

$(-3hk) + 10hk = 7hk$
$\therefore 2h^2 + 7hk - 15k^2 = (2h - 3k)(h + 5k)$

(d) $3m^2 = 3m \times m$
$-12n^2 = n \times (-12n)$ or $(-n) \times 12n$
$= 2n \times (-6n)$ or $(-2n) \times 6n$
$= 3n \times (-4n)$ or $(-3n) \times 4n$

$\times$	m	$-6n$
$3m$	$3m^2$	$-18mn$
$2n$	$2mn$	$-12n^2$

$2mn + (-18mn) = -16mn$
$\therefore 3m^2 - 16mn - 12n^2 = (3m + 2n)(m - 6n)$

(e) $3p^2 + 15pq + 18q^2 = 3(p^2 + 5pq + 6q^2)$
$p^2 = p \times p$
$6q^2 = q \times 6q$ or $(-q) \times (-6q)$
$= 2q \times 3q$ or $(-2q) \times (-3q)$

$\times$	p	$3q$
p	p^2	$3pq$
$2q$	$2pq$	$6p^2$

$2pq + 3pq = 5pq$
$\therefore 3p^2 + 15pq + 18q^2 = 3(p + 2q)(p + 3q)$

(f) $2r^2t - 9rst + 10s^2t = t(2r^2 - 9rs + 10s^2)$
$2r^2 = 2r \times r$
$10s^2 = s \times 10s$ or $(-s) \times (-10s)$
$= 2s \times 5s$ or $(-2s) \times (-5s)$

$\times$	r	$-2s$
$2r$	$2r^2$	$-4rs$
$-5s$	$-5rs$	$10s^2$

$(-5rs) + (-4rs) = -9rs$
$\therefore 2r^2t - 9rst + 10s^2t = t(2r - 5s)(r - 2s)$

12. $\left(\frac{1}{4}x^2y\right) \times \left(\frac{16}{5}yz^3\right) = \left(\frac{1}{4} \times x \times x \times y\right) \times \left(\frac{16}{5} \times y \times z \times z \times z\right)$

$= \left(\frac{1}{4} \times \frac{16}{5}\right) \times (x \times x) \times (y \times y) \times (z \times z \times z)$

$= \frac{4}{5}x^2y^2z^3$

13. (a) $(8x - y)(x + 3y) - (4x + y)(9y - 2x)$
$= 8x(x + 3y) - y(x + 3y) - [4x(9y - 2x) + y(9y - 2x)]$
$= 8x^2 + 24xy - xy - 3y^2 - (36xy - 8x^2 + 9y^2 - 2xy)$
$= 8x^2 + 24xy - xy - 3y^2 - 36xy + 8x^2 - 9y^2 + 2xy$
$= 8x^2 + 8x^2 + 24xy - xy - 36xy + 2xy - 3y^2 - 9y^2$
$= 16x^2 - 11xy - 12y^2$

(b) $(10x + y)(3x + 2y) - (5x - 4y)(-x - 6y)$
$= 10x(3x + 2y) + y(3x + 2y) - [5x(-x - 6y) - 4y(-x - 6y)]$
$= 30x^2 + 20xy + 3xy + 2y^2 -(-5x^2 - 30xy + 4xy + 24y^2)$
$= 30x^2 + 20xy + 3xy + 2y^2 + 5x^2 + 30xy - 4xy - 24y^2$
$= 30x^2 + 5x^2 + 20xy + 3xy + 30xy - 4xy + 2y^2 - 24y^2$
$= 35x^2 + 49xy - 22y^2$

14. (a) $(2x - 3y)(x + 5y - 2)$
$= 2x(x + 5y - 2) - 3y(x + 5y - 2)$
$= 2x^2 + 10xy - 4x - 3xy - 15y^2 + 6y$
$= 2x^2 - 4x + 10xy - 3xy + 6y - 15y^2$
$= 2x^2 - 4x + 7xy + 6y - 15y^2$

(b) $(x + 4)(x^2 - 5x + 7)$
$= x(x^2 - 5x + 7) + 4(x^2 - 5x + 7)$
$= x^3 - 5x^2 + 7x + 4x^2 - 20x + 28$
$= x^3 - 5x^2 + 4x^2 + 7x - 20x + 28$
$= x^3 - x^2 - 13x + 28$

(c) $(x - 1)(x^2 + 2x - 1)$
$= x(x^2 + 2x - 1) - (x^2 + 2x - 1)$
$= x^3 + 2x^2 - x - x^2 - 2x + 1$
$= x^3 + 2x^2 - x^2 - x - 2x + 1$
$= x^3 + x^2 - 3x + 1$

(d) $(3x^2 - 3x + 4)(3 - x)$
$= 3x^2 (3 - x) - 3x(3 - x) + 4(3 - x)$
$= 9x^2 - 3x^3 - 9x + 3x^2 + 12 - 4x$
$= -3x^3 + 9x^2 + 3x^2 - 9x - 4x + 12$
$= -3x^3 + 12x^2 - 13x + 12$

15. (a) $x^2y^2 = xy \times xy$
$-15 = 1 \times (-15)$ or $(-1) \times 15$
$= 3 \times (-5)$ or $(-3) \times 5$

$\times$	xy	5
xy	x^2y^2	$5xy$
-3	$-3xy$	-15

$(-3xy) + 5xy = 2xy$
$\therefore x^2y^2 + 2xy - 15 = (xy - 3)(xy + 5)$

(b) $12x^2y^2 = 12xy \times xy$ or $6xy \times 2xy$ or $4xy \times 3xy$
$-40 = 1 \times (-40)$ or $(-1) \times 40$
$= 2 \times (-20)$ or $(-2) \times 20$
$= 4 \times (-10)$ or $(-4) \times 10$
$= 5 \times (-8)$ or $(-5) \times 8$

$\times$	$3xy$	-8
$4xy$	$12x^2y^2$	$-32xy$
5	$15xy$	-40

$15xy + (-32xy) = -17xy$
$\therefore 12x^2y^2 - 17xy - 40 = (4xy + 5)(3xy - 8)$

(c) $4x^2y^2z - 22xyz + 24z = 2z(2x^2y^2 - 11xy + 12)$
$2x^2y^2 = 2xy \times xy$
$12 = 1 \times 12$ or $(-1) \times (-12)$
$= 2 \times 6$ or $(-2) \times (-6)$
$= 3 \times 4$ or $(-3) \times (-4)$

$\times$	xy	-4
$2xy$	$2x^2y^2$	$-8xy$
-3	$-3xy$	12

$(-3xy) + (-8xy) = -11xy$
$\therefore 4x^2y^2z - 22xyz + 24z = 2z(2xy - 3)(xy - 4)$

(d) $2x^2 + \frac{5}{3}xy - 2y^2 = \frac{1}{3}(6x^2 + 5xy + 6y^2)$
$6x^2 = 6x \times x$ or $3x \times 2x$
$-6y^2 = y \times (-6y)$ or $(-y) \times 6y$
$= 2y \times (-3y)$ or $(-2y) \times 3y$

$\times$	$2x$	$3y$
$3x$	$6x^2$	$9xy$
$-2y$	$-4xy$	12

$(-4xy) + 9xy = 5xy$
$\therefore 2x^2 + \frac{5}{3}xy - 2y^2 = \frac{1}{3}(3x - 2y)(2x + 3y)$

Exercise 4B

1. (a) $(a + 4)^2 = a^2 + 2(a)(4) + 4^2$
$= a^2 + 8a + 16$

(b) $(3b + 2)^2 = (3b)^2 + 2(3b)(2) + 2^2$
$= 9b^2 + 12b + 4$

(c) $(c + 4d)^2 = c^2 + 2(c)(4d) + (4d)^2$
$= c^2 + 8cd + 16d^2$

(d) $(9h + 2k)^2 = (9h)^2 + 2(9h)(2k) + (2k)^2$
$= 81h^2 + 36hk + 4k^2$

2. (a) $(m - 9)^2 = m^2 - 2(m)(9) + 9^2$
$= m^2 - 18m + 81$

(b) $(5n - 4)^2 = (5n)^2 - 2(5n)(4) + 4^2$
$= 25n^2 - 40n + 16$

(c) $(9 - 5p)^2 = 9^2 - 2(9)(5p) + (5p)^2$
$= 81 - 90p + 25p^2$

(d) $(3q - 8r)^2 = (3q)^2 - 2(3q)(8r) + (8r)^2$
$= 9q^2 - 48qr + 64r^2$

3. (a) $(s-5)(s+5) = s^2 - 5^2$
$= s^2 - 25$

(b) $(2t+11)(2t-11) = (2t)^2 - 11^2$
$= 4t^2 - 121$

(c) $(7+2u)(7-2u) = 7^2 - (2u)^2$
$= 49 - 4u^2$

(d) $(w-10x)(w+10x) = w^2 - (10x)^2$
$= w^2 - 100x^2$

4. (a) $1203^2 = (1200+3)^2$
$= 1200^2 + 2(1200)(3) + 3^2$
$= 1\,440\,000 + 7200 + 9$
$= 1\,447\,209$

(b) $892^2 = (900-8)^2$
$= 900^2 - 2(900)(8) + 8^2$
$= 810\,000 - 14\,400 + 64$
$= 795\,664$

(c) $1998 \times 2002 = (2000-2)(2000+2)$
$= 2000^2 - 2^2$
$= 4\,000\,000 - 4$
$= 3\,999\,996$

5. $(x-y)^2 = x^2 - 2xy + y^2$
$= x^2 + y^2 - 2xy$
Since $x^2 + y^2 = 80$ and $xy = 12$,
$\therefore (x-y)^2 = 80 - 2(12)$
$= 56$

6. $x^2 - y^2 = (x+y)(x-y)$
Since $x + y = 10$ and $x - y = -4$,
$\therefore x^2 - y^2 = 10 \times (-4)$
$= -40$

7. (a) $\left(\frac{1}{5}a + 3b\right)^2 = \left(\frac{1}{5}a\right)^2 + 2\left(\frac{1}{5}a\right)(3b) + (3b)^2$
$= \frac{1}{25}a^2 + \frac{6}{5}ab + 9b^2$

(b) $\left(\frac{1}{2}c + \frac{2}{3}d\right)^2 = \left(\frac{1}{2}c\right)^2 + 2\left(\frac{1}{2}c\right)\left(\frac{2}{3}d\right) + \left(\frac{2}{3}d\right)^2$
$= \frac{1}{4}c^2 + \frac{2}{3}cd + \frac{4}{9}d^2$

8. (a) $\left(\frac{3}{2}h - 5k\right)^2 = \left(\frac{3}{2}h\right)^2 - 2\left(\frac{3}{2}h\right)(5k) + (5k)^2$
$= \frac{9}{4}h^2 - 15hk + 25k^2$

(b) $\left(-\frac{6}{5}m - 3n\right)^2 = \left(-\frac{6}{5}m\right)^2 - 2\left(-\frac{6}{5}m\right)(3n) + (3n)^2$
$= \frac{36}{25}m^2 + \frac{36}{5}mn + 9n^2$

9. (a) $(6p+5)(5-6p) = (5+6p)(5-6p)$
$= 5^2 - (6p)^2$
$= 25 - 36p^2$
$= -36p^2 + 25$

(b) $\left(9r - \frac{4}{5}q\right)\left(9r + \frac{4}{5}q\right) = (9r)^2 - \left(\frac{4}{5}q\right)^2$
$= 81r^2 - \frac{16}{25}q^2$

(c) $\left(\frac{s}{2} + \frac{t}{3}\right)\left(\frac{t}{3} - \frac{s}{2}\right) = \left(\frac{t}{3} + \frac{s}{2}\right)\left(\frac{t}{3} - \frac{s}{2}\right)$
$= \left(\frac{t}{3}\right)^2 - \left(\frac{s}{2}\right)^2$
$= \frac{t^2}{9} - \frac{s^2}{4}$
$= -\frac{s^2}{4} + \frac{t^2}{9}$

(d) $(u+2)(u-2)(u^2+4) = (u^2 - 2^2)(u^2+4)$
$= (u^2-4)(u^2+4)$
$= (u^2)^2 - 4^2$
$= u^4 - 16$

10. (a) $4(x+3)^2 - 3(x+4)(x-4)$
$= 4[x^2 + 2(x)(3) + 3^2] - 3(x^2 - 4^2)$
$= 4(x^2 + 6x + 9) - 3(x^2 - 16)$
$= 4x^2 + 24x + 36 - 3x^2 + 48$
$= 4x^2 - 3x^2 + 24x + 36 + 48$
$= x^2 + 24x + 84$

(b) $(5x-7y)(5x+7y) - 2(x-2y)^2$
$= (5x)^2 - (7y)^2 - 2[x^2 - 2(x)(2y) + (2y)^2]$
$= 25x^2 - 49y^2 - 2(x^2 - 4xy + 4y^2)$
$= 25x^2 - 49y^2 - 2x^2 + 8xy - 8y^2$
$= 25x^2 - 2x^2 + 8xy - 49y^2 - 8y^2$
$= 23x^2 + 8xy - 57y^2$

11. $\left(\frac{1}{2}x + \frac{1}{2}y\right)^2 = \left(\frac{1}{2}x\right)^2 + 2\left(\frac{1}{2}x\right)\left(\frac{1}{2}y\right) + \left(\frac{1}{2}y\right)^2$
$= \frac{1}{4}x^2 + \frac{1}{2}xy + \frac{1}{4}y^2$
$= \frac{1}{4}x^2 + \frac{1}{4}y^2 + \frac{1}{2}xy$
$= \frac{1}{4}(x^2 + y^2) + \frac{1}{2}xy$
Since $x^2 + y^2 = 14$ and $xy = 5$,
$\therefore \left(\frac{1}{2}x + \frac{1}{2}y\right)^2 = \frac{1}{4}(14) + \frac{1}{2}(5)$
$= 6$

12. $2x^2 - 2y^2 = 125$
$2(x^2 - y^2) = 125$
$2(x+y)(x-y) = 125$
Since $x - y = 2.5$,
$\therefore 2(x+y)(2.5) = 125$
$5(x+y) = 125$
$x + y = 25$

13. $\left(\frac{1}{16}x^2 + \frac{1}{25}y^2\right)\left(\frac{1}{4}x + \frac{1}{5}y\right)\left(\frac{1}{4}x - \frac{1}{5}y\right)$
$= \left(\frac{1}{16}x^2 + \frac{1}{25}y^2\right)\left[\left(\frac{1}{4}x\right)^2 - \left(\frac{1}{5}y\right)^2\right]$
$= \left(\frac{1}{16}x^2 + \frac{1}{25}y^2\right)\left(\frac{1}{16}x^2 - \frac{1}{25}y^2\right)$
$= \left(\frac{1}{16}x^2\right)^2 - \left(\frac{1}{25}y^2\right)^2$
$= \frac{1}{256}x^4 - \frac{1}{625}y^4$

14. **(i)** $(p-2q)^2 - p(p-4q)$
$= p^2 - 2(p)(2q) + (2q)^2 - p^2 + 4pq$
$= p^2 - 4pq + 4q^2 - p^2 + 4pq$
$= p^2 - p^2 - 4pq + 4pq + 4q^2$
$= 4q^2$

(ii) Let $p = 5330$ and $q = 10$,
$5310^2 - 5330 \times 5290$
$= [5330 - 2(10)]^2 - 5330[5330 - 4(10)]$
$= 4(10)^2$ (From **(i)**)
$= 400$

15. **(i)** $n^2 - (n-a)(n+a) = n^2 - (n^2 - a^2)$
$= n^2 - n^2 + a^2$
$= a^2$

(ii) Let $n = 16\,947$ and $a = 3$,
$16\,947^2 - 16\,944 \times 16\,950$
$= 16\,947^2 - (16\,947 - 3)(16\,947 + 3)$
$= 3^2$ (From **(i)**)
$= 9$

Exercise 4C

1. **(a)** $a^2 + 14a + 49 = a^2 + 2(a)(7) + 7^2$
$= (a+7)^2$

(b) $4b^2 + 4b + 1 = (2b)^2 + 2(2b)(1) + 1^2$
$= (2b+1)^2$

(c) $c^2 + 2cd + d^2 = (c+d)^2$

(d) $4h^2 + 20hk + 25k^2 = (2h)^2 + 2(2h)(5k) + (5k)^2$
$= (2h+5k)^2$

2. **(a)** $m^2 - 10m + 25 = m^2 - 2(m)(5) + 5^2$
$= (m-5)^2$

(b) $169n^2 - 52n + 4 = (13n)^2 - 2(13n)(2) + 2^2$
$= (13n-2)^2$

(c) $81 - 180p + 100p^2 = 9^2 - 2(9)(10p) + (10p)^2$
$= (9-10p)^2$

(d) $49q^2 - 42qr + 9r^2 = (7q)^2 - 2(7q)(3r) + (3r)^2$
$= (7q-3r)^2$

3. **(a)** $s^2 - 144 = s^2 - 12^2$
$= (s+12)(s-12)$

(b) $36t^2 - 25 = (6t)^2 - 5^2$
$= (6t+5)(6t-5)$

(c) $225 - 49u^2 = 15^2 - (7u)^2$
$= (15+7u)(15-7u)$

(d) $49w^2 - 81x^2 = (7w)^2 - (9x)^2$
$= (7w+9x)(7w-9x)$

4. **(a)** $59^2 - 41^2 = (59+41)(59-41)$
$= 100 \times 18$
$= 1800$

(b) $7.7^2 - 2.3^2 = (7.7+2.3)(7.7-2.3)$
$= 10 \times 5.4$
$= 54$

5. **(a)** $3a^2 + 12a + 12 = 3(a^2 + 4a + 4)$
$= 3[a^2 + 2(a)(2) + 2^2]$
$= 3(a+2)^2$

(b) $25b^2 + 5bc + \frac{1}{4}c^2 = (5b)^2 + 2(5b)\left(\frac{1}{2}c\right) + \left(\frac{1}{2}c\right)^2$
$= \left(5b + \frac{1}{2}c\right)^2$

(c) $\frac{16}{49}d^2 + \frac{8}{35}df + \frac{1}{25}f^2 = \left(\frac{4}{7}d\right)^2 + 2\left(\frac{4}{7}d\right)\left(\frac{1}{5}f\right) + \left(\frac{1}{5}f\right)^2$
$= \left(\frac{4}{7}d + \frac{1}{5}f\right)^2$

(d) $h^4 + 2h^2k + k^2 = (h^2)^2 + 2(h^2)(k) + k^2$
$= (h^2 + k)^2$

6. **(a)** $36m^2 - 48mn + 16n^2 = 4(9m^2 - 12mn + 4n^2)$
$= 4[(3m)^2 - 2(3m)(2n) + (2n)^2]$
$= 4(3m-2n)^2$

(b) $\frac{1}{3}p^2 - \frac{2}{3}pq + \frac{1}{3}q^2 = \frac{1}{3}(p^2 - 2pq + q^2)$
$= \frac{1}{3}[p^2 - 2(p)(q) + q^2]$
$= \frac{1}{3}(p-q)^2$

(c) $16r^2 - rs + \frac{1}{64}s^2 = (4r)^2 - 2(4r)\left(\frac{1}{8}s\right) + \left(\frac{1}{8}s\right)^2$
$= \left(4r - \frac{1}{8}s\right)^2$

(d) $25 - 10tu + t^2u^2 = 5^2 - 2(5)(tu) + (tu)^2$
$= (5-tu)^2$

7. **(a)** $32a^2 - 98b^2 = 2(16a^2 - 49b^2)$
$= 2[(4a)^2 - (7b)^2]$
$= 2(4a+7b)(4a-7b)$

(b) $c^2 - \frac{1}{4}d^2 = c^2 - \left(\frac{1}{2}d\right)^2$
$= \left(c + \frac{1}{2}d\right)\left(c - \frac{1}{2}d\right)$

(c) $\frac{9h^2}{100} - 16k^2 = \left(\frac{3h}{10}\right)^2 - (4k)^2$
$= \left(\frac{3h}{100} + 4k\right)\left(\frac{3h}{100} - 4k\right)$

(d) $m^2 - 64n^4 = m^2 - (8n)^2$
$= (m+8n)(m-8n)$

8. **(a)** $(a+3)^2 - 9 = (a+3)^2 - 3^2$
$= [(a+3)+3][(a+3)-3]$
$= a(a+6)$

(b) $16 - 25(b+3)^2 = -\{[5(b+3)]^2 - 4^2\}$
$= -[5(b+3)+4][5(b+3)-4]$
$= -(5b+19)(5b+11)$

(c) $c^2 - (d+2)^2 = [c+(d+2)][c-(d+2)]$
$= (c+d+2)(c-d-2)$

(d) $(2h-1)^2 - 4k^2 = (2h-1)^2 - (2k)^2$
$= (2h-1+2k)(2h-1-2k)$

(e) $25m^2 - (n-1)^2 = (5m)^2 - (n-1)^2$
$= [5m+(n-1)][5m-(n-1)]$
$= (5m+n-1)(5m-n+1)$

(f) $(p + 1)^2 - (p - 1)^2 = [(p + 1) + (p - 1)][(p + 1) - (p - 1)]$
$= 2p(2)$
$= 4p$

9. (i) Let the length of the cube be l cm.
$l^2 = x^2 + 4x + 4$
$= x^2 + 2(x)(2) + 2^2$
$= (x + 2)^2$
$l = \sqrt{(x+2)^2} \quad (l > 0)$
$= x + 2$
$\therefore$ The length of the cube is $(x + 2)$ cm.

(ii) Volume of the cube
$= l^3$
$= l(l^2)$
$= (x + 2)(x^2 + 4x + 4)$
$= x(x^2 + 4x + 4) + 2(x^2 + 4x + 4)$
$= x^3 + 4x^2 + 4x + 2x^2 + 8x + 8$
$= x^3 + 4x^2 + 2x^2 + 4x + 8x + 8$
$= (x^3 + 6x^2 + 12x + 8)$ cm^3
$\therefore$ The volume of the cube is $(x^3 + 6x^2 + 12x + 8)$ cm^3.

10. (a) $4(x - 1)^2 - 81(x + 1)^2$
$= [2(x - 1)]^2 - [9(x + 1)]^2$
$= [2(x - 1) + 9(x + 1)][2(x - 1) - 9(x + 1)]$
$= (2x - 2 + 9x + 9)(2x - 2 - 9x - 9)$
$= (11x + 7)(-7x - 11)$
$= -(11x + 7)(7x + 11)$

(b) $16x^2 + 8x + 1 - 9y^2$
$= [(4x)^2 + 2(4x)(1) + 1^2] - (3y)^2$
$= (4x + 1)^2 - (3y)^2$
$= (4x + 1 + 3y)(4x + 1 - 3y)$

(c) $4x^2 - y^2 + 4y - 4$
$= 4x^2 - (y^2 - 4y + 4)$
$= (2x)^2 - [y^2 - 2(y)(2) + 2^2]$
$= (2x)^2 - (y - 2)^2$
$= [2x + (y - 2)][2x - (y - 2)]$
$= 2(x + y - 2)(2x - y + 2)$

(d) $13x^2 + 26xy + 13y^2 - 13$
$= 13(x^2 + 2xy + y^2 - 1)$
$= 13\{[x^2 + 2(x)(y) + y^2] - 1^2\}$
$= 13[(x + y)^2 - 1^2]$
$= 13(x + y + 1)(x + y - 1)$

Exercise 4D

1. (a) $45x^2 - 81xy = 9x(5x - 9y)$
(b) $39xy - 15x^2z = 3x(13y - 5xz)$
(c) $xy^2z^2 - x^2y^3 = xy^2(z^2 - xy)$
(d) $-15\pi x^3y - 10\pi x^3 = -5\pi x^3(3y + 2)$

2. (a) $6a(x - 2y) + 5(x - 2y) = (x - 2y)(6a + 5)$
(b) $2b(x + 3y) - c(3y + x) = 2b(x + 3y) - c(x + 3y)$
$= (x + 3y)(2b - c)$
(c) $3d(5x - y) - 4f(5x - y) = (5x - y)(3d - 4f)$
(d) $5h(x + 3y) + 10k(x + 3y) = 5[h(x + 3y) + 2k(x + 3y)]$
$= 5(x + 3y)(h + 2k)$

3. (a) $ax - 5a + 4x - 20 = a(x - 5) + 4(x - 5)$
$= (x - 5)(a + 4)$
(b) $ax + bx + ay + by = x(a + b) + y(a + b)$
$= (a + b)(x + y)$
(c) $x + xy + 2y + 2y^2 = x(1 + y) + 2y(1 + y)$
$= (1 + y)(x + 2y)$
(d) $x^2 - 3x + 2xy - 6y = x(x - 3) + 2y(x - 3)$
$= (x - 3)(x + 2y)$

4. (a) $(x + y)(a + b) - (y + z)(a + b)$
$= (a + b)[(x + y) - (y + z)]$
$= (a + b)(x + y - y - z)$
$= (a + b)(x - z)$

(b) $(c + 2d)^2 - (c + 2d)(3c - 7d)$
$= (c + 2d)[(c + 2d) - (3c - 7d)]$
$= (c + 2d)(c + 2d - 3c + 7d)$
$= (c + 2d)(-2c + 9d)$

(c) $x(2h - k) + 3y(k - 2h)$
$= x(2h - k) - 3y(2h - k)$
$= (2h - k)(x - 3y)$

(d) $6x(4m - n) - 2y(n - 4m)$
$= 2[3x(4m - n) - y(n - 4m)]$
$= 2[3x(4m - n) + y(4m - n)]$
$= 2(4m - n)(3x + y)$

5. (a) $3ax + 28by + 4ay + 21bx$
$= 3ax + 4ay + 21bx + 28by$
$= a(3x + 4y) + 7b(3x + 4y)$
$= (3x + 4y)(a + 7b)$

(b) $12cy + 20c - 15 - 9y$
$= 4c(3y + 5) - 3(5 + 3y)$
$= 4c(3y + 5) - 3(3y + 5)$
$= (3y + 5)(4c - 3)$

(c) $dy + fy - fz - dz = y(d + f) - z(f + d)$
$= y(d + f) - z(d + f)$
$= (d + f)(y - z)$

(d) $3x^2 + 6xy - 4xz - 8yz$
$= 3x(x + 2y) - 4z(x + 2y)$
$= (x + 2y)(3x - 4z)$

(e) $2xy - 8x + 12 - 3y = 2x(y - 4) + 3(4 - y)$
$= 2x(y - 4) - 3(y - 4)$
$= (y - 4)(2x - 3)$

(f) $5xy - 25x^2 + 50x - 10y$
$= 5(xy - 5x^2 + 10x - 2y)$
$= 5[x(y - 5x) + 2(5x - y)]$
$= 5[x(y - 5x) - 2(y - 5x)]$
$= 5(y - 5x)(x - 2)$

(g) $x^2y^2 - 5x^2y - 5xy^2 + xy^3$
$= xy(xy - 5x - 5y + y^2)$
$= xy[x(y - 5) + y(-5 + y)]$
$= xy[x(y - 5) + y(y - 5)]$
$= xy(y - 5)(x + y)$

(h) $kx + hy - hx - ky = kx - hx - ky + hy$
$= x(k - h) + y(-k + h)$
$= x(k - h) - y(k - h)$
$= (k - h)(x - y)$

6. **(a)** $144p(y - 5x^2) - 12q(10x^2 - 2y)$
$= 144p(y - 5x^2) + 24q(y - 5x^2)$
$= 24[6p(y - 5x^2) + q(y - 5x^2)]$
$= 24(y - 5x^2)(6p + q)$

(b) $2(5x + 10y)(2y - x)^2 - 4(6y + 3x)(x - 2y)$
$= 10(x + 2y)(2y - x^2) - 12(2y + x)(x - 2y)$
$= 10(x + 2y)(x - 2y)^2 - 12(x + 2y)(x - 2y)$
$= 2(x + 2y)(x - 2y)[5(x - 2y) - 6]$
$= 2(x + 2y)(x - 2y)(5x - 10y - 6)$

7. **(i)** $\frac{1}{3}p^2q + \frac{4}{3}p^2r = \frac{1}{3}p^2(q + 4r)$

(ii) When $p = 1.2$, $q = 36$ and $r = 16$,
$\frac{1}{3} \times 1.2^2 \times 36 + \frac{4}{3} \times 1.2^2 \times 16$
$= \frac{1}{3}(1.2)^2[36 + 4(16)]$
$= \frac{1}{3}(1.44)(36 + 64)$
$= \frac{1}{3}(1.44)(100)$
$= \frac{1}{3} \times 144$
$= 48$

8. **(i)** $x^3 + 3x - x^2 - 3$
$= x(x^2 + 3) - (x^2 + 3)$
$= (x^2 + 3)(x - 1)$

(ii) $(x^2 - 3)^3 - (2 - x^2)^2 + 3(x^2 - 3)$
$= (x^2 - 3)^3 + 3(x^2 - 3) - (x^2 - 2)^2$
$= (x^2 - 3)^3 + 3(x^2 - 3) - [(x^2 - 3) + 1]^2$
$= (x^2 - 3)^3 + 3(x^2 - 3) - [(x^2 - 3)^2 + 2(x^2 - 3) + 1]$
$= (x^2 - 3)^3 + 3(x^2 - 3) - (x^2 - 3)^2 - 2(x^2 - 3) - 1$
$= (x^2 - 3)^3 + (x^2 - 3) - (x^2 - 3)^2 - 1$
$= (x^2 - 3)^3 + 3(x^2 - 3) - (x^2 - 3)^2 - 3 - 2(x^2 - 3) + 2$
$= [(x^2 - 3)^3 + 3(x^2 - 3) - (x^2 - 3)^2 - 3] - 2[(x^2 - 3) - 1]$
$= [(x^2 - 3)^2 + 3](x^2 - 3 - 1) - 2(x^2 - 4)$ (From **(i)**)
$= [(x^2)^2 - 2(x^2)(3) + 3^2 + 3](x^2 - 4) - 2(x^2 - 4)$
$= (x^4 - 6x^2 + 12)(x^2 - 4) - 2(x^2 - 4)$
$= (x^4 - 6x^2 + 12 - 2)(x^2 - 4)$
$= (x^4 - 6x^2 + 10)(x^2 - 4)$

Review Exercise 4

1. **(a)** $-2a(a - 5b + 7) = -2a^2 + 10ab - 14a$

(b) $(2c + 3d)(3c + 4d) = 2c(3c + 4d) + 3d(3c + 4d)$
$= 6c^2 + 8cd + 9cd + 12d^2$
$= 6c^2 + 17cd + 12d^2$

(c) $(k + 3h)(5h - 4k) = k(5h - 4k) + 3h(5h - 4k)$
$= 5hk - 4k^2 + 15h^2 - 12hk$
$= -4k^2 + 5hk - 12hk + 15h^2$
$= -4k^2 - 7hk + 15h^2$

(d) $(2m + 1)(m^2 + 3m - 1) = 2m(m^2 + 3m - 1) + (m^2 + 3m - 1)$
$= 2m^3 + 6m^2 - 2m + m^2 + 3m - 1$
$= 2m^3 + 6m^2 + m^2 - 2m + 3m - 1$
$= 2m^3 + 7m^2 + m - 1$

2. **(a)** $2p(3p - 5q) - q(2q - 3p) = 6p^2 - 10pq - 2q^2 + 3pq$
$= 6p^2 - 10pq + 3pq - 2q^2$
$= 6p^2 - 7pq - 2q^2$

(b) $-4s(3s + 4r) - 2r(2r - 5s) = -12s^2 - 16sr - 4r^2 + 10sr$
$= -12s^2 - 16sr + 10sr - 4r^2$
$= -12s^2 - 6sr - 4r^2$

(c) $(8t - u)(t + 9u) - t(2u - 7t) = 8t(t + 9u) - u(t + 9u) - t(2u - 7t)$
$= 8t^2 + 72tu - tu - 9u^2 - 2tu + 7t^2$
$= 8t^2 + 7t^2 + 72tu - tu - 2tu - 9u^2$
$= 15t^2 + 69tu - 9u^2$

(d) $(2w + 3x)(w - 5x) - (3w + 7x)(w - 7x)$
$= 2w(w - 5x) + 3x(w - 5x) - [3w(w - 7x) + 7x(w - 7x)]$
$= 2w^2 - 10wx + 3wx - 15x^2 - (3w^2 - 21wx + 7wx - 49x^2)$
$= 2w^2 - 10wx + 3wx - 15x^2 - 3w^2 + 21wx - 7wx + 49x^2$
$= 2w^2 - 3w^2 - 10wx + 3wx + 21wx - 7wx - 15x^2 + 49x^2$
$= -w^2 + 7wx - 34x^2$

3. **(a)** $x^2 = x \times x$
$-63y^2 = y \times (-63y)$ or $(-y) \times 63y$
$= 3y \times (-21y)$ or $(-3y) \times 21y$
$= 7y \times (-9y)$ or $(-7y) \times 9y$

$\times$	x	$9y$
x	x^2	$9xy$
$-7y$	$-7xy$	$-63y^2$

$(-7xy) + 9xy = 2xy$
$\therefore x^2 + 2xy - 63y^2 = (x - 7y)(x + 9y)$

(b) $2x^2 = 2x \times x$
$3y^2 = y \times 3y$ or $(-y) \times (-3y)$

$\times$	x	y
$2x$	$2x^2$	$2xy$
$3y$	$3xy$	$3y^2$

$3xy + 2xy = 5xy$
$\therefore 2x^2 + 5xy + 3y^2 = (2x + 3y)(x + y)$

(c) $6x^2y^2 = 6xy \times xy$ or $3xy \times 2xy$
$-4 = 1 \times (-4)$ or $(-1) \times 4$
$= 2 \times (-2)$ or $(-2) \times 2$

$\times$	$2xy$	1
$3xy$	$6x^2y^2$	$3xy$
-4	$-8xy$	-4

$(-8xy) + 3xy = -5xy$
$\therefore 6x^2y^2 - 5xy - 4 = (3xy - 4)(2xy + 1)$

(d) $3z - 8xyz + 4x^2y^2z = z(3 - 8xy + 4x^2y^2)$

$3 = 3 \times 1$

$4x^2y^2 = xy \times 4xy$ or $(-xy) \times (4xy)$

$= 2xy \times 2xy$ or $(-2xy) \times (-2xy)$

$\times$	1	$-2xy$
3	3	$-6xy$
$-2xy$	$-2xy$	$4x^2y^2$

$(-2xy) + (-6xy) = -8xy$

$\therefore 3z - 8xyz + 4x^2y^2z = z(3 - 2xy)(1 - 2xy)$

4. (a) $(-x + 5y)^2 = (-x)^2 + 2(-x)(5y) + (5y)^2$

$= x^2 - 10xy + 25y^2$

(b) $(x^2 + y)(x^2 - y) = (x^2)^2 - y^2$

$= x^4 - y^2$

(c) $\left(3x + \frac{4}{5}y\right)^2 = (3x)^2 + 2(3x)\left(\frac{4}{5}y\right) + \left(\frac{4}{5}y\right)^2$

$= 9x^2 + \frac{24}{5}xy + \frac{16}{25}y^2$

(d) $\left(-\frac{1}{4}x - \frac{1}{6}y\right)^2 = \left(-\frac{1}{4}x\right)^2 + 2\left(-\frac{1}{4}x\right)\left(-\frac{1}{6}y\right) + \left(-\frac{1}{6}y\right)^2$

$= \frac{1}{16}x^2 + \frac{1}{12}xy + \frac{1}{36}y^2$

(e) $\left(5x - \frac{7}{4}y\right)\left(5x + \frac{7}{4}y\right) = (5x)^2 - \left(\frac{7}{4}y\right)^2$

$= 25x^2 - \frac{49}{16}y^2$

(f) $\left(\frac{3}{4}xy + \frac{1}{3}z\right)\left(\frac{3}{4}xy - \frac{1}{3}z\right) = \left(\frac{3}{4}xy\right)^2 - \left(\frac{1}{3}z\right)^2$

$= \frac{9}{16}x^2y^2 - \frac{1}{9}z^2$

5. (a) $1 - 121x^2 = 1^2 - (11x)^2$

$= (1 + 11x)(1 - 11x)$

(b) $x^2 + 6xy + 9y^2 = x^2 + 2(x)(3y) + (3y)^2$

$= (x + 3y)^2$

(c) $25x^2 - 100xy + 100y^2 = 25(x^2 - 4xy + 4y^2)$

$= 25[x^2 - 2(x)(2y) + (2y)^2]$

$= 25(x - 2y)^2$

(d) $36y^2 - 49(x + 1)^2 = (6y)^2 - [7(x + 1)]^2$

$= [6y + 7(x + 1)][6y - 7(x + 1)]$

$= (6y + 7x + 7)(6y - 7x - 7)$

6. (a) $-14xy - 21y^2 = -7y(2x + 3y)$

(b) $9xy^2 - 36x^2y = 9xy(y - 4x)$

(c) $(2x - 3y)(a + b) + (x - y)(b + a)$

$= (2x - 3y)(a + b) + (x - y)(a + b)$

$= (a + b)(2x - 3y + x - y)$

$= (a + b)(3x - 4y)$

(d) $5(x - 2y) - (x - 2y)^2 = (x - 2y)[5 - (x - 2y)]$

$= (x - 2y)(5 - x + 2y)$

(e) $x^2 + 3xy + 2x + 6y = x(x + 3y) + 2(x + 3y)$

$= (x + 3y)(x + 2)$

(f) $3x^3 - 2x^2 + 3x - 2 = x^2(3x - 2) + (3x - 2)$

$= (3x - 2)(x^2 + 1)$

(g) $4cx - 6cy - 8dx + 12dy = 2(2cx - 3cy - 4dx + 6dy)$

$= 2[c(2x - 3y) - 2d(2x - 3y)]$

$= 2(2x - 3y)(c - 2d)$

(h) $5xy - 10x - 12y + 6y^2 = 5x(y - 2) + 6y(-2 + y)$

$= 5x(y - 2) + 6y(y - 2)$

$= (y - 2)(5x + 6y)$

7. $x^3 + x^2 - 4x - 4 = x^2(x + 1) - 4(x + 1)$

$= (x + 1)(x^2 - 4)$

$= (x + 1)(x^2 - 2^2)$

$= (x + 1)(x + 2)(x - 2)$

8. (a) $899^2 = (900 - 1)^2$

$= 900^2 - 2(900) + 1^2$

$= 810\,000 - 1800 + 1$

$= 808\,201$

(b) $659^2 - 341^2 = (659 + 341)(659 - 341)$

$= 1000 \times 318$

$= 318\,000$

9. $2(x - y)^2 = 116$

$(x - y)^2 = 58$

$x^2 - 2xy + y^2 = 58$

Since $xy = 24$,

$\therefore x^2 - 2(24) + y^2 = 58$

$x^2 - 48 + y^2 = 58$

$x^2 + y^2 = 106$

10. (i) $(f + 3)^2 = f^2 + 2(f)(3) + 3^2$

$= f^2 + 6f + 9$

(ii) From **(i)**,

$[(2h + k) + 3]^2 = (2h + k)^2 + 6(2h + k) + 9$

$= (2h)^2 + 2(2h)(k) + k^2 + 12h + 6k + 9$

$= 4h^2 + 4hk + k^2 + 12h + 6k + 9$

Challenge Yourself

1. $(a + b)^2 = a^2 + b^2$

$a^2 + 2ab + b^2 = a^2 + b^2$

$2ab = 0$

$ab = 0$

$\therefore \sqrt{ab} = 0$

2. Let $a = h^2 + k^2$ and $b = m^2 + n^2$.

$h^2 + k^2 - m^2 - n^2 = 15$

$h^2 + k^2 - (m^2 + n^2) = 15$

$a - b = 15$

$(h^2 + k^2)^2 + (m^2 + n^2)^2 = 240.5$

$a^2 + b^2 = 240.5$

$(a - b)^2 = a^2 - 2ab + b^2$

$= a^2 + b^2 - 2ab$

$15^2 = 240.5 - 2ab$

$2ab = 240.5 - 225$

$= 15.5$

$(a + b)^2 = a^2 + 2ab + b^2$

$= a^2 + b^2 + 2ab$

$= 240.5 + 15.5$

$= 256$

$\therefore h^2 + k^2 + m^2 + n^2 = a + b$

$= \sqrt{256} \quad (h^2 + k^2 + m^2 + n^2 > 0)$

$= 16$

Revision Exercise A1

1. When $x = 67$, $y = 2$,

$2 = k\sqrt{67 - 3}$

$= k\sqrt{64}$

$= 8k$

$k = \frac{1}{4}$

When $x = 39$, $y = p$,

$p = \frac{1}{4}\sqrt{39 - 3}$

$= \frac{1}{4}\sqrt{36}$

$= \frac{1}{4} \times 6$

$= 1.5$

When $x = q$, $y = 2.75$,

$2.75 = \frac{1}{4}\sqrt{q - 3}$

$\sqrt{q - 3} = 11$

$q - 3 = 121$

$q = 124$

$\therefore k = \frac{1}{4}, p = 1.5, q = 124$

2. Since R is inversely proportional to d^2,

then $R = \frac{k}{d^2}$, where k is a constant.

When $d = 2.5$, $R = 20$,

$20 = \frac{k}{2.5^2}$

$\therefore k = 125$

$\therefore R = \frac{125}{d^2}$

When $R = 31.25$,

$31.25 = \frac{125}{d^2}$

$d^2 = \frac{125}{31.25}$

$= 4$

$d = \sqrt{4}$ $(d > 0)$

$= 2$

$\therefore$ The diameter of the wire is 2 mm.

3. $8x + 3y = 14$ — (1)

$2x + y = 4$ — (2)

From (2), $y = 4 - 2x$ — (3)

Substitute (3) into (1):

$8x + 3(4 - 2x) = 14$

$8x + 12 - 6x = 14$

$2x = 2$

$x = 1$

Substitute $x = 1$ into (3):

$y = 4 - 2(1)$

$= 2$

$\therefore$ The solution is $x = 1$ and $y = 2$.

4. Let the number of protons in an iron atom be x,
the number of protons in an oxygen atom be y.

$2x + 3y = 76$ — (1)

$x + y = 34$ — (2)

$3 \times (2)$: $3x + 3y = 102$ — (3)

(3) – (1):

$(3x + 3y) - (2x + 3y) = 102 - 76$

$3x + 3y - 2x - 3y = 26$

$x = 26$

Substitute $x = 26$ into (2):

$26 + y = 34$

$y = 8$

$\therefore$ There are 26 protons in an iron atom and 8 protons in an oxygen atom.

5. **(a)** $(2a + 5b)^2 - (a + 3b)(a - 6b)$

$= (2a)^2 + 2(2a)(5b) + (5b)^2 - (a^2 - 6ab + 3ab - 18b^2)$

$= 4a^2 + 20ab + 25b^2 - a^2 + 6ab - 3ab + 18b^2$

$= 4a^2 - a^2 + 20ab + 6ab - 3ab + 25b^2 + 18b^2$

$= 3a^2 + 23ab + 43b^2$

(b) $(4c + d)(4c - d) - \left(2c - \frac{1}{5}d\right)^2$

$= (16c^2 - d^2) - \left[(2c)^2 - 2(2c)\left(\frac{1}{5}d\right) + \left(\frac{1}{5}d\right)^2\right]$

$= 16c^2 - d^2 - \left(4c^2 - \frac{4}{5}cd + \frac{1}{25}d^2\right)$

$= 16c^2 - d^2 - 4c^2 + \frac{4}{5}cd - \frac{1}{25}d^2$

$= 16c^2 - 4c^2 + \frac{4}{5}cd - d^2 - \frac{1}{25}d^2$

$= 12c^2 + \frac{4}{5}cd - \frac{26}{25}d^2$

6. **(a)** $4f^2 - 10f + 6 = 2(2f^2 - 5f + 3)$

$2f^2 = 2f \times f$

$3 = 1 \times 3$ or $(-1) \times (-3)$

$\times$	f	-1
$2f$	$8h^2$	$-2f$
-3	$-3f$	3

$(-3f) + (-2f) = -5f$

$\therefore 4f^2 - 10f + 6 = 2(2f - 3)(f - 1)$

(b) $1 - 12hk + 36h^2k^2 = 1^2 - 2(1)(6hk) + (6hk)^2$

$= (1 - 6hk)^2$

(c) $5m^2n - 15mn^2 - 25mn = 5mn(m - 3n - 5)$

(d) $2px + 3qy - 2py - 3qx$

$= 2px - 2py - 3qx + 3qy$

$= 2p(x - y) - 3q(x - y)$

$= (x - y)(2p - 3q)$

7. Area of the trapezium

$= \frac{1}{2} \times \text{(sum of lengths of parallel sides)} \times \text{height}$

$= \frac{1}{2} \times [3x + (5x + 6)] \times (4x - 3)$

$= \frac{1}{2} \times (3x + 5x + 6) \times (4x - 3)$

$= \frac{1}{2} \times (8x + 6) \times (4x - 3)$

$= \frac{1}{2} \times 2 \times (4x + 3) \times (4x - 3)$

$= (4x)^2 - 3^2$

$= (16x^2 - 9)$ cm^2 (shown)

Revision Exercise A2

1. Since E is directly proportional to d^2,
then $E = kd^2$, where k is a constant.
When $d = d_1$, $E = E_1$,
$E_1 = kd_1^2$
When $d_2 = (100 - 25)\% \times d_1$, $E = E_2$

$$= 75\% \times d_1$$
$$= \frac{75}{100} \times d_1$$
$$= \frac{3}{4} d_1$$

$$E_2 = k\left(\frac{3}{4}d_1\right)^2$$
$$= k\left(\frac{9}{16}d_1^2\right)$$
$$= \frac{9}{16}kd_1^2$$
$$= \frac{9}{16}E_1$$

$\therefore$ Percentage change in energy stored $= \dfrac{E_2 - E_1}{E_1} \times 100\%$

$$= \frac{\frac{9}{16}E_1 - E_1}{E_1} \times 100\%$$
$$= -\frac{7}{16} \times 100\%$$
$$= -43.75\%$$

$\therefore$ The percentage decrease in energy stored is 43.75%.

2. **(i)** Since y is inversely proportional to $2x^2 + 5$,
then $y = \dfrac{k}{2x^2 + 5}$, where k is a constant.
When $x = 2$, $y = 7$,

$$7 = \frac{k}{2(2^2) + 5}$$
$$7 = \frac{k}{13}$$

$\therefore k = 91$

$$\therefore y = \frac{91}{2x^2 + 5}$$

(ii) When $x = 8$,

$$y = \frac{91}{2(8^2) + 5}$$
$$= \frac{91}{133}$$
$$= \frac{13}{19}$$

(iii) When $y = 5.2$,

$$5.2 = \frac{91}{2x^2 + 5}$$
$$2x^2 + 5 = \frac{91}{5.2}$$
$$= 17.5$$
$$2x^2 = 12.5$$
$$x^2 = 6.25$$
$$x = \pm\sqrt{6.25}$$
$$= \pm 2.5$$

3. Since the lines pass through $(2, -3)$,

$p(2) + (-3) = 3$
$2p - 3 = 3$
$2p = 6$
$p = 3$

$2 + 2(-3) = q$
$q = 2 - 6$
$= -4$

$\therefore p = 3$, $q = -4$

4. $x + y = 70$ — (1)
$2x + 4y = 196$ — (2)
From (2), $x + 2y = 98$ — (3)
(3) – (1):
$(x + 2y) - (x + y) = 98 - 70$
$x + 2y - x - y = 98 - 70$
$y = 28$
Substitute $y = 28$ into (1):
$x + 28 = 70$
$x = 42$
$\therefore$ There are 42 chickens and 28 rabbits on the farm.

5. **(a)** $(a^2 - 7a + 6)(3a - 2) - a(2a^2 - 7)$
$= (3a^3 - 2a^2 - 21a^2 + 14a + 18a - 12) - (2a^3 - 7a)$
$= 3a^3 - 2a^2 - 21a^2 + 14a + 18a - 12 - 2a^3 + 7a$
$= 3a^3 - 2a^3 - 2a^2 - 21a^2 + 14a + 18a + 7a - 12$
$= a^3 - 23a^2 + 39a - 12$

(b) $\left(b^2 + \frac{1}{9}\right)\left(b + \frac{1}{3}\right)\left(b - \frac{1}{3}\right) = \left(b^2 + \frac{1}{9}\right)\left[b^2 - \left(\frac{1}{3}\right)^2\right]$

$$= \left(b^2 + \frac{1}{9}\right)\left(b^2 - \frac{1}{9}\right)$$
$$= (b^2)^2 - \left(\frac{1}{9}\right)^2$$
$$= b^4 - \frac{1}{81}$$

6. **(a)** $2c^2d^2 + 5cd - 12 = 2(cd)^2 + 5cd - 12$
$2(cd)^2 = 2cd \times cd$
$-12 = 1 \times (-12)$ or $(-1) \times 12$
$= 2 \times (-6)$ or $(-2) \times 6$
$= 3 \times (-4)$ or $(-3) \times 4$

×	cd	4
$2cd$	c^2	$8cd$
–3	$-3cd$	–12

$(-3cd) + 8cd = 5cd$
$\therefore 2c^2d^2 + 5cd - 12 = (2cd - 3)(cd + 4)$

(b) $25h^2k^2 + 10hk + 1 = (5hk)^2 + 2(5hk)(1) + 1^2$
$= (5hk + 1)^2$

(c) $16 - 4(m + 2)^2 = 4[4 - (m + 2)^2]$
$= 4[2^2 - (m + 2)^2]$
$= 4[2 + (m + 2)][2 - (m + 2)]$
$= 4(2 + m + 2)(2 - m - 2)$
$= -4m(4 + m)$

(d) $3pr - ps + 6qr - 2qs = p(3r - s) + 2q(3r - s)$
$= (3r - s)(p + 2q)$

7. **(a)** $805^2 = (800 + 5)^2$

$= 800^2 + 2(800)(5) + 5^2$

$= 640\ 000 + 8000 + 25$

$= 648\ 025$

(b) $903^2 - 97^2 = (903 + 97)(903 - 97)$

$= 1000 \times 806$

$= 806\ 000$

Chapter 5 Quadratic Equations and Graphs

TEACHING NOTES

Suggested Approach

To solve quadratic equations by factorisation, students should be familiar with the "zero product" property, i.e., if 2 factors P and Q are such that $P \times Q = 0$, then either $P = 0$ or $Q = 0$ or both P and Q are equal to 0.

Teachers should use graphing software to explore the 2 important properties of quadratic graphs - symmetry and the maximum/minimum point.

This chapter exposes students to mathematical and real-life problems that can be solved using quadratic equations and that involve the graphs of quadratic functions.

Section 5.1: Solving Quadratic Equations by Factorisation

Teachers may need to recap the steps involved in factorising quadratic expressions using the "multiplication frame" method with students before illustrating how to solve quadratic equations by factorisation together with the "zero product" property.

Errors that students may make in solving quadratic equations should be highlighted (see Thinking Time on page 141 of the textbook).

Section 5.2: Applications of Quadratic Equations in Real-World Contexts

In this section, students will practise applying quadratic equations in solving mathematical problems in the real-world contexts.

Teachers should remind students to check for any unsuitable answers (e.g. negative values for positive quantities such as length and mass etc.) and indicate the rejection of such values in their answers.

Section 5.3: Graphs of Quadratic Functions

As a lesson introduction, students should go through the activity on Page 147 (see: Investigation: Relationship Between the Area of a Square and the Length of its side). Students are expected to deduce that the shape of the graph is non-linear. As such, for each value of x, there is exactly one corresponding value of A, then $A = x^2$ is the equation of the function.

Students should also explore the effect of the values of a, b and c in the quadratic graphs of equation $y = ax^2 + bx + c$ where $c \neq 0$ (see Investigation: Graphs of $y = x^2$ and $y = -x^2$)

In Secondary One, students have learnt to plot linear function graphs. Using given table of values and scales, teachers should provide students with ample practices to plot different types of quadratic function graphs. Besides plotting the quadratic graphs, students should be able to recognise and write down the equation of the line of symmetry of the plotted quadratic graphs. Students are to understand that for a given y value, there could be 2 possible x values which will satisfy the quadratic equations. Teachers can emphasise this by asking students to state the values of x for a given value of y by highlighting the 2 possible answers.

Teachers should also highlight to students that quadratic graphs have either a maximum or minimum y value. Students have to observe that the maximum/minimum point is also on the line of symmetry of the graph. Teachers may pose questions about maximum/minimum points in different forms to give students added practice, e.g.

- State the maximum/minimum point;
- State the maximum/minimum value of y;
- Sate the value of x for which y is maximum/minimum.

Teachers should go through the mathematical and real-life problems that involve the graphs of quadratic functions (see Worked Example 10 on page 155 of the textbook).

Challenge Yourself

Question 2 is from an Additional Mathematics topic: Quadratic Equations, Inequalities and Modulus Functions. The proof can be easily derived by working backwards.

WORKED SOLUTIONS

Thinking Time (Page 141)

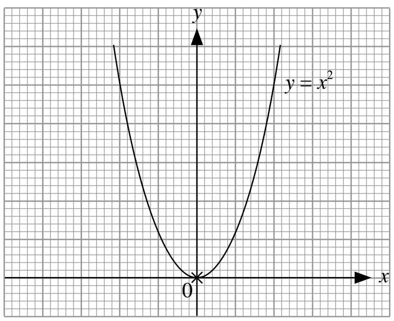
Since we assumed that $x = y$, then $x - y = 0$.
Therefore, the step 'Divide by $x - y$ on both sides' is an invalid step, as we are dividing both sides by 0.

Investigation (Relationship Between the Area of a Square and the Length of its Side)

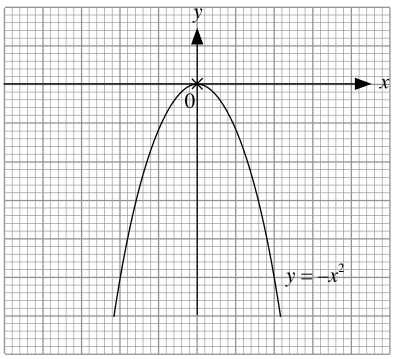
1. For each value of x, there is exactly one corresponding value of A. $A = x^2$ is the equation of a function.
2. The shape of the graph is a hyperbola and not a straight line. Hence, the graph of the function is non-linear.

Investigation (Graphs of $y = x^2$ and $y = -x^2$)

1. **(a)**



(b)



2. **(a)** The coordinates of this point are (0, 0).
   (b) For $y = x^2$, the lowest point of the graph is (0, 0).
   There is no highest point.
   For $y = -x^2$, the highest point of each graph is (0, 0).
   There is no lowest point.
   (c) Both graphs are symmetrical about the y-axis. The equation of the line of symmetry of both graphs is $x = 0$.

Investigation (Graphs of $y = ax^2 + bx + c$, where $a \neq 0$)

1. As the value of a increases, the shape of the graph becomes narrower.
2. As the value of a decreases but remaining positive, the shape becomes wider.
3. As the value of a decreases until it becomes negative, the shape of the graph changes from one that opens upwards, to one that opens downwards.
4. The value of a determines whether the shape of the graph becomes narrower or wider.
   When a is positive, the curve opens upwards indefinitely and when a is negative, the curve opens downwards indefinitely.
5. As the value of c increases, the position of the graph changes by shifting upwards.
6. As the value of c decreases, the position of the graph changes by shifting downwards.
7. The value of c affects the distance the graph is from the x-axis.
8. The shape of the graph opens upwards indefinitely. As the value of b increases, the graph shifts from right to left.
9. The shape of the graph opens downwards indefinitely. As the value of b increases, the graph shifts from left to right.
10. The value of b determines the position the graph is on the left or the right of the y-axis, and the distance from it.
11.

Quadratic Graph	Coefficient of x^2	Opens upwards / downwards	Coordinates of minimum / maximum point	Equation of line of symmetry	x-intercept (s)	y-intercept
$y = x^2 - 4x + 3$	1	Opens upwards	(2, –1)	$x = 2$	1, 3	3
$y = -x^2 - 2x + 3$	–1	Opens downwards	(–1, 4)	$x = -1$	–3, –1	3
$y = x^2 - 4x + 4$	1	Opens upwards	(2, 0)	$x = 2$	2	4
$y = -4x^2 + 12x - 9$	–4	Opens downwards	(1.5, 0)	$x = 1.5$	1.5	–9
$y = 2x^2 + 2x + 1$	2	Opens upwards	(–0.5, 0.5)	$x = -0.5$	None	1
$y = -3x^2 + x - 4$	–3	Opens downwards	$\left(\frac{1}{6}, 3\frac{11}{12}\right)$	$x = \frac{1}{6}$	None	–4

Table 5.1

Practise Now 1

1. **(a)** $x(x + 2) = 0$
   $x = 0$ or $x + 2 = 0$
   $\therefore x = 0$ or $x = -2$
   (b) $3x(x - 1) = 0$
   $3x = 0$ or $x - 1 = 0$
   $\therefore x = 0$ or $= 1$
   (c) $(x + 5)(x - 7) = 0$
   $x + 5 = 0$ or $x - 7 = 0$
   $\therefore x = -5$ or $x = 7$
   (d) $(3x + 2)(4x - 5) = 0$
   $3x + 2 = 0$ or $4x - 5 = 0$
   $\therefore x = -\frac{2}{3}$ or $x = 1\frac{1}{4}$

Practise Now 2

1. **(a)** $3x^2 - 15x = 0$
   $3x(x - 5) = 0$
   $3x = 0$ or $x - 5 = 0$
   $\therefore x = 0$ or $x = 5$

(b) $x^2 + 8x + 16 = 0$
$x^2 + 2(x)(4) + 4^2 = 0$
$(x + 4)^2 = 0$
$x + 4 = 0$
$\therefore x = -4$

(c) $x^2 + 5x + 4 = 0$
$(x + 1)(x + 4) = 0$
$x + 1 = 0$ or $x + 4 = 0$
$\therefore x = -1$ or $x = -4$

(d) $3x^2 - 17x + 10 = 0$
$(3x - 2)(x - 5) = 0$
$3x - 2 = 0$ or $x - 5 = 0$
$\therefore x = \frac{2}{3}$ or $x = 5$

2. **(i)** $3x^2 - 10x + 8 = 0$
$(3x - 4)(x - 2) = 0$
$3x - 4 = 0$ or $x - 2 = 0$
$\therefore x = 1\frac{1}{3}$ or $x = 2$

(ii) $3(y + 1)^2 - 10(y + 1) + 8 = 0$
From **(i)**, letting $x = y + 1$,
$y + 1 = 1\frac{1}{3}$ or $y + 1 = 2$
$\therefore y = \frac{1}{3}$ or $y = 1$

3. **(i)** When $x = -2$,
$(-2)^2 + p(-2) + 8 = 0$
$4 - 2p + 8 = 0$
$2p = 12$
$\therefore p = 6$

(ii) $x^2 + 6x + 8 = 0$
$(x + 2)(x + 4) = 0$
$x + 2 = 0$ or $x + 4 = 0$
$\therefore x = -2$ or $x = -4$
$\therefore$ The other solution is $x = -4$.

Practise Now 3

1. **(a)** $x(x + 1) = 6$
$x^2 + x - 6 = 0$
$(x - 2)(x + 3) = 0$
$x - 2 = 0$ or $x + 3 = 0$
$\therefore x = 2$ or $x = -3$

(b) $9y(1 - y) = 2$
$9y - 9y^2 - 2 = 0$
$9y^2 - 9y + 2 = 0$
$(3y - 1)(3y - 2) = 0$
$3y - 1 = 0$ or $3y - 2 = 0$
$\therefore y = \frac{1}{3}$ or $y = \frac{2}{3}$

Practise Now 4

1. Let the smaller number be x.
Then the next consecutive even number is $x + 2$.
$\therefore x^2 + (x + 2)^2 = 164$
$x^2 + x^2 + 4x + 4 = 164$
$2x^2 + 4x - 160 = 0$
$x^2 + 2x - 80 = 0$
$(x - 8)(x + 10) = 0$
$x - 8 = 0$ or $x + 10 = 0$
$\therefore x = 8$ or $x = -10$ (rejected since $x > 0$)
When $x = 8$,
$x + 2 = 8 + 2$
$= 10$
The two consecutive positive even numbers are 8 and 10.

2. Let the smaller number be x.
Then the bigger number is $x + 5$.
$\therefore [x + (x + 5)]^2 = 169$
$(2x + 5)^2 = 169$
$4x^2 + 20x + 25 = 169$
$4x^2 + 20x - 144 = 0$
$x^2 + 5x - 36 = 0$
$(x - 4)(x + 9) = 0$
$x - 4 = 0$ or $x + 9 = 0$
$\therefore x = 4$ or $x = -9$ (rejected since $x > 0$)
When $x = 4$,
$x + 5 = 4 + 5$
$= 9$
The two numbers are 4 and 9.

Practise Now 5

Let the length of the rectangle garden be x cm.
Then the breadth of the rectangle is $\left(\frac{20 - 2x}{2}\right)$ cm = $(10 - x)$ cm.
$\therefore x(10 - x) = 24$
$10x - x^2 = 24$
$x^2 - 10x + 24 = 0$
$(x - 4)(x - 6) = 0$
$x - 4 = 0$ or $x - 6 = 0$
$\therefore x = 4$ or $x = 6$
When $x = 4$,
Breadth of rectangle = $10 - 4$
= 6 cm
When $x = 6$,
Breadth of rectangle = $10 - 6$
= 4 cm
Since the length of a rectangle usually refers to the longer side,
Length of rectangle = 6 cm
Breadth of rectangle = 4 cm

Practise Now 6

1. **(i)** When $t = 12$,

$y = 96(12) - 4(12^2)$
$= 576$

$\therefore$ Height of rocket 12 seconds after it leaves the ground = 576 m.

(ii) Let $y = 0$.

$96t - 4t^2 = 0$
$4t(24 - t) = 0$
$4t = 0$ or $24 - t = 0$
$\therefore t = 0$ or $t = 24$

$\therefore$ The rocket will strike the ground again 24 seconds after it leaves the ground.

2. **(i)** When $t = 0$,

$h = 18 + 6(0) - 4(0^2)$
$= 18$

$\therefore$ The school building is 18 m tall.

(ii) Let $h = 0$.

$18 + 6t - 4t^2 = 0$
$4t^2 - 6t - 18 = 0$
$2t^2 - 3t - 9 = 0$
$(2t + 3)(t - 3) = 0$
$2t + 3 = 0$ or $t - 3 = 0$
$\therefore t = -1\frac{1}{2}$ or $t = 3$
(rejected, since $t > 0$)

$\therefore$ The ball will strike the ground 3 seconds after it is thrown.

Practise Now 7

Let $y = 0$.

$2x^2 - x - 6 = 0$
$(2x + 3)(x - 2) = 0$
$2x + 3 = 0$ or $x - 2 = 0$
$\therefore x = -1\frac{1}{2}$ or $x = 2$

$\therefore$ The coordinates of A and B are $\left(-1\frac{1}{2}, 0\right)$ and $(2, 0)$ respectively.

When $x = 0$,

$y = 2(0^2) - 0 - 6$
$= -6$

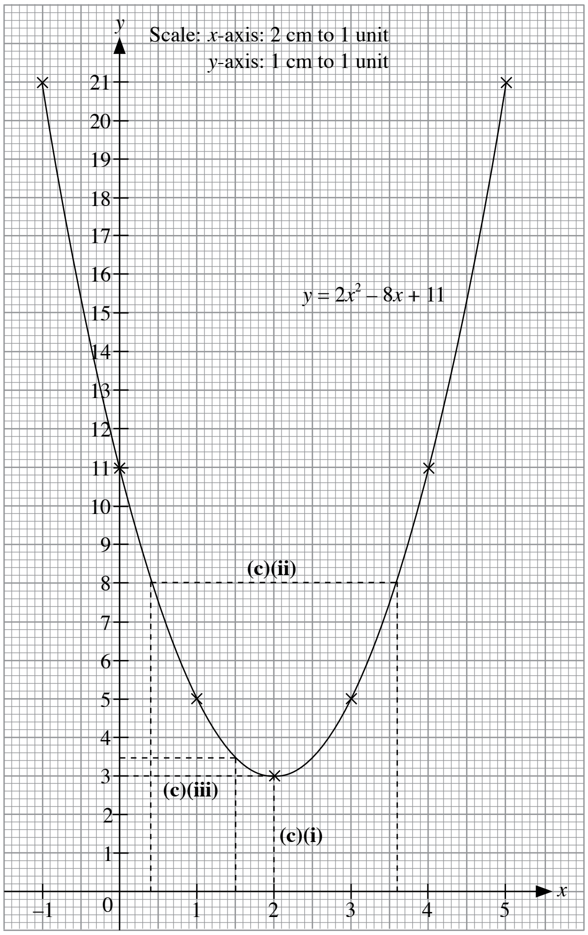
$\therefore$ The coordinates of C are $(0, -6)$.

Practise Now 8

(a) When $x = 3$,

$y = 2(3^2) - 8(3) + 11$
$= 5$
$\therefore q = 5$

(b)



(c) **(i)** When $x = 1.5$,

$y \approx 3.5$

(ii) When $y = 8$,

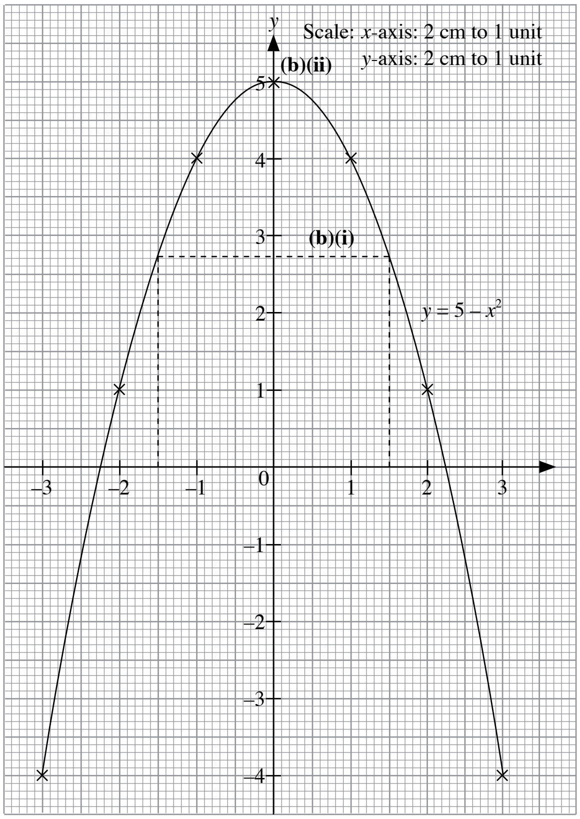
$x \approx 0.4$ or 3.6

(iii) Minimum value of $y = 3$

Minimum value of y occurs when $x = 2$

Practise Now 9

(a)



(b) **(i)** When $y = 2.75$,

$x \approx \pm 1.5$

(ii) Maximum value of $y = 5$

(c) The equation of the line of symmetry of the graph is $x = 0$.

Practise Now 10

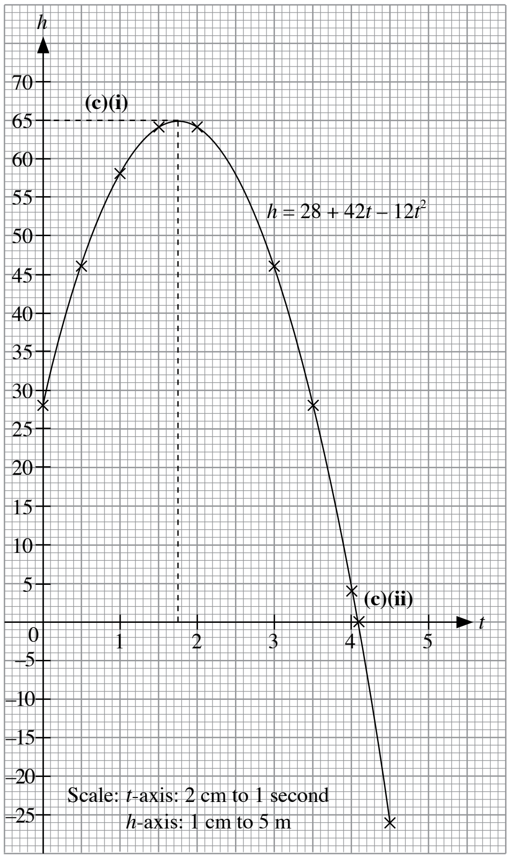
1. **(a)** When $t = 0$,

$h = 28 + 42(0) - 12(0^2)$

$= 28$

(b)

t	0	0.5	1	1.5	2	3	3.5	4	4.5
h	28	46	58	64	64	46	28	4	–26



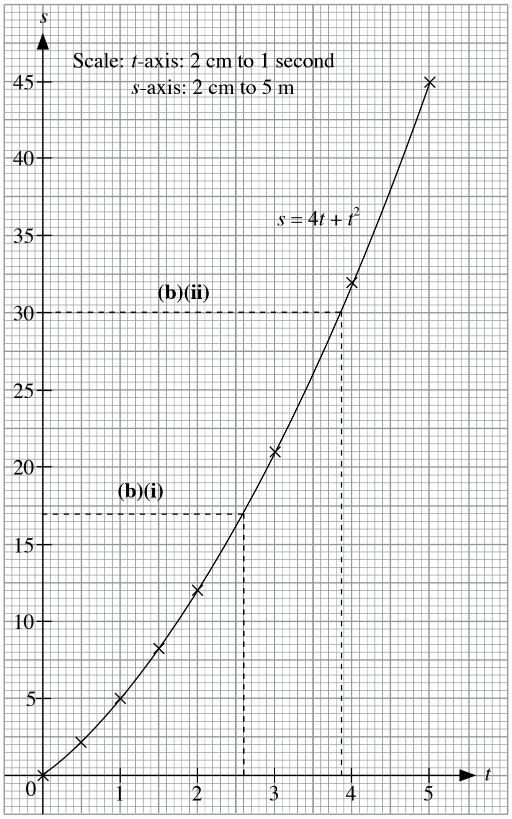
(c) **(i)** Maximum height of stone = 65 m

Time at which maximum height of stone occurs ≈ 1.75 s

(ii) The stone will strike level ground approximately 4.05 seconds after it is thrown.

2. **(a)**

t	0	0.5	1	1.5	2	3	4	5
s	0	2.25	5	8.25	12	21	32	45



(b) **(i)** Distance object travels after 2.6 seconds = 17.25 m

(ii) Time taken for object to travel 30 m = 3.85 s

Exercise 5A

1. **(a)** $a(a-9)=0$

$a=0$ or $a-9=0$

$\therefore a=0$ or $a=9$

(b) $b(b+7)=0$

$b=0$ or $b+7=0$

$\therefore b=0$ or $b=-7$

(c) $5c(c+1)=0$

$5c=0$ or $c+1=0$

$\therefore c=0$ or $c=-1$

(d) $2d(d-6)=0$

$2d=0$ or $d-6=0$

$\therefore d=0$ or $d=6$

(e) $-f(3f-7)=0$

$-f=0$ or $3f-7=0$

$\therefore f=0$ or $f=2\frac{1}{3}$

(f) $-\frac{1}{2}h(2h+3)=0$

$-\frac{1}{2}h=0$ or $2h+3=0$

$h=0$ or $h=-1\frac{1}{2}$

2. **(a)** $(k-4)(k-9)=0$

$k-4=0$ or $k-9=0$

$\therefore k=4$ or $k=9$

(b) $(m-3)(m+5)=0$

$m-3=0$ or $m+5=0$

$\therefore m=3$ or $m=-5$

(c) $(n+4)(n-11)=0$

$n+4=0$ or $n-11=0$

$\therefore n=-4$ or $n=11$

(d) $(p+1)(p+2)=0$

$p+1=0$ or $p+2=0$

$p=-1$ or $p=-2$

(e) $(7q-6)(4q-5)=0$

$7q-6=0$ or $4q-5=0$

$\therefore q=\frac{6}{7}$ or $q=1\frac{1}{4}$

(f) $(3r-5)(2r+1)=0$

$3r-5=0$ or $2r+1=0$

$\therefore r=1\frac{2}{3}$ or $r=-\frac{1}{2}$

(g) $(5s+3)(2-s)=0$

$5s+3=0$ or $2-s=0$

$\therefore s=-\frac{3}{5}$ or $s=2$

(h) $(-2t-5)(8t-5)=0$

$-2t-5=0$ or $8t-5=0$

$\therefore t=-2\frac{1}{2}$ or $t=\frac{5}{8}$

3. **(a)** $a^2+9a=0$

$a(a+9)=0$

$a=0$ or $a+9=0$

$\therefore a=0$ or $a=-9$

(b) $b^2-7b=0$

$b(b-7)=0$

$b=0$ or $b-7=0$

$\therefore b=0$ or $b=7$

(c) $5c^2+25c=0$

$5c(c+5)=0$

$5c=0$ or $c+5=0$

$\therefore c=0$ or $c=-5$

(d) $3d^2-4d=0$

$d(3d-4)=0$

$d=0$ or $3d-4=0$

$\therefore d=0$ or $d=1\frac{1}{3}$

(e) $3f-81f^2=0$

$3f(1-27f)=0$

$3f=0$ or $1-27f=0$

$\therefore f=0$ or $f=\frac{1}{27}$

(f) $-4h^2-16h=0$

$-4h(1+4h)=0$

$-4h=0$ or $1+4h=0$

$\therefore h=0$ or $h=-\frac{1}{4}$

4. **(a)** $k^2 + 12k + 36 = 0$

$k^2 + 2(k)(6) + 6^2 = 0$

$(k + 6)^2 = 0$

$k + 6 = 0$

$\therefore k = -6$

(b) $m^2 - 16m + 64 = 0$

$m^2 - 2(m)(8) + 8^2 = 0$

$(m - 8)^2 = 0$

$m - 8 = 0$

$\therefore m = 8$

(c) $n^2 - 16 = 0$

$n^2 - 4^2 = 0$

$(n + 4)(n - 4) = 0$

$n + 4 = 0$ or $n - 4 = 0$

$\therefore n = -4$ or $n = 4$

(d) $25p^2 + 70p + 49 = 0$

$(5p)^2 + 2(5p)(7) + 7^2 = 0$

$(5p + 7)^2 = 0$

$5p + 7 = 0$

$\therefore p = -1\frac{2}{5}$

(e) $4q^2 - 12q + 9 = 0$

$(2q)^2 - 2(2q)(3) + 3^2 = 0$

$(2q - 3)^2 = 0$

$2q - 3 = 0$

$\therefore q = 1\frac{1}{2}$

(f) $4r^2 - 100 = 0$

$(2r)^2 - 10^2 = 0$

$(2r + 10)(2r - 10) = 0$

$2r + 10 = 0$ or $2r - 10 = 0$

$\therefore r = -5$ or $r = 5$

5. **(a)** $s^2 + 10s + 21 = 0$

$(s + 3)(s + 7) = 0$

$s + 3 = 0$ or $s + 7 = 0$

$\therefore s = -3$ or $s = -7$

(b) $t^2 - 16t + 63 = 0$

$(t - 7)(t - 9) = 0$

$t - 7 = 0$ or $t - 9 = 0$

$\therefore t = 7$ or $t = 9$

(c) $u^2 + 6u - 27 = 0$

$(u - 3)(u + 9) = 0$

$u - 3 = 0$ or $u + 9 = 0$

$\therefore u = 3$ or $u = -9$

(d) $v^2 - 5v - 24 = 0$

$(v + 3)(v - 8) = 0$

$v + 3 = 0$ or $v - 8 = 0$

$\therefore v = -3$ or $v = 8$

(e) $3w^2 + 49w + 60 = 0$

$(3w + 4)(w + 15) = 0$

$3w + 4 = 0$ or $w + 15 = 0$

$\therefore w = -1\frac{1}{3}$ or $w = -15$

(f) $6x^2 - 29x + 20 = 0$

$(6x - 5)(x - 4) = 0$

$6x - 5 = 0$ or $x - 4 = 0$

$\therefore x = \frac{5}{6}$ or $x = 4$

(g) $3y^2 + 7y - 6 = 0$

$(3y - 2)(y + 3) = 0$

$3y - 2 = 0$ or $y + 3 = 0$

$\therefore y = \frac{2}{3}$ or $y = -3$

(h) $2z^2 - 3z - 14 = 0$

$(2z - 7)(z + 2) = 0$

$2z - 7 = 0$ or $z + 2 = 0$

$\therefore z = 3\frac{1}{2}$ or $z = -2$

6. $7x^3 + 21x^2 = 0$

$7x^2(x + 3) = 0$

$7x^2 = 0$ or $x + 3 = 0$

$\therefore x = 0$ or $x = -3$

7. **(a)** $121 - a^2 = 0$

$11^2 - a^2 = 0$

$(11 + a)(11 - a) = 0$

$11 + a = 0$ or $11 - a = 0$

$\therefore a = -11$ or $a = 11$

(b) $128 - 2b^2 = 0$

$64 - b^2 = 0$

$8^2 - b^2 = 0$

$(8 + b)(8 - b) = 0$

$8 + b = 0$ or $8 - b = 0$

$\therefore b = -8$ or $b = 8$

(c) $c^2 - \frac{1}{4} = 0$

$c^2 - \left(\frac{1}{2}\right)^2 = 0$

$\left(c + \frac{1}{2}\right)\left(c - \frac{1}{2}\right) = 0$

$c + \frac{1}{2} = 0$ or $c - \frac{1}{2} = 0$

$\therefore c = -\frac{1}{2}$ or $c = \frac{1}{2}$

(d) $\frac{4}{9} - \frac{d^2}{25} = 0$

$\left(\frac{2}{3}\right)^2 - \left(\frac{d}{5}\right)^2 = 0$

$\left(\frac{2}{3} + \frac{d}{5}\right)\left(\frac{2}{3} - \frac{d}{5}\right) = 0$

$\frac{2}{3} + \frac{d}{5} = 0$ or $\frac{2}{3} - \frac{d}{5} = 0$

$\therefore d = -3\frac{1}{3}$ or $d = 3\frac{1}{3}$

8. **(a)** $7f + f^2 = 60$

$f^2 + 7f - 60 = 0$

$(f - 5)(f + 12) = 0$

$f - 5 = 0$ or $f + 12 = 0$

$\therefore f = 5$ or $f = -12$

(b) $15 = 8h^2 - 2h$

$8h^2 - 2h - 15 = 0$

$(4h + 5)(2h - 3) = 0$

$4h + 5 = 0$ or $2h - 3 = 0$

$\therefore h = -1\frac{1}{4}$ or $h = 1\frac{1}{2}$

9. **(a)** $k(2k + 5) = 3$

$2k^2 + 5k = 3$

$2k^2 + 5k - 3 = 0$

$(2k - 1)(k + 3) = 0$

$2k - 1 = 0$ or $k + 3 = 0$

$\therefore k = \frac{1}{2}$ or $k = -3$

(b) $2m(m - 5) = 5m - 18$

$2m^2 - 10m = 5m - 18$

$2m^2 - 15m + 18 = 0$

$(2m - 3)(m - 6) = 0$

$2m - 3 = 0$ or $m - 6 = 0$

$\therefore m = 1\frac{1}{2}$ or $m = 6$

(c) $(n - 2)(n + 4) = 27$

$n^2 + 4n - 2n - 8 = 27$

$n^2 + 2n - 35 = 0$

$(n - 5)(n + 7) = 0$

$n - 5 = 0$ or $n + 7 = 0$

$\therefore n = 5$ or $n = -7$

(d) $(p - 1)(p - 6) = 126$

$p^2 - 6p - p + 6 = 126$

$p^2 - 7p - 120 = 0$

$(p + 8)(p - 15) = 0$

$p + 8 = 0$ or $p - 15 = 0$

$\therefore p = -8$ or $p = 15$

(e) $(2q - 3)(q - 4) = 18$

$2q^2 - 8q - 3q + 12 = 18$

$2q^2 - 11q - 6 = 0$

$(2q + 1)(q - 6) = 0$

$2q + 1 = 0$ or $q - 6 = 0$

$\therefore q = -\frac{1}{2}$ or $q = 6$

(f) $3r^2 - 5(r + 1) = 7r + 58$

$3r^2 - 5r - 5 = 7r + 58$

$3r^2 - 12r - 63 = 0$

$r^2 - 4r - 21 = 0$

$(r + 3)(r - 7) = 0$

$r + 3 = 0$ or $r - 7 = 0$

$\therefore r = -3$ or $r = 7$

(g) $(3s + 1)(s - 4) = -5(s - 1)$

$3s^2 - 12s + s - 4 = -5s + 5$

$3s^2 - 6s - 9 = 0$

$s^2 - 2s - 3 = 0$

$(s + 1)(s - 3) = 0$

$s + 1 = 0$ or $s - 3 = 0$

$\therefore s = -1$ or $s = 3$

(h) $(t + 2)(t - 3) = t + 2$

$t^2 - 3t + 2t - 6 = t + 2$

$t^2 - 2t - 8 = 0$

$(t + 2)(t - 4) = 0$

$t + 2 = 0$ or $t - 4 = 0$

$\therefore t = -2$ or $t = 4$

10. **(i)** $6x^2 - x - 15 = 0$

$(3x - 5)(2x + 3) = 0$

$3x - 5 = 0$ or $2x + 3 = 0$

$\therefore x = 1\frac{2}{3}$ or $x = -1\frac{1}{2}$

(ii) $6(y - 3)^2 - (y - 3) - 15 = 0$

From **(i)**, letting $x = y - 3$,

$y - 3 = 1\frac{2}{3}$ or $y - 3 = -1\frac{1}{2}$

$y = 4\frac{2}{3}$ or $y = 1\frac{1}{2}$

11. **(a)** $\frac{1}{2}x^2 - \frac{11}{4}x + \frac{5}{4} = 0$

$2x^2 - 11x + 5 = 0$

$(2x - 1)(x - 5) = 0$

$2x - 1 = 0$ or $x - 5 = 0$

$\therefore x = \frac{1}{2}$ or $x = 5$

(b) $2 - 3.5y - 9.75y^2 = 0$

$8 - 14y - 39y^2 = 0$

$(4 - 13y)(2 + 3y) = 0$

$4 - 13y = 0$ or $2 + 3y = 0$

$\therefore y = \frac{4}{13}$ or $y = -\frac{2}{3}$

12. $9x^2y^2 - 12xy + 4 = 0$

$(3xy)^2 - 2(3xy)(2) + 2^2 = 0$

$(3xy - 2)^2 = 0$

$3xy - 2 = 0$

$\therefore y = \frac{2}{3x}$

13. $x - (2x - 3)^2 = -6(x^2 + x - 2)$

$x - (4x^2 - 12x + 9) = -6x^2 - 6x + 12$

$x - 4x^2 + 12x - 9 = -6x^2 - 6x + 12$

$2x^2 + 19x - 21 = 0$

$(2x + 21)(x - 1) = 0$

$2x + 21 = 0$ or $x - 1 = 0$

$\therefore x = -10\frac{1}{2}$ or $x = 1$

14. (i) When $x = 5$,

$5^2 - q(5) + 10 = 0$

$25 - 5q + 10 = 0$

$5q = 35$

$\therefore q = 7$

(ii) $x^2 - 7x + 10 = 0$

$(x - 2)(x - 5) = 0$

$x - 2 = 0$ or $x - 5 = 0$

$\therefore x = 2$ or $x = 5$

$\therefore$ The other solution is $x = 2$.

Exercise 5B

1. Let the number be x.

$\therefore x + 2x^2 = 10$

$2x^2 + x - 10 = 0$

$(2x + 5)(x - 2) = 0$

$2x + 5 = 0$ or $x - 2 = 0$

$\therefore x = -2\frac{1}{2}$ or $x = 2$

(rejected, since x is whole number)

$\therefore$ The number is 2.

2. Let the number be x.

$\therefore 3x^2 - 4x = 15$

$3x^2 - 4x - 15 = 0$

$(3x + 5)(x - 3) = 0$

$3x + 5 = 0$ or $x - 3 = 0$

$\therefore x = -1\frac{2}{3}$ or $x = 3$

(rejected, since x is whole number)

$\therefore$ The number is 3.

3. Let the smaller number be x.

Then the next consecutive positive number is $x + 1$.

$\therefore x^2 + (x + 1)^2 = 113$

$x^2 + x^2 + 2x + 1 = 113$

$2x^2 + 2x - 112 = 0$

$x^2 + x - 56 = 0$

$(x - 7)(x + 8) = 0$

$x - 7 = 0$ or $x + 8 = 0$

$x = 7$ or $x = -8$ (rejected, since $x > 0$)

When $x = 7$,

$x + 1 = 7 + 1$

$= 8$

$\therefore$ The two numbers are 7 and 8.

4. Let the smaller number be x.

Then the greater positive number is $x + 7$.

$\therefore [x + (x + 7)]^2 = 289$

$(2x + 7)^2 = 289$

$4x^2 + 28x + 49 = 289$

$4x^2 + 28x - 240 = 0$

$x^2 + 7x - 60 = 0$

$(x - 5)(x + 12) = 0$

$x - 5 = 0$ or $x + 12 = 0$

$\therefore x = 5$ or $x = -12$ (rejected, since $x > 0$)

When $x = 5$,

$x + 7 = 5 + 7$

$= 12$

$\therefore$ The two numbers are 5 and 12.

5. Let the smaller number be x.

Then the greater number is $x + 9$.

$\therefore x(x + 9) = 162$

$x^2 + 9x = 162$

$x^2 + 9x - 162 = 0$

$(x - 9)(x + 18) = 0$

$x - 9 = 0$ or $x + 18 = 0$

$\therefore x = 9$ or $x = -18$

When $x = 9$,

$x + 9 = 9 + 9$

$= 18$

When $x = -18$,

$x + 9 = -18 + 9$

$= -9$

$\therefore$ The two numbers are 9 and 18, or -18 and -9.

6. Let the length of the rectangular campsite be x m. Then the breadth of the rectangular campsite is $\left(\frac{64 - 2x}{2}\right)$ m $= (32 - x)$ m.

$\therefore x(32 - x) = 27$

$32x - x^2 = 207$

$x^2 - 32x + 207 = 0$

$(x - 9)(x - 23) = 0$

$x - 9 = 0$ or $x - 23 = 0$

$\therefore x = 9$ or $x = 23$

When $x = 9$,

Breadth of garden $= 32 - 9$

$= 23$ m

When $x = 23$,

Breadth of garden $= 32 - 23$

$= 9$ m

Since the length of a rectangle usually refers to the longer side,

Length of campsite = 23 m

Breadth of campsite = 9 m

7. Let the width of the path be x m.

Then the length of the path is $(2x + 70)$ m and the breadth of the path is $(2x + 50)$ m.

Area of the rectangular field $= 70 \times 50$

$= 3500 \text{ m}^2$

$\therefore (2x + 70)(2x + 50) - 3500 = 1024$

$4x^2 + 100x + 140x + 3500 - 3500 = 1024$

$4x^2 + 240x - 1024 = 0$

$x^2 + 60x - 256 = 0$

$(x - 4)(x + 64) = 0$

$x - 4 = 0$ or $x + 64 = 0$

$\therefore x = 4$ or $x = -64$

(rejected, since $x > 0$)

$\therefore$ The width of the path is 4 m.

8. $\frac{1}{2}(x + 3)(2x - 5) = 20$

$\frac{1}{2}(2x^2 - 5x + 6x - 15) = 20$

$2x^2 + x - 15 = 40$

$2x^2 + x - 55 = 0$

$(2x + 11)(x - 5) = 0$

$2x + 11 = 0$ or $x - 5 = 0$

$\therefore x = -5\frac{1}{2}$ or $x = 5$

(rejected, since $x > 0$)

$\therefore$ The value of x is 5.

9. Let the perimeter of the smaller square be x cm.

Then the perimeter of the larger square is $(44 - x)$ cm.

$\therefore \left(\frac{x}{4}\right)^2 + \left(\frac{44 - x}{4}\right)^2 = 65$

$\frac{x^2}{16} + \frac{1936 - 88x + x^2}{16} = 65$

$x^2 + 1936 - 88x + x^2 = 1040$

$2x^2 - 88x + 896 = 0$

$x^2 - 44x + 448 = 0$

$(x - 16)(x - 28) = 0$

$x - 16 = 0$ or $x - 28 = 0$

$\therefore x = 16$ or $x = 28$

When $x = 16$,

Perimeter of the larger square $= 44 - 16$

$= 28$ cm

When $x = 28$,

Perimeter of the smaller square $= 44 - 28$

$= 16$ cm

$\therefore$ The perimeter of each square is 16 cm and 28 cm.

10. (i) Area of circle $= \pi(3^2)$

$= 9\pi$

Area of square removed from circle $= x^2$

$\therefore 9\pi - x^2 = 7\pi$

$2\pi - x^2 = 0$ (shown)

(ii) $2\pi - x^2 = 0$

$x^2 = 2\pi$

$\therefore x = \pm\sqrt{2\pi}$

$= \pm 2.51$ (to 3 s.f.)

(iii) Since $x > 0$, $x = \sqrt{2\pi}$.

Perimeter of square $= 4\sqrt{2\pi}$

$= 10.0$ cm (to 3 s.f.)

$\therefore$ The perimeter of the square is 10.0 cm.

11. (i) Distance Amirah walks $= (x + 1)x$

$= (x^2 + x)$ km

Distance Amirah cycles $= (2x + 5)(x - 1)$

$= 2x^2 - 2x + 5x - 5$

$= (2x^2 + 3x - 5)$ km

$\therefore (x^2 + x) + (2x^2 + 3x - 5) = 90$

$3x^2 + 4x - 5 = 90$

$3x^2 + 4x - 95 = 0$ (shown)

(ii) $3x^2 + 4x - 95 = 0$

$(3x + 19)(x - 5) = 0$

$3x + 19 = 0$ or $x - 5 = 0$

$\therefore x = -6\frac{1}{3}$ or $x = 5$

(iii) When $x = -6\frac{1}{3}$,

Speed at which Amirah walks $= -6\frac{1}{3} + 1$

$= -5\frac{1}{3}$ km/h < 0

$\therefore x = 5$

Time taken for her entire journey $= x + (x - 1)$

$= 5 + (5 - 1)$

$= 9$ hours

$\therefore$ The time taken for her entire journey is 9 hours.

12. (i) When $t = 0$,

$h = 4 + 11(0) - 3(0^2)$

$= 4$

$\therefore$ The hawk drops its prey 4 m above the ground.

(ii) When $h = 0$,

$4 + 11t - 3t^2 = 0$

$3t^2 - 11t - 4 = 0$

$(3t + 1)(t - 4) = 0$

$3t + 1 = 0$ or $t - 4 = 0$

$t = -\frac{1}{3}$ or $t = 4$

(rejected, since $t > 0$)

$\therefore$ The prey will fall onto the ground after 4 s.

13. $\frac{(x + 1)^2 - (x - 3)}{x - 2} = 16$

$(x + 1)^2 - (x - 3) = 16(x - 2)$

$x^2 + 2x + 1 - x + 3 = 16x - 32$

$x^2 - 15x + 36 = 0$

$(x - 3)(x - 12) = 0$

$x - 3 = 0$ or $x - 12 = 0$

$\therefore x = 3$ or $x = 12$

14. (i) When $t = 2.5$,

$y = 17(2.5) - 5(2.5^2)$

$= 11.25$

$\therefore$ The height of the object 2.5 seconds after it leaves the ground is 11.25 m.

(ii) When $y = 0$,

$17t - 5t^2 = 0$

$t(17 - 5t) = 0$

$t = 0$ or $17 - 5t = 0$

$\therefore t = 0$ or $t = 3.4$

$\therefore$ The object will strike the ground again after 3.4 s.

(iii) When $t = 1$,

$y = 17(1) - 5(1^2)$

$= 12$

From **(i)**,

Distance between the two objects $= 12 - 11.25$

$= 0.75$ m

$\therefore$ The distance between two objects is 0.75 m

15. (i) When $t = 3.5$,

$h = 56(3.5) - 7(3.5^2)$

$= 110.25$

$\therefore$ The height of the ball 3.5 seconds after it leaves the ground is 110.25 m.

(ii) When $t = 0$,

$56t - 7t^2 = 0$

$7t(8 - t) = 0$

$7t = 0$ or $8 - t = 0$

$\therefore t = 0$ or $t = 8$

$\therefore$ The ball will strike the ground again after 8 s.

(iii) When $h = 49$,

$56t - 7t^2 = 49$

$7t^2 - 56t + 49 = 0$

$t^2 - 8t + 7 = 0$

$(t - 1)(t - 7) = 0$

$t - 1 = 0$ or $t - 7 = 0$

$\therefore t = 1$ or $t = 7$

$\therefore$ The ball will be 49 m above the ground after 1 s and 7 s. The first time refers to when the ball is going upwards while the second time refers to when the ball is coming down. Hence, there are two possible answers.

Exercise 5C

1. (a) When $x = -4$, $y = a$,

$a = (-4)^2 + 2(-4) - 8$

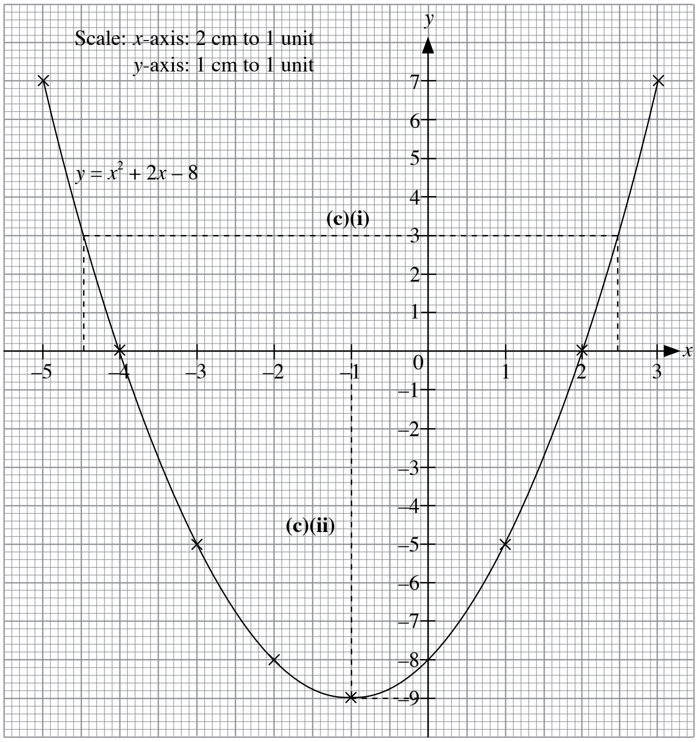
$= 0$

When $x = 1$, $y = b$,

$b = 1^2 + 2(1) - 8$

$= -5$

(b)



(c) (i) When $y = 3$,

$x \approx 2.45$ or -4.45

(ii) Minimum value of $y = -9$

(d) The equation of the line of symmetry of the graph is $x = -1$.

2. (a) When $x = -3$, $y = p$,

$p = 2 - 3(-3) - 2(-3)^2$

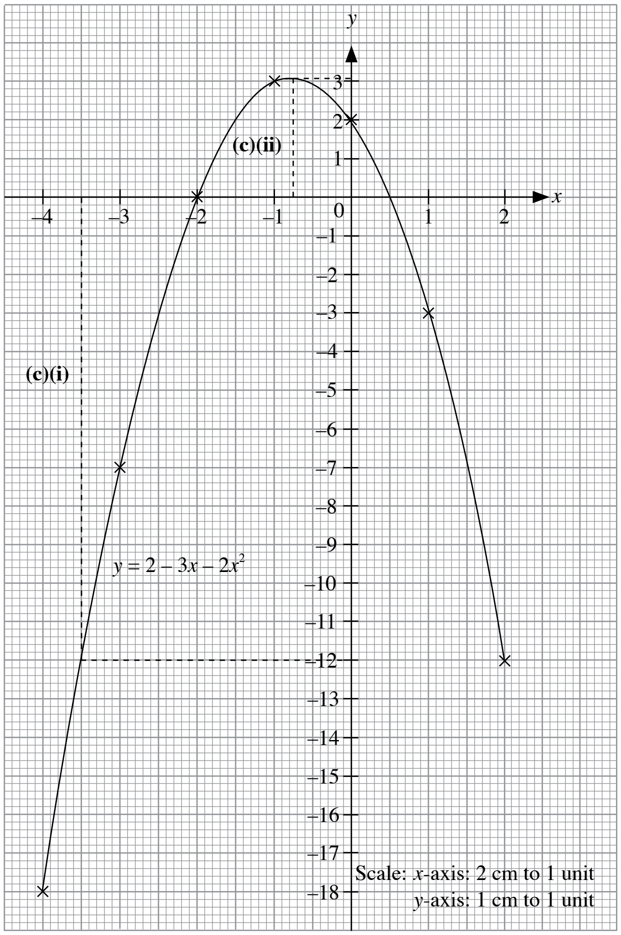
$= 7$

When $x = 1$, $y = q$,

$q = 2 - 3(1) - 2(1^2)$

$= -3$

(b)



(c) (i) When $x = -3.5$,

$y = -12$

(ii) Maximum value of $y \approx 3.1$

Maximum value of y occurs when $x = -0.75$

3. (i) Let $y = 0$.

$-x^2 - x + 20 = 0$

$x^2 + x - 20 = 0$

$(x + 5)(x - 4) = 0$

$x + 5 = 0$ or $x - 4 = 0$

$\therefore x = -5$ or $x = 4$

$\therefore$ The coordinates of A and B are $(-5, 0)$ and $(4, 0)$ respectively.

When $x = 0$,

$y = -0^2 - 0 + 20$

$= 20$

$\therefore$ The coordinates of C are $(0, 20)$.

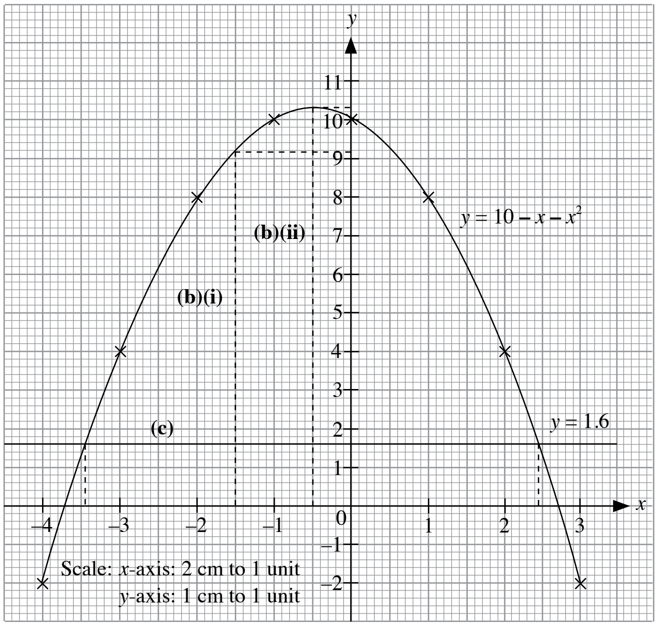
(ii) When $x = 3$, $y = h$,

$h = -3^2 - 3 + 20$

$= 8$

$\therefore$ The value of h is 8.

4. (a)



(b) (i) When $x = -1.5$,

$y \approx 9.3$

(ii) Maximum value of $y \approx 10.3$

Maximum value of y occurs when $x = -0.5$

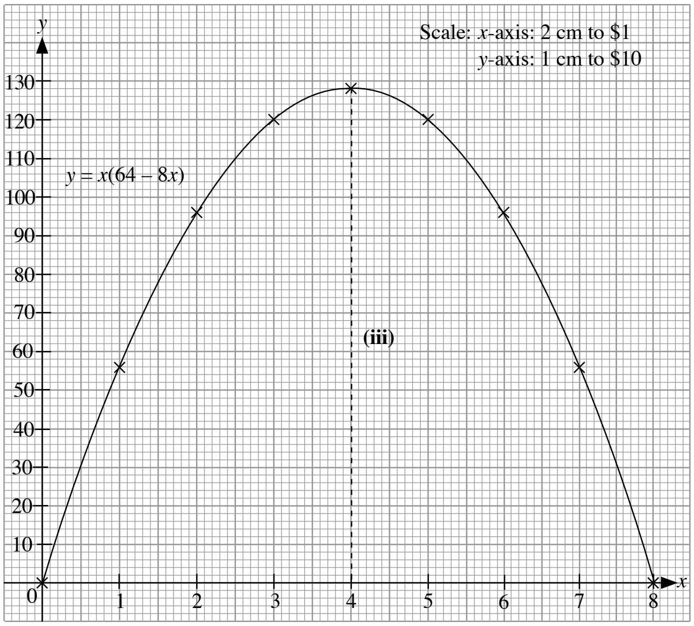
(c) By drawing $y = 1.6$,

Solution is $10 - x - x^2 = 1.6$: $x \approx -3.45$ or 2.45

5. (i) $y = x(64 - 8x)$

(ii)

x	0	1	2	3	4	5	6	7	8
y	0	56	96	120	128	120	96	56	0



(iii) The amount at which the store should price each book is $4.

6. (i) $A = \frac{1}{2}(x + 2)(7 - x)$

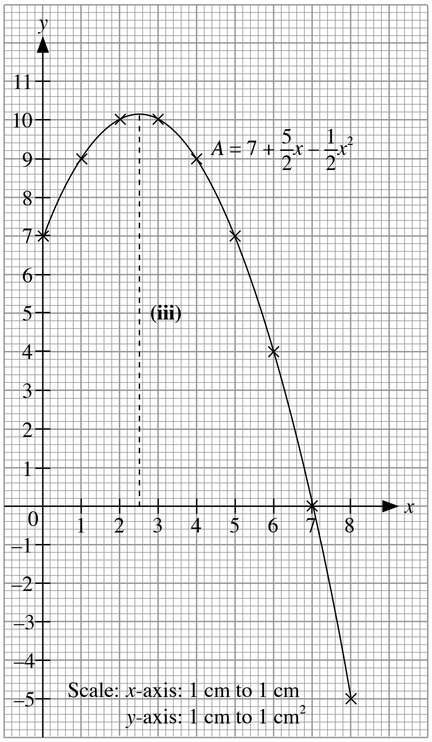
$= \frac{1}{2}(7x - x^2 + 14 - 2x)$

$= \frac{1}{2}(14 + 5x - x^2)$

$= 7 + \frac{5}{2}x - \frac{1}{2}x^2$ (shown)

(ii)

x	0	1	2	3	4	5	6	7	8
y	7	9	10	10	9	7	4	0	–5



(iii) Maximum area occurs when $x = 2.5$

Length of side of triangle $= 2.5 + 2$

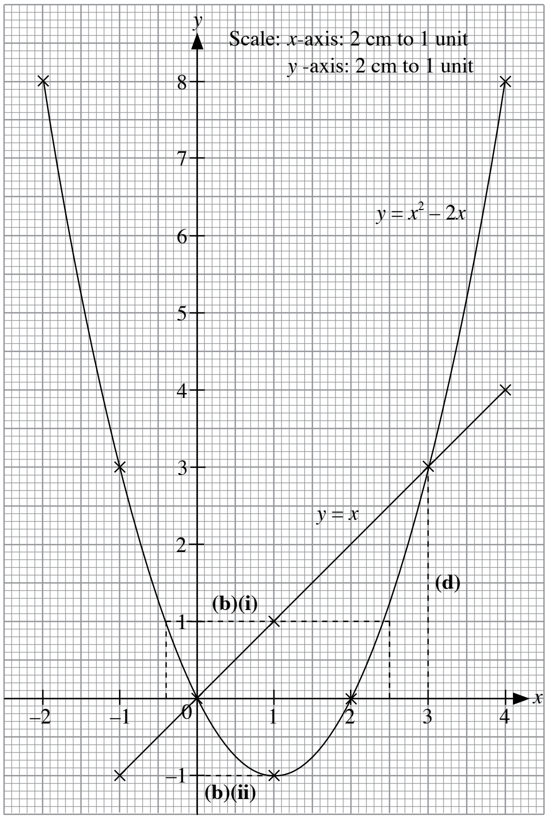
$= 4.5$ cm

Corresponding height $= 7 - 2.5$

$= 4.5$ cm

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7. (a)



(b) (i) When $y = 1$,

$x = -0.4$ or 2.4

(ii) Minimum value of $y = -1$

(c) The line of symmetry of the graph is $x = 1$.

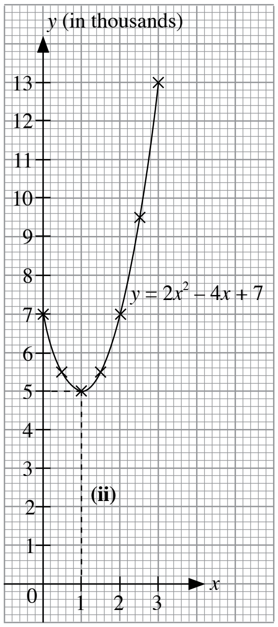
(d) By drawing $y = x$,

x	–1	1	4
y	–1	1	4

Solution to $x^2 - 2x = x$: $x = 0$ or 3

8. (i)

x	0	0.5	1	1.5	2	2.5	3
y	7	5.5	5	5.5	7	9.5	13



Scale: x-axis: 1 cm to 1 year

y-axis: 1 cm to \$1000

(ii) Minimum value of the asset = \$5000

Time at which minimum value of assets occurs = 1 year.

Review Exercise 5

1. (a) $-6a(-5 - 2a) = 0$

$-6a = 0$ or $-5 - 2a = 0$

$\therefore a = 0$ or $a = -2\frac{1}{2}$

(b) $(4b + 11)(3b - 7) = 0$

$4b + 11 = 0$ or $3b - 7 = 0$

$\therefore b = -2\frac{3}{4}$ or $b = 2\frac{1}{3}$

(c) $4c^2 = 7c$

$4c^2 - 7c = 0$

$c(4c - 7) = 0$

$c = 0$ or $4c - 7 = 0$

$\therefore c = 0$ or $c = 1\frac{3}{4}$

(d) $9 + 6d + d^2 = 0$

$d^2 + 2(d)(3) + 3^2 = 0$

$(d + 3)^2 = 0$

$d + 3 = 0$

$\therefore d = -3$

(e) $49f^2 - 140f + 100 = 0$

$(7f)^2 - 2(7f)(10) + 10^2 = 0$

$(7f - 10)^2 = 0$

$7f - 10 = 0$

$\therefore f = 1\frac{3}{7}$

(f) $147 - 3h^2 = 0$

$-3(h^2 - 49) = 0$

$h^2 - 49 = 0$

$h^2 - 7^2 = 0$

$(h + 7)(h - 7) = 0$

$h + 7 = 0$ or $h - 7 = 0$

$\therefore h = -7$ or $h = 7$

2. **(a)** $6k^2 + 11k - 10 = 0$

$(3k - 2)(2k + 5) = 0$

$3k - 2 = 0$ or $2k + 5 = 0$

$\therefore k = \frac{2}{3}$ or $k = -2\frac{1}{2}$

(b) $3m(8 - 3m) = 16$

$24m - 9m^2 = 16$

$9m^2 - 24m + 16 = 0$

$(3m)^2 - 2(3m)(4) + 4^2 = 0$

$(3m - 4)^2 = 0$

$3m - 4 = 0$

$\therefore m = 1\frac{1}{3}$

(c) $(3n - 1)^2 = 12n + 8$

$(3n)^2 - 2(3n)(1) + 1^2 = 12n + 8$

$9n^2 - 6n + 1 = 12n + 8$

$9n^2 - 18n - 7 = 0$

$(3n + 1)(3n - 7) = 0$

$3n + 1 = 0$ or $3n - 7 = 0$

$\therefore n = -\frac{1}{3}$ or $n = 2\frac{1}{3}$

(d) $(5p + 1)(p + 4) = 2(7p + 5)$

$5p^2 + 20p + p + 4 = 14p + 10$

$5p^2 + 7p - 6 = 0$

$(5p - 3)(p + 2) = 0$

$5p - 3 = 0$ or $p + 2 = 0$

$\therefore p = \frac{3}{5}$ or $p = -2$

3. **(i)** $2x^2 - 11x - 21 = 0$

$(2x + 3)(x - 7) = 0$

$2x + 3 = 0$ or $x - 7 = 0$

$\therefore x = -1\frac{1}{2}$ or $x = 7$

(ii) $2(y + 2)^2 - 11(y + 2) - 21 = 0$

From **(i)**, letting $x = y + 1$,

$y + 1 = -1\frac{1}{2}$ or $y + 1 = 7$

$\therefore y = -2\frac{1}{2}$ or $y = 6$

4. **(i)** When $x = 3$,

$2(3^2) - 5(3) + k = 0$

$18 - 15 + k = 0$

$\therefore k = -3$

(ii) $2x^2 - 5x - 3 = 0$

$(2x + 1)(x - 3) = 0$

$2x + 1 = 0$ or $x - 3 = 0$

$\therefore x = -\frac{1}{2}$ or $x = 3$

5. Let the smaller number be x.

Then the greater number is $x + 3$.

$\therefore x^2 = 4(x + 3)$

$= 4x + 12$

$x^2 - 4x - 12 = 0$

$(x + 2)(x - 6) = 0$

$x + 2 = 0$ or $x - 6 = 0$

$\therefore x = -2$ or $x = 6$

When $x = -2$,

$x + 3 = -2 + 3$

$= 1$

When $x = 6$,

$x + 3 = 6 + 3$

$= 9$

$\therefore$ The two numbers are –2 and 1, or 6 and 9.

6. **(i)** $x^2 + 4x = 2(x + 4x)$

$= 2(5x)$

$= 10x$

$\therefore x^2 - 6x = 0$ (shown)

(ii) $x^2 - 6x = 0$

$x(x - 6) = 0$

$x = 0$ or $x - 6 = 0$

$\therefore x = 0$ or $x = 6$

(iii) $x = 0$ is rejected, since $x > 0$.

Age of Ethan's father when Ethan was born

$= x^2 - x$

$= 6^2 - 6$

$= 30$ years

7. **(i)** $(2x + 5)(2x - 1) = 3(x + 1)^2$

$4x^2 - 2x + 10x - 5 = 3(x^2 + 2x + 1)$

$4x^2 + 8x - 5 = 3x^2 + 6x + 3$

$\therefore x^2 + 2x - 8 = 0$ (shown)

(ii) $x^2 + 2x - 8 = 0$

$(x - 2)(x + 4) = 0$

$x - 2 = 0$ or $x + 4 = 0$

$\therefore x = 2$ or $x = -4$

(iii) When $x = -4$,

$2x - 1 = 2(-4) - 1$

$= -9 < 0$

$\therefore x = 2$

Perimeter of rectangle $= 2[(2x + 5) + (2x - 1)]$

$= 2(4x + 4)$

$= 2 \times [4(2) + 4]$

$= 24$ cm

8. Let the number of students in the class be x.

Each student will send out $(x - 1)$ cards.

$\therefore x(x - 1) = 870$

$x^2 - x - 870 = 0$

$(x - 30)(x + 29) = 0$

$x - 30 = 0$ or $x + 29 = 0$

$\therefore x = 30$ or $x = -29$ (rejected, since $x > 0$)

$\therefore$ The number of students in the class in 30.

9. (i) When $t = 2$,

$y = 20(2) - 5(2^2)$

$= 20$

$\therefore$ The height of the object 2 seconds after it leaves the ground is 20 m.

(ii) When $y = 15$,

$20t - 5t^2 = 15$

$5t^2 - 20t + 15 = 0$

$t^2 - 4t + 3 = 0$

$(t - 1)(t - 3) = 0$

$t - 1 = 0$ or $t - 3 = 0$

$\therefore t = 1$ $t = 3$

$\therefore$ The object will be 15 m above the ground after 1 s and 3 s.

10. Let $y = 0$.

$(1 - x)(x + 5) = 0$

$1 - x = 0$ or $x + 5 = 0$

$\therefore x = 1$ or $x = -5$

$\therefore$ The coordinates of A and B are $(-5, 0)$ and $(1, 0)$ respectively.

When $x = 0$,

$y = (1 - 0)(0 + 5)$

$= 5$

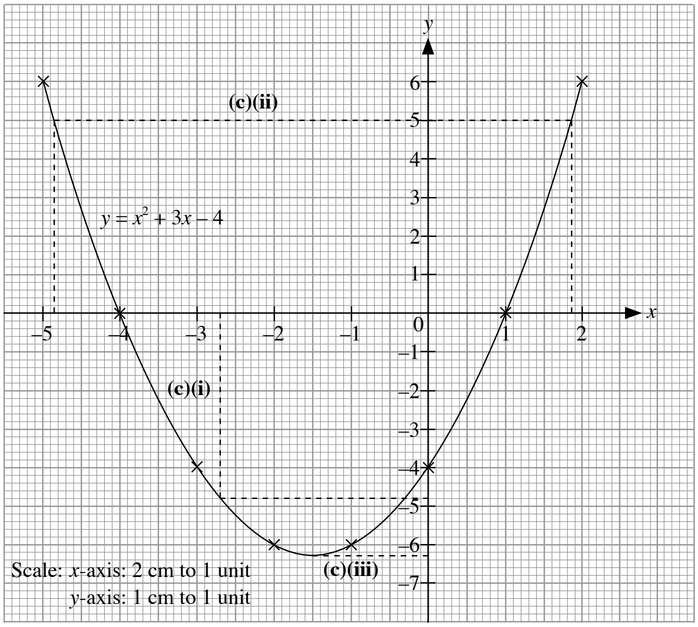
$\therefore$ The coordinates of C are $(0, 5)$.

11. (a) When $x = -5$, $y = p$,

$p = (-5)^2 + 3(-5) - 4$

$= 6$

(b)



(c) (i) When $x = -2.7$,

$y = 4.8$

(ii) When $y = 5$,

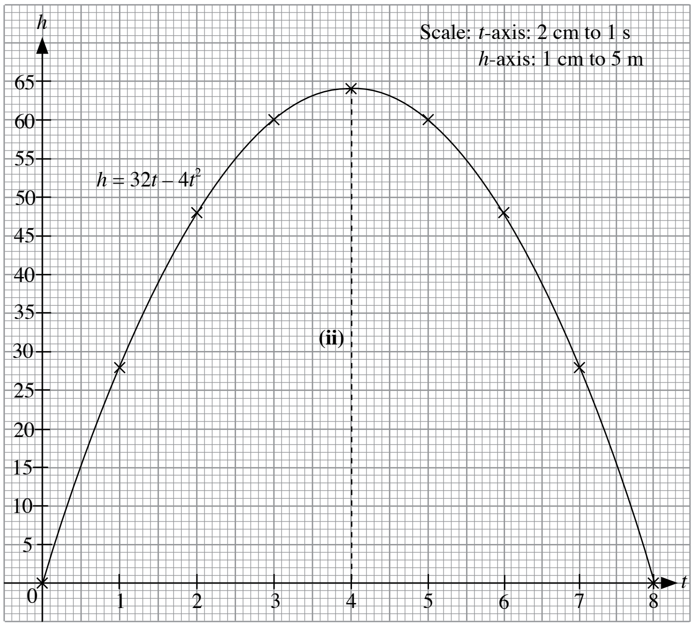
$x \approx -4.85$ or 1.85

(iii) Minimum value of $y = -6.3$

(d) The line of symmetry of the graph is $x = -1.5$.

12. (i)

t	0	1	2	3	4	5	6	7	8
h	0	28	48	60	64	60	48	28	0



(ii) Maximum height of the ball above the ground = 64 m

Time maximum height occurs = 4 s

Challenge Yourself

1. Let the smaller solution of $2x^2 - 6x - k = 0$ be n.

Then the other solution is $n + 5$.

$\therefore (x + n)(x + n + 5) = 0$

$x^2 + (n + 5)x + nx + n(n + 5) = 0$

$x^2 + (2n + 5)x + n(n + 5) = 0$

$2x^2 + 2(2n + 5)x + 2n(n + 5) = 0$

Comparing terms with $2x^2 - 6x - k = 0$,

$2(2n + 5) = -6$

$2n + 5 = -3$

$2n = -8$

$\therefore n = -4$

When $n = -4$,

$-k = 2n(n + 5)$

$= 2(-4)(-4 + 5)$

$= -8$

$\therefore k = 8$

2. **(i)** Since α and β are the solutions of a quadratic equation,

$$(x - \alpha)(x - \beta) = 0$$
$$x^2 - \beta x - \alpha x + \alpha\beta = 0$$
$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$
$$ax^2 - a(\alpha + \beta)x + a\alpha\beta = 0$$

Comparing terms with $ax^2 + bx + c = 0$,

$$b = -a(\alpha + \beta)$$
$$\alpha + \beta = -\frac{b}{a} \text{ (shown)}$$
$$c = a\alpha\beta$$
$$\alpha\beta = \frac{c}{a} \text{ (shown)}$$

(ii) From **(i)**,

$$p + q = -\frac{10}{2}$$
$$= -5$$
$$pq = \frac{7}{2}$$
$$= 3\frac{1}{2}$$
$$(p - q)^2 = p^2 - 2pq + q^2$$
$$= p^2 + 2pq + q^2 - 4pq$$
$$= (p + q)^2 - 4pq$$
$$= (-5)^2 - 4\left(3\frac{1}{2}\right)$$
$$= 11$$

Since $p > q$, $p - q > 0$.

$$\therefore p - q = \sqrt{11}$$
$$= 3.32 \text{ (to 3 s.f.)}$$

Chapter 6 Algebraic Fractions and Formulae

TEACHING NOTES

Suggested Approach

Students would have learnt how to expand, factorise and solve linear and quadratic equations by now. Teachers may want to relate algebraic fractions with numerical fractions by simply replacing numerals with algebraic expressions. Students are expected to know how to manipulate algebraic fractions, perform the operations on algebraic fractions and express one variable in terms of the other variables at the end of this chapter.

Section 6.1: Algebraic Fractions

Students are to recall that the value of a fraction does not change when multiplied or divided by a non-zero number or expression. The same applies to algebraic fractions. The usual rules of expansion and factorisation can be used as well.

Section 6.2: Multiplication and Division of Algebraic Fractions

Teachers can let students observe that the multiplication and division of algebraic fractions are very much similar to the multiplication and division of numerical fractions. The notion of reciprocals should be illustrated. Students may need reminders to check that their final expression are in its simplest form.

Section 6.3: Addition and Subtraction of Algebraic Fractions

In Secondary One, students have learnt to manipulate linear expressions with fractional coefficients. Here, the scope is extended to algebraic fractions. Teachers should highlight the important step of converting algebraic fractions to like fractions, and after this step, the fractions can be combined to a single fraction and then simplified.

Section 6.4: Manipulation of Algebraic Formulae

Teachers can work through the worked examples together with the students so as to illustrate how one variable can be made the subject of a formula.

Teachers should emphasise that when making a single variable the subject of a formula, the single variable must be on the left-hand side of the equation while all other variables must be on the right-hand side of the equation. Often, students may make mistakes whereby the subject of the formula is also found on the right-hand side of the equation.

Another error students may make is to substitute the wrong values when evaluating the expression, particularly when there are multiple variables in the equation.

Challenge Yourself

Questions 1 and 2 require an application of the law of indices. Question 3 requires an algebraic manipulation of the given condition to derive the proof. (Hint: $\frac{a+b-c}{a} = 1 + \frac{b-c}{a}$)

WORKED SOLUTIONS

Class Discussion (Finding the Value of an Unknown in a Formula)

1. Yes
2. Yes. There is no difference whether we find the value of the unknown by first substituting the known values or making the subject of the formula and then substitute the known values to find the unknown.

Practise Now 1

(a) $\dfrac{\overset{2}{\cancel{8}}\,\overset{x^2}{\cancel{x^5}}\,\overset{1}{\cancel{y}}}{\underset{3}{\cancel{12}}\,\underset{1}{\cancel{x^3}}\,\underset{y^3}{\cancel{y^4}}} = \dfrac{2x^2}{3y^3}$

(b) $\dfrac{\overset{1}{\cancel{9}}\,\overset{x^2}{\cancel{x^4}}\,\overset{(x-y)^2}{\cancel{(x-y)^3}}}{\underset{3}{\cancel{27}}\,\underset{1}{\cancel{x^2}}\,y^3\,\underset{1}{\cancel{(x-y)}}} = \dfrac{x^2(x-y)^2}{3y^2}$

Practise Now 2

(a) $\dfrac{h^2+7hk}{5hk} = \dfrac{\cancel{h}(h+7k)}{5\cancel{h}k} = \dfrac{h+7k}{5k}$

(b) $\dfrac{15p}{10p^2-5p} = \dfrac{\overset{3}{\cancel{15}}\,\cancel{p}}{\cancel{5}\,\cancel{p}(2p-1)} = \dfrac{3}{2p-1}$

(c) $\dfrac{z^2-4z}{4z-16} = \dfrac{z\,\cancel{(z-4)}}{4\,\cancel{(z-4)}} = \dfrac{z}{4}$

Practise Now 3

1. **(a)** $\dfrac{3v^2-9v}{v^2-9} = \dfrac{3v\,\cancel{(v-3)}}{(v+3)\,\cancel{(v-3)}} = \dfrac{3v}{v+3}$

   (b) $\dfrac{p^2-7pq+12q^2}{5p^2-20pq} = \dfrac{(p-3q)\,\cancel{(p-4q)}}{5p\,\cancel{(p-4q)}} = \dfrac{p-3q}{5p}$

   (c) $\dfrac{x^2-3xy+2xz-6yz}{xy-2xz-3y^2+6yz} = \dfrac{x(x-3y)+2z(x-3y)}{x(y-2z)-3y(y-2z)}$

   $= \dfrac{\cancel{(x-3y)}\,(x+2z)}{(y-2z)\,\cancel{(x-3y)}}$

   $= \dfrac{x+2z}{y-2z}$

2. $\dfrac{n^4-5n^2+6}{n^4-9} = \dfrac{(n^2-2)\,\cancel{(n^2-3)}}{(n^2+3)\,\cancel{(n^2-3)}} = \dfrac{n^2-2}{n^2+3}$

Practise Now 4

1. **(a)** $\dfrac{\cancel{2}\,\cancel{a^2}}{\cancel{5}\,\underset{c^2}{\cancel{c^3}}} \times \dfrac{\overset{3}{\cancel{15}}\,\cancel{c}}{\underset{4}{\cancel{8}}\,\underset{a^2}{\cancel{a^4}}} = \dfrac{3}{4a^2c^2}$

   (b) $\dfrac{3p^2}{15q^3} \times \dfrac{35qr^2}{12pr} \div \dfrac{5r^4}{6p^3q^2} = \dfrac{3p^2}{15q^3} \times \dfrac{35qr^2}{12pr} \times \dfrac{6p^3q^2}{5r^4}$

   $= \dfrac{\overset{7}{\cancel{630}}\,\overset{p^4}{\cancel{p^5}}\,\cancel{q^3}\,\cancel{r^2}}{\underset{10}{\cancel{900}}\,\cancel{p}\,\cancel{q^3}\,\underset{r^3}{\cancel{r^5}}}$

   $= \dfrac{7p^4}{10r^3}$

   (c) $\dfrac{2x-6}{5x+5y} \div \dfrac{3}{7y+7x} = \dfrac{2(x-3)}{5\,\cancel{(x+y)}} \times \dfrac{7\,\cancel{(x+y)}}{3}$

   $= \dfrac{14(x-3)}{15}$

   (d) $\dfrac{h^2-6h+9}{h^2-2h} \times \dfrac{h-2}{h-3} = \dfrac{\overset{h-3}{\cancel{(h-3)^2}}}{h\,\cancel{(h-2)}} \times \dfrac{\cancel{h-2}}{\cancel{h-3}}$

   $= \dfrac{h-3}{h}$

2. $\dfrac{3m-n}{n+m} \div \dfrac{2n-6m}{m+n} = \dfrac{\cancel{3m-n}}{\cancel{m+n}} \times \dfrac{\cancel{m+n}}{-2\,\cancel{(3m-n)}}$

   $= -\dfrac{1}{2}$

Practise Now 5

1. **(a)** $\dfrac{6}{5a} + \dfrac{3}{8a} = \dfrac{48}{40a} + \dfrac{15}{40a}$

   $= \dfrac{48+15}{40a}$

   $= \dfrac{63}{40a}$

   (b) $\dfrac{4}{2b+3c} - \dfrac{7}{6b+9c} = \dfrac{4}{2b+3c} - \dfrac{7}{3(2b+3c)}$

   $= \dfrac{12}{3(2b+3c)} - \dfrac{7}{3(2b+3c)}$

   $= \dfrac{5}{3(2b+3c)}$

   (c) $\dfrac{h}{2-3k} - \dfrac{3h}{3k-2} = \dfrac{h}{2-3k} - \dfrac{3h}{-(2-3k)}$

   $= \dfrac{h}{2-3k} + \dfrac{3h}{2-3k}$

   $= \dfrac{h+3h}{2-3k}$

   $= \dfrac{4h}{2-3k}$

2. (a) $$\frac{2m+3n}{3m} - \frac{m-n}{2n} = \frac{2n(2m+3n)}{6mn} - \frac{3m(m-n)}{6mn}$$
$$= \frac{2n(2m+3n)-3m(m-n)}{6mn}$$
$$= \frac{4mn+6n^2-3m^2+3mn}{6mn}$$
$$= \frac{6n^2+7mn-3m^2}{6mn}$$

(b) $$\frac{3p}{4p-4q} - \frac{5p-2q}{3p-3q} = \frac{3p}{4(p-q)} - \frac{5p-2q}{3(p-q)}$$
$$= \frac{9p}{12(p-q)} - \frac{4(5p-2q)}{12(p-q)}$$
$$= \frac{9p-4(5p-2q)}{12(p-q)}$$
$$= \frac{9p-20p+8q}{12(p-q)}$$
$$= \frac{8q-11p}{12(p-q)}$$

(c) $$\frac{5x}{4x-3y} - \frac{7y}{6y-8x} = \frac{5x}{4x-3y} - \frac{7y}{-2(4x-3y)}$$
$$= \frac{10x}{2(4x-3y)} + \frac{7y}{2(4x-3y)}$$
$$= \frac{10x+7y}{2(4x-3y)}$$

Practise Now 6

1. (a) $$\frac{2}{x+1} - \frac{3}{2x-5} = \frac{2(2x-5)}{(x+1)(2x-5)} - \frac{3(x+1)}{(x+1)(2x-5)}$$
$$= \frac{2(2x-5)-3(x+1)}{(x+1)(2x-5)}$$
$$= \frac{4x-10-3x-3}{(x+1)(2x-5)}$$
$$= \frac{x-13}{(x+1)(2x-5)}$$

(b) $$\frac{2}{y^2-9} - \frac{y}{y-3} = \frac{2}{(y+3)(y-3)} - \frac{y}{y-3}$$
$$= \frac{2}{(y+3)(y-3)} - \frac{y(y+3)}{(y+3)(y-3)}$$
$$= \frac{2-y(y+3)}{(y+3)(y-3)}$$
$$= \frac{2-y^2-3y}{(y+3)(y-3)}$$

(c) $$\frac{1}{z+5} - \frac{1}{z-5} + \frac{2z}{z^2-25}$$
$$= \frac{1}{z+5} - \frac{1}{z-5} + \frac{2z}{(z+5)(z-5)}$$
$$= \frac{z-5}{(z+5)(z-5)} - \frac{z+5}{(z+5)(z-5)} + \frac{2z}{(z+5)(z-5)}$$
$$= \frac{z-5-z-5+2z}{(z+5)(z-5)}$$
$$= \frac{2z-10}{(z+5)(z-5)}$$
$$= \frac{2(z-5)}{(z+5)(z-5)}$$
$$= \frac{2}{z+5}$$

Practise Now 7

1. (i) $$v = u + at$$
$$v - u = u + at - u$$
$$v - u = at$$
$$\frac{v-u}{t} = \frac{at}{t}$$
$$\therefore a = \frac{v-u}{t}$$

(ii) When $t = 4$, $u = 10$ and $v = 50$,
$$a = \frac{50-10}{4}$$
$$= \frac{40}{4}$$
$$= 10$$

2. (i) $$I = \frac{PRT}{100}$$
$$I \times 100 = \frac{PRT}{100} \times 100$$
$$100I = PRT$$
$$\frac{100I}{PR} = \frac{PRT}{PR}$$
$$\therefore T = \frac{100I}{PR}$$

(ii) When $P = 50\ 000$, $R = 2$ and $I = 4000$,
$$T = \frac{100 \times 4000}{50\ 000 \times 2}$$
$$= \frac{400\ 000}{100\ 000}$$
$$= 4$$
$\therefore$ The number of years is 4 years.

Practise Now 8

1. (i) $$y = \frac{2x+5}{3x-7}$$
$$(3x-7) \times y = (3x-7) \times \frac{2x+5}{3x-7}$$
$$y(3x-7) = 2x+5$$
$$3xy - 7y = 2x + 5$$
$$3xy - 7y - 2x = 2x + 5 - 2x$$
$$3xy - 7y - 2x = 5$$
$$3xy - 7y - 2x + 7y = 5 + 7y$$
$$3xy - 2x = 7y + 5$$
$$x(3y-2) = 7y + 5$$
$$\frac{x(3y-2)}{3y-2} = \frac{7y+5}{3y-2}$$
$$\therefore x = \frac{7y+5}{3y-2}$$

(ii) When $y = –3$,

$$x = \frac{7(-3) + 5}{3(-3) - 2}$$
$$= \frac{-21 + 5}{-9 - 2}$$
$$= \frac{-16}{-11}$$
$$= 1\frac{5}{11}$$

2. (i)
$$p = a + \frac{bx^2}{3k}$$
$$p - a = a + \frac{bx^2}{3k} - a$$
$$p - a = \frac{bx^2}{3k}$$
$$3k \times (p - a) = 3k \times \frac{bx^2}{3k}$$
$$3k(p - a) = bx^2$$
$$\frac{3k(p-a)}{3(p-a)} = \frac{bx^2}{3(p-a)}$$
$$\therefore k = \frac{bx^2}{3(p-a)}$$

(ii) When $a = 1$, $b = –2$, $p = 3$ and $x = 9$,

$$k = \frac{-2(9^2)}{3(3-1)}$$
$$= \frac{-2(81)}{3(2)}$$
$$= -\frac{81}{3}$$
$$= -27$$

Practise Now 9

1. (i)
$$3y = \sqrt{b^2 - 4ax}$$
$$9y^2 = b^2 - 4ax$$
$$9y^2 + 4ax = b^2 - 4ax + 4ax$$
$$9y^2 + 4ax = b^2$$
$$9y^2 + 4ax - 9y^2 = b^2 - 9y^2$$
$$4ax = b^2 - 9y^2$$
$$\frac{4ax}{4a} = \frac{b^2 - 9y^2}{4a}$$
$$\therefore x = \frac{b^2 - 9y^2}{4a}$$

(ii) When $a = –5$, $b = 4$ and $y = 2$,

$$x = \frac{4^2 - 9(2^2)}{4(-5)}$$
$$= \frac{16 - 9(4)}{-20}$$
$$= \frac{16 - 36}{-20}$$
$$= \frac{-20}{-20}$$
$$= 1$$

Practise Now 10

1. (a) When $x = 5$,

$$y = \sqrt{\frac{5+7}{5-2}}$$
$$= \sqrt{\frac{12}{3}}$$
$$= \sqrt{4}$$
$$= 2$$

(b) When $y = 4$,

$$4 = \sqrt{\frac{x+7}{x-2}}$$
$$16 = \frac{x+7}{x-2}$$
$$16(x - 2) = x + 7$$
$$16x - 32 = x + 7$$
$$16x = x + 39$$
$$15x = 39$$
$$\therefore x = 2\frac{3}{5}$$

2. When $a = 3$,

$$3 = \sqrt[3]{\frac{5b+16}{2b-23}}$$
$$27 = \frac{5b+16}{2b-23}$$
$$27(2b - 23) = 5b + 16$$
$$54b - 621 = 5b + 16$$
$$54b = 5b + 637$$
$$49b = 637$$
$$\therefore b = 13$$

3. When $y = 4$ and $z = 3$,

$$\sqrt[3]{\frac{x+4}{x-4}} = 3$$
$$\frac{x+4}{x-4} = 27$$
$$x + 4 = 27(x - 4)$$
$$x + 4 = 27x - 108$$
$$x + 112 = 27x$$
$$112 = 26x$$
$$\therefore x = 4\frac{4}{13}$$

Practise Now 11

(a)
$$\frac{a-3}{5} + \frac{2a-1}{3} = 4$$
$$3(a - 3) + 5(2a - 1) = 60$$
$$3a - 9 + 10a - 5 = 60$$
$$13a - 14 = 60$$
$$13a = 74$$
$$\therefore a = 5\frac{9}{13}$$

(b) $\frac{3}{2b+3} - \frac{5}{3b-4} = 0$

$3(3b-4) - 5(2b+3) = 0$

$9b - 12 - 10b - 15 = 0$

$-b - 27 = 0$

$-b = 27$

$\therefore b = -27$

Exercise 6A

1. **(a)** $\frac{\overset{1}{\cancel{4}}\,\overset{1}{\cancel{x^4}}}{\underset{3}{\cancel{12}}\,\underset{x}{\cancel{x^5}}\,y} = \frac{1}{3xy}$

(b) $\frac{\overset{2}{\cancel{16}}\,\overset{1}{\cancel{a^3}}\,\overset{b^2}{\cancel{b^4}}}{\underset{3}{\cancel{24}}\,\underset{a^2}{\cancel{a^5}}\,\underset{1}{\cancel{b^2}}} = \frac{2b^2}{3a^2}$

(c) $\frac{\overset{1}{\cancel{23}}\,\overset{q^2}{\cancel{q^3}}\,\overset{1}{\cancel{r}}}{\underset{3}{\cancel{69}}\,\underset{1}{\cancel{q}}\,\underset{r^2}{\cancel{r^3}}\,s} = \frac{q^2}{3r^2 s}$

(d) $\frac{\overset{1}{\cancel{3}}\,\overset{1}{\cancel{m}}\,\overset{n}{\cancel{n^2}}\,\overset{1}{\cancel{p^3}}}{\underset{6}{\cancel{18}}\,\underset{m^2}{\cancel{m^3}}\,\underset{1}{\cancel{n}}\,\underset{p^3}{\cancel{p^6}}} = \frac{n}{6m^2p^3}$

(e) $\frac{\overset{1}{\cancel{15}}\,\overset{1}{\cancel{a}}\,\overset{c^2}{\cancel{c^3}}}{\underset{5}{\cancel{75}}\,\underset{a}{\cancel{a^2}}\,b^4\,\underset{1}{\cancel{c}}} = \frac{c^2}{5ab^4}$

(f) $\frac{\overset{1}{\cancel{16}}\,\overset{1}{\cancel{x}}\,\overset{1}{\cancel{y^3}}\,\overset{1}{\cancel{z}}}{\underset{4}{\cancel{64}}\,\underset{x}{\cancel{x^2}}\,\underset{y}{\cancel{y^4}}\,\underset{z^2}{\cancel{z^3}}} = \frac{1}{4xyz^2}$

2. **(a)** $\frac{xy+3y}{4x+12} = \frac{y\,\cancel{(x+3)}}{4\,\cancel{(x+3)}}$

$= \frac{y}{4}$

(b) $\frac{8a+4b}{bc+2ac} = \frac{4(2a+b)}{c(b+2a)}$

$= \frac{4\,\cancel{(2a+b)}}{c\,\cancel{(2a+b)}}$

$= \frac{4}{c}$

(c) $\frac{a^2+2ab}{6a} = \frac{\cancel{a}(a+2b)}{6\cancel{a}}$

$= \frac{a+2b}{6}$

(d) $\frac{c^2}{c^2-cd} = \frac{\overset{c}{\cancel{c^2}}}{\cancel{c}(c-d)}$

$= \frac{c}{c-d}$

(e) $\frac{(m-n)^2}{m^2-mn} = \frac{\overset{m-n}{\cancel{(m-n)^2}}}{m\,\cancel{(m-n)}}$

$= \frac{m-n}{m}$

(f) $\frac{5pq}{15p-10pq} = \frac{\cancel{5}\,\cancel{p}\,q}{\cancel{5}\,\cancel{p}\,(3-2q)}$

$= \frac{q}{3-2q}$

3. **(a)** $\frac{2a+b}{4a^2-b^2} = \frac{\cancel{2a+b}}{\cancel{(2a+b)}\,(2a-b)}$

$= \frac{1}{2a-b}$

(b) $\frac{c^2+2cd-15d^2}{4c^2+20cd} = \frac{(c-3d)\,\cancel{(c+5d)}}{4c\,\cancel{(c+5d)}}$

$= \frac{c-3d}{4c}$

(c) $\frac{3a-6}{a^2+a-6} = \frac{3\,\cancel{(a-2)}}{\cancel{(a-2)}\,(a+3)}$

$= \frac{3}{a+3}$

(d) $\frac{x^2+6x-7}{x^2-x} = \frac{\cancel{(x-1)}\,(x+7)}{x\,\cancel{(x-1)}}$

$= \frac{x+7}{x}$

(e) $\frac{k^2-9}{k^2-7k+12} = \frac{(k+3)\,\cancel{(k-3)}}{\cancel{(k-3)}\,(k-4)}$

$= \frac{k+3}{k-4}$

(f) $\frac{mk+8k}{m^2+4m-32} = \frac{k\,\cancel{(m+8)}}{(m-4)\,\cancel{(m+8)}}$

$= \frac{k}{m-4}$

4. **(a)** $\frac{15a^2}{8ab^3c} \times \frac{4c}{5ab} = \frac{\overset{3}{\cancel{60}}\,\cancel{a^2}\,\cancel{c}}{\underset{2}{\cancel{40}}\,\cancel{a^2}\,b^4\,\cancel{c}}$

$= \frac{3}{2b^4}$

(b) $\frac{3(c+d)}{c-d} \times \frac{2c-2d}{8c+8d} = \frac{3\,\cancel{(c+d)}}{\cancel{c-d}} \times \frac{\cancel{2}\,\cancel{(c-d)}}{\underset{4}{\cancel{8}}\,\cancel{(c+d)}}$

$= \frac{3}{4}$

(c) $\frac{a-2b}{16} \div \frac{4a-8b}{24} = \frac{a-2b}{16} \times \frac{24}{4a-8b}$

$= \frac{a-2b}{16} \times \frac{24}{4(a-2b)}$

$= \frac{\overset{3}{\cancel{24}}\,\cancel{(a-2b)}}{\underset{8}{\cancel{64}}\,\cancel{(a-2b)}}$

$= \frac{3}{8}$

(d) $\frac{8c^3}{6(c+d)} \div \frac{2c^2}{3c+3d} = \frac{8c^3}{6(c+d)} \times \frac{3c+3d}{2c^2}$

$= \frac{8c^3}{6(c+d)} \times \frac{3(c+d)}{2c^2}$

$= \frac{\overset{2}{\cancel{24}}\,\overset{c}{\cancel{c^3}}\,\cancel{(c+d)}}{\cancel{12}\,\cancel{c^2}\,\cancel{(c+d)}}$

$= 2c$

5. **(a)** $\dfrac{\cancel{9}\,\cancel{x}\,\cancel{(a-b)^2}}{\cancel{27}\,\cancel{x^3}\,\cancel{(a-b)^3}} = \dfrac{1}{3x^2(a-b)}$

(b) $\dfrac{\cancel{7}\,\cancel{a^3}\,\cancel{(a-3b)^4}}{\cancel{21}\,\cancel{a^2}\,b\,\cancel{(a-3b)^2}} = \dfrac{a(a-3b)^2}{3b}$

(c) $\dfrac{8ab^3(2a+3b)^2}{32a^2b(3b+2a)} = \dfrac{\cancel{8}\,\cancel{a}\,\cancel{b^3}\,\cancel{(2a+3b)^2}}{4\,\cancel{32}\,\cancel{a^2}\,\cancel{b}\,\cancel{(2a+3b)}}$

$= \dfrac{b^2(2a+3b)}{4a}$

(d) $\dfrac{8an^3(b+c)}{96a^2n(c+b)^2} = \dfrac{\cancel{8}\,\cancel{a}\,\cancel{n^3}\,\cancel{(b+c)}}{12\,\cancel{96}\,\cancel{a^2}\,\cancel{n}\,\cancel{(b+c)^2}}$

$= \dfrac{n^2}{12a(b+c)}$

(e) $\dfrac{y^2-2y-15}{y^2-3y-10} = \dfrac{\cancel{(y-5)}\,(y+3)}{\cancel{(y-5)}\,(y+2)}$

$= \dfrac{y+3}{y+2}$

(f) $\dfrac{8-2m-m^2}{2m^2-3m-2} = \dfrac{-(m^2+2m-8)}{2m^2-3m-2}$

$= \dfrac{-\cancel{(m-2)}\,(m+4)}{(2m+1)\,\cancel{(m-2)}}$

$= \dfrac{-(m+4)}{2m+1}$

$= \dfrac{-m-4}{2m+1}$

(g) $\dfrac{9x^2-y^2}{y^2-2xy-3x^2} = \dfrac{(3x+y)(3x-y)}{(y-3x)(y+x)}$

$= \dfrac{-(y+3x)\,\cancel{(y-3x)}}{\cancel{(y-3x)}\,(y+x)}$

$= \dfrac{-(y+3x)}{y+x}$

(h) $\dfrac{3x^2+5xy-2y^2}{4x^2+7xy-2y^2} = \dfrac{(3x-y)\,\cancel{(x+2y)}}{(4x-y)\,\cancel{(x+2y)}}$

$= \dfrac{3x-y}{4x-y}$

(i) $\dfrac{b^2-a^2}{2a^2+ab-3b^2} = \dfrac{(b+a)(b-a)}{(2a+3b)(a-b)}$

$= \dfrac{-(a+b)\,\cancel{(a-b)}}{(2a+3b)\,\cancel{(a-b)}}$

$= \dfrac{-(a+b)}{2a+3b}$

$= \dfrac{-a-b}{2a+3b}$

(j) $\dfrac{y^2-6y-7}{2y^2-17y+21} = \dfrac{(y+1)\,\cancel{(y-7)}}{(2y-3)\,\cancel{(y-7)}}$

$= \dfrac{y+1}{2y-3}$

(k) $\dfrac{3x-3y}{ax-ay-x+y} = \dfrac{3(x-y)}{a(x-y)-(x-y)}$

$= \dfrac{3\,\cancel{(x-y)}}{\cancel{(x-y)}\,(a-1)}$

$= \dfrac{3}{a-1}$

(l) $\dfrac{a^2-ab-ac+bc}{a^2+ab-ac-bc} = \dfrac{a(a-b)-c(a-b)}{a(a+b)-c(a+b)}$

$= \dfrac{(a-b)\,\cancel{(a-c)}}{(a+b)\,\cancel{(a-c)}}$

$= \dfrac{a-b}{a+b}$

(m) $\dfrac{a^2+am-an-mn}{a^2+am+an+mn} = \dfrac{a(a+m)-n(a+m)}{a(a+m)+n(a+m)}$

$= \dfrac{\cancel{(a+m)}\,(a-n)}{\cancel{(a+m)}\,(a+n)}$

$= \dfrac{a-n}{a+n}$

6. **(a)** $\dfrac{5a}{9b} \times \dfrac{ac^2}{2b} \times \dfrac{c^3}{8b^4} = \dfrac{5a^2c^5}{144b^6}$

(b) $\dfrac{6df}{9f^3} \times \dfrac{3f}{16d^2} \div \dfrac{8d^3f^2}{27d} = \dfrac{6df}{9f^3} \times \dfrac{3f}{16d^2} \times \dfrac{27d}{8d^3f^2}$

$= \dfrac{\cancel{486}\,\cancel{d^2}\,\cancel{f^2}}{\cancel{1152}\,\cancel{d^5}\,\cancel{f^5}}$

$= \dfrac{27}{64d^3f^3}$

(c) $2y \div \dfrac{4y}{5xy} \times \dfrac{64xy}{100x^3y^4} = 2y \times \dfrac{5xy}{4y} \times \dfrac{64xy}{100x^3y^4}$

$= \dfrac{\cancel{640}\,\cancel{x^2}\,\cancel{y^3}}{\cancel{400}\,\cancel{x^3}\,\cancel{y^5}}$

$= \dfrac{8}{5xy^2}$

(d) $\dfrac{3ps}{4pqr} \div \dfrac{3pq^2}{12p^3q} \times \dfrac{14s^3}{7qr} = \dfrac{3ps}{4pqr} \times \dfrac{12p^3q}{3pq^2} \times \dfrac{14s^3}{7qr}$

$= \dfrac{\cancel{504}\,\cancel{p^4}\,\cancel{q}\,s^4}{\cancel{84}\,\cancel{p^2}\,\cancel{q^3}\,r^2}$

$= \dfrac{6p^2s^4}{q^2r^2}$

(e) $\frac{3w-7}{5w^3} \div \frac{21-9w}{27w} = \frac{3w-7}{5w^3} \times \frac{27w}{21-9w}$

$= \frac{3w-7}{5w^3} \times \frac{27w}{-3(3w-7)}$

$= \frac{\overset{9}{\cancel{27}}\ \cancel{w}\ \cancel{(3w-7)}}{\underset{-5}{\cancel{-15}}\ \underset{w^2}{\cancel{w^3}}\ \cancel{(3w-7)}}$

$= -\frac{9}{5w^2}$

(f) $\frac{6x^2y}{16y-8x} \times \frac{12x-24y}{4xy^2} = \frac{6x^2y}{8(2y-x)} \times \frac{12(x-2y)}{4xy^2}$

$= \frac{6x^2y}{-8(x-2y)} \times \frac{12(x-2y)}{4xy^2}$

$= \frac{\overset{9}{\cancel{72}}\ \overset{x}{\cancel{x^2}}\ \cancel{y}\ \cancel{(x-2y)}}{\underset{-4}{\cancel{-32}}\ \cancel{x}\ \underset{y}{\cancel{y^2}}\ \cancel{(x-2y)}}$

$= -\frac{9x}{4y}$

(g) $\frac{h^2-h-6}{h^2-9} \times \frac{h^2}{h^2+2h} = \frac{\cancel{(h-3)}\ \cancel{(h+2)}}{(h+3)\cancel{(h-3)}} \times \frac{\overset{h}{\cancel{h^2}}}{\cancel{h}\ \cancel{(h+2)}}$

$= \frac{h}{h+3}$

(h) $\frac{c^2-d^2}{c^2-2cd+d^2} \div \frac{1}{cd+d^2} = \frac{c^2-d^2}{c^2-2cd+d^2} \times cd+d^2$

$= \frac{(c+d)\cancel{(c-d)}}{\underset{(c-d)}{\cancel{(c-d)^2}}} \times d(c+d)$

$= \frac{d(c+d)^2}{c-d}$

(i) $\frac{m^2-4}{m^2-3m+2} \div \frac{m}{m-1} = \frac{m^2-4}{m^2-3m+2} \times \frac{m-1}{m}$

$= \frac{(m+2)\cancel{(m-2)}}{\cancel{(m-1)}\ \cancel{(m-2)}} \times \frac{\cancel{m-1}}{m}$

$= \frac{m+2}{m}$

(j) $\frac{z^2}{z^2-4} \div \frac{3z-z^2}{z^2-5z+6} = \frac{z^2}{z^2-4} \times \frac{z^2-5z+6}{3z-z^2}$

$= \frac{\overset{z}{\cancel{z^2}}}{(z+2)\cancel{(z-2)}} \times \frac{\cancel{(z-2)}\ \cancel{(z-3)}}{_{-1}\cancel{-z}\ \cancel{(z-3)}}$

$= -\frac{z}{z+2}$

(k) $a^2-4b^2 \div \frac{a^2+2ab}{ab} = a^2-4b^2 \times \frac{ab}{a^2+2ab}$

$= \cancel{(a+2b)}(a-2b) \times \frac{\cancel{a}b}{\cancel{a}\ \cancel{(a+2b)}}$

$= b(a-2b)$

(l) $\frac{y^2-4y+4}{2-6y} \times \frac{2y+4}{3y^2-12} = \frac{(y-2)^2}{2(1-3y)} \times \frac{2(y+2)}{3(y^2-4)}$

$= \frac{\overset{(y-2)}{\cancel{(y-2)^2}}}{\cancel{2}(1-3y)} \times \frac{\cancel{2}\ \cancel{(y+2)}}{3\cancel{(y+2)}\ \cancel{(y-2)}}$

$= \frac{y-2}{3(1-3y)}$

7. $\frac{x^2+y^2-z^2+2xy}{x^2-y^2-z^2-2yz} = \frac{(x^2+2xy+y^2)-z^2}{x^2-(y^2+2yz+z^2)}$

$= \frac{(x+y)^2-z^2}{x^2-(y+z)^2}$

$= \frac{[(x+y)+z][(x+y)-z]}{[x+(y+z)][x-(y+z)]}$

$= \frac{\cancel{(x+y+z)}(x+y-z)}{\cancel{(x+y+z)}(x-y-z)}$

$= \frac{x+y-z}{x-y-z}$

Exercise 6B

1. (a) $\frac{7}{6a} + \frac{4}{9a} = \frac{63}{54a} + \frac{24}{54a}$

$= \frac{63+24}{54a}$

$= \frac{87}{54a}$

$= \frac{29}{18a}$

(b) $\frac{3}{2b} + \frac{1}{3b} - \frac{5}{6b} = \frac{9}{6b} + \frac{2}{6b} - \frac{5}{6b}$

$= \frac{9+2-5}{6b}$

$= \frac{6}{6b}$

$= \frac{1}{b}$

(c) $\frac{1}{3c} - \frac{1}{3d} = \frac{d}{3cd} - \frac{c}{3cd}$

$= \frac{d-c}{3cd}$

(d) $\frac{f-4h}{3k} - \frac{2f-5h}{8k} = \frac{8(f-4h)}{24k} - \frac{3(2f-5h)}{24k}$

$= \frac{8f-32h-6f+15h}{24k}$

$= \frac{2f-17h}{24k}$

(e) $\frac{4a}{x-3y} + \frac{3a}{3x-9y} = \frac{12a}{3x-9y} + \frac{3a}{3x-9y}$

$= \frac{12a+3a}{3x-9y}$

$= \frac{15a}{3x-9y}$

$= \frac{15a}{3(x-3y)}$

$= \frac{5a}{x-3y}$

(f)
$$\begin{aligned}\frac{p+3}{2z}+\frac{p-1}{6z}-\frac{2p+1}{3z} &= \frac{3(p+3)}{6z}+\frac{p-1}{6z}-\frac{2(2p+1)}{6z}\\ &= \frac{3(p+3)+(p-1)-2(2p+1)}{6z}\\ &= \frac{3p+9+p-1-4p-2}{6z}\\ &= \frac{6}{6z}\\ &= \frac{1}{z}\end{aligned}$$

2. (a)
$$\begin{aligned}\frac{5}{a}+\frac{3}{a+4} &= \frac{5(a+4)}{a(a+4)}+\frac{3a}{a(a+4)}\\ &= \frac{5(a+4)+3a}{a(a+4)}\\ &= \frac{5a+20+3a}{a(a+4)}\\ &= \frac{8a+20}{a(a+4)}\end{aligned}$$

(b)
$$\begin{aligned}\frac{1}{2b}-\frac{3}{b+c} &= \frac{b+c}{2b(b+c)}-\frac{3(2b)}{2b(b+c)}\\ &= \frac{b+c-3(2b)}{2b(b+c)}\\ &= \frac{b+c-6b}{2b(b+c)}\\ &= \frac{c-5b}{2b(b+c)}\end{aligned}$$

(c)
$$\begin{aligned}\frac{4}{d-5}+\frac{2}{2d+3} &= \frac{4(2d+3)}{(d-5)(2d+3)}+\frac{2(d-5)}{(d-5)(2d+3)}\\ &= \frac{4(2d+3)+2(d-5)}{(d-5)(2d+3)}\\ &= \frac{8d+12+2d-10}{(d-5)(2d+3)}\\ &= \frac{10d+2}{(d-5)(2d+3)}\end{aligned}$$

(d)
$$\begin{aligned}\frac{2}{f+5}-\frac{3}{f-1} &= \frac{2(f-1)}{(f+5)(f-1)}-\frac{3(f+5)}{(f+5)(f-1)}\\ &= \frac{2(f-1)-3(f+5)}{(f+5)(f-1)}\\ &= \frac{2f-2-3f-15}{(f+5)(f-1)}\\ &= \frac{-f-17}{(f+5)(f-1)}\end{aligned}$$

(e)
$$\begin{aligned}\frac{11}{3h-7}+\frac{2}{6-5h} &= \frac{11(6-5h)}{(3h-7)(6-5h)}+\frac{2(3h-7)}{(3h-7)(6-5h)}\\ &= \frac{11(6-5h)+2(3h-7)}{(3h-7)(6-5h)}\\ &= \frac{66-55h+6h-14}{(3h-7)(6-5h)}\\ &= \frac{52-49h}{(3h-7)(6-5h)}\end{aligned}$$

(f)
$$\begin{aligned}\frac{3}{k^2-1}+\frac{2}{k-1} &= \frac{3}{(k+1)(k-1)}+\frac{2}{k-1}\\ &= \frac{3}{(k+1)(k-1)}+\frac{2(k+1)}{(k+1)(k-1)}\\ &= \frac{3+2(k+1)}{(k+1)(k-1)}\\ &= \frac{3+2k+2}{(k+1)(k-1)}\\ &= \frac{2k+5}{(k+1)(k-1)}\end{aligned}$$

(g)
$$\begin{aligned}\frac{3}{4m^2-1}-\frac{5}{2m+1} &= \frac{3}{(2m+1)(2m-1)}-\frac{5}{2m+1}\\ &= \frac{3}{(2m+1)(2m-1)}-\frac{5(2m-1)}{(2m+1)(2m-1)}\\ &= \frac{3-5(2m-1)}{(2m+1)(2m-1)}\\ &= \frac{3-10m+5}{(2m+1)(2m-1)}\\ &= \frac{8-10m}{(2m+1)(2m-1)}\end{aligned}$$

(h)
$$\begin{aligned}\frac{2}{n-2}+\frac{3}{(n-2)^2} &= \frac{2(n-2)}{(n-2)^2}+\frac{3}{(n-2)^2}\\ &= \frac{2(n-2)+3}{(n-2)^2}\\ &= \frac{2n-4+3}{(n-2)^2}\\ &= \frac{2n-1}{(n-2)^2}\end{aligned}$$

3. (a)
$$\begin{aligned}\frac{5}{2(a-b)}+\frac{4}{3(b-a)} &= \frac{5}{2(a-b)}+\frac{4}{-3(a-b)}\\ &= \frac{5}{2(a-b)}-\frac{4}{3(a-b)}\\ &= \frac{15}{6(a-b)}-\frac{8}{6(a-b)}\\ &= \frac{15-8}{6(a-b)}\\ &= \frac{7}{6(a-b)}\end{aligned}$$

(b)
$$\begin{aligned}\frac{c-1}{3c-7}-\frac{1}{14-6c} &= \frac{c-1}{3c-7}-\frac{1}{-2(3c-7)}\\ &= \frac{c-1}{3c-7}+\frac{1}{2(3c-7)}\\ &= \frac{2(c-1)}{2(3c-7)}+\frac{1}{2(3c-7)}\\ &= \frac{2(c-1)+1}{2(3c-7)}\\ &= \frac{2c-2+1}{2(3c-7)}\\ &= \frac{2c-1}{2(3c-7)}\end{aligned}$$

(c)
$$\begin{aligned}\frac{4f}{10f-5d}+\frac{2d}{6f-3d} &= \frac{4f}{5(2f-d)}+\frac{2d}{3(2f-d)}\\ &= \frac{12f}{15(2f-d)}+\frac{10d}{15(2f-d)}\\ &= \frac{12f+10d}{15(2f-d)}\end{aligned}$$

(d) $\dfrac{u+1}{2u-8} - \dfrac{u+2}{12-3u} = \dfrac{u+1}{2(u-4)} - \dfrac{u+2}{-3(u-4)}$

$= \dfrac{u+1}{2(u-4)} + \dfrac{u+2}{3(u-4)}$

$= \dfrac{3(u+1)}{6(u-4)} + \dfrac{2(u+2)}{6(u-4)}$

$= \dfrac{3(u+1)+2(u+2)}{6(u-4)}$

$= \dfrac{3u+3+2u+4}{6(u-4)}$

$= \dfrac{5u+7}{6(u-4)}$

(e) $\dfrac{2m-5}{9n-6} - \dfrac{m+3}{4-6n} = \dfrac{2m-5}{3(3n-2)} - \dfrac{m+3}{-2(3n-2)}$

$= \dfrac{2m-5}{3(3n-2)} + \dfrac{m+3}{2(3n-2)}$

$= \dfrac{2(2m-5)}{6(3n-2)} + \dfrac{3(m+3)}{6(3n-2)}$

$= \dfrac{2(2m-5)+3(m+3)}{6(3n-2)}$

$= \dfrac{4m-10+3m+9}{6(3n-2)}$

$= \dfrac{7m-1}{6(3n-2)}$

(f) $\dfrac{h+k}{p-q} + \dfrac{3h+k}{8q-8p} = \dfrac{h+k}{p-q} + \dfrac{3h+k}{-8(p-q)}$

$= \dfrac{h+k}{p-q} - \dfrac{3h+k}{8(p-q)}$

$= \dfrac{8(h+k)}{8(p-q)} - \dfrac{3h+k}{8(p-q)}$

$= \dfrac{8(h+k)-(3h+k)}{8(p-q)}$

$= \dfrac{8h+8k-3h-k}{8(p-q)}$

$= \dfrac{5h+7k}{8(p-q)}$

(g) $\dfrac{5x^2}{6x-6y} - \dfrac{2x^2}{3y-3x} = \dfrac{5x^2}{6(x-y)} - \dfrac{2x^2}{-3(x-y)}$

$= \dfrac{5x^2}{6(x-y)} + \dfrac{2x^2}{3(x-y)}$

$= \dfrac{5x^2}{6(x-y)} + \dfrac{4x^2}{6(x-y)}$

$= \dfrac{5x^2+4x^2}{6(x-y)}$

$= \dfrac{9x^2}{6(x-y)}$

$= \dfrac{3x^2}{2(x-y)}$

(h) $\dfrac{3x}{4y-2z} - \dfrac{2x}{z-2y} + \dfrac{5}{3z-6y}$

$= \dfrac{3x}{-2(z-2y)} - \dfrac{2x}{z-2y} + \dfrac{5}{3(z-2y)}$

$= -\dfrac{3x}{2(z-2y)} - \dfrac{2x}{z-2y} + \dfrac{5}{3(z-2y)}$

$= -\dfrac{9x}{6(z-2y)} - \dfrac{12x}{6(z-2y)} + \dfrac{10}{6(z-2y)}$

$= \dfrac{-9x-12x+10}{6(z-2y)}$

$= \dfrac{10-21x}{6(z-2y)}$

4. (a) $\dfrac{3a}{3a-5} + \dfrac{4a}{4a-1} = \dfrac{3a(4a-1)}{(3a-5)(4a-1)} + \dfrac{4a(3a-5)}{(3a-5)(4a-1)}$

$= \dfrac{3a(4a-1)+4a(3a-5)}{(3a-5)(4a-1)}$

$= \dfrac{12a^2-3a+12a^2-20a}{(3a-5)(4a-1)}$

$= \dfrac{24a^2-23a}{(3a-5)(4a-1)}$

(b) $\dfrac{5}{2b+1} - \dfrac{2b}{(2b+1)^2} = \dfrac{5(2b+1)}{(2b+1)^2} - \dfrac{2b}{(2b+1)^2}$

$= \dfrac{5(2b+1)-2b}{(2b+1)^2}$

$= \dfrac{10b+5-2b}{(2b+1)^2}$

$= \dfrac{8b+5}{(2b+1)^2}$

(c) $\dfrac{h+5}{h^2-6h} - \dfrac{3}{h-6} = \dfrac{h+5}{h(h-6)} - \dfrac{3}{h-6}$

$= \dfrac{h+5}{h(h-6)} - \dfrac{3h}{h(h-6)}$

$= \dfrac{h+5-3h}{h(h-6)}$

$= \dfrac{5-2h}{h(h-6)}$

(d) $\dfrac{1}{m} + \dfrac{2}{m-4} + \dfrac{3}{m-3}$

$= \dfrac{(m-4)(m-3)}{m(m-4)(m-3)} + \dfrac{2m(m-3)}{m(m-4)(m-3)} + \dfrac{3m(m-4)}{m(m-4)(m-3)}$

$= \dfrac{(m-4)(m-3)+2m(m-3)+3m(m-4)}{m(m-4)(m-3)}$

$= \dfrac{m^2-3m-4m+12+2m^2-6m+3m^2-12m}{m(m-4)(m-3)}$

$= \dfrac{6m^2-25m+12}{m(m-4)(m-3)}$

(e) $\frac{x+y}{x-y}+\frac{x^2-4y^2}{x^2-y^2}-\frac{x-3y}{x+y}$

$=\frac{x+y}{x-y}+\frac{x^2-4y^2}{(x+y)(x-y)}-\frac{x-3y}{x+y}$

$=\frac{(x+y)(x+y)}{(x+y)(x-y)}+\frac{x^2-4y^2}{(x+y)(x-y)}-\frac{(x-3y)(x-y)}{(x+y)(x-y)}$

$=\frac{(x+y)(x+y)+x^2-4y^2-(x-3y)(x-y)}{(x+y)(x-y)}$

$=\frac{x^2+xy+xy+y^2+x^2-4y^2-(x^2-xy-3xy+3y^2)}{(x+y)(x-y)}$

$=\frac{x^2+xy+xy+y^2+x^2-4y^2-x^2+4xy-3y^2}{(x+y)(x-y)}$

$=\frac{x^2+6xy-6y^2}{(x+y)(x-y)}$

(f) $\frac{1}{2z-3}-\frac{2}{3-2z}+\frac{18}{9-4z^2}$

$=\frac{1}{2z-3}-\frac{2}{-(2z-3)}+\frac{18}{-(2z+3)(2z-3)}$

$=\frac{1}{2z-3}+\frac{2}{2z-3}-\frac{18}{(2z+3)(2z-3)}$

$=\frac{1+2}{2z-3}-\frac{18}{(2z+3)(2z-3)}$

$=\frac{3(2z+3)}{(2z+3)(2z-3)}-\frac{18}{(2z+3)(2z-3)}$

$=\frac{3(2z+3)-18}{(2z+3)(2z-3)}$

$=\frac{6z+9-18}{(2z+3)(2z-3)}$

$=\frac{6z-9}{(2z+3)(2z-3)}$

$=\frac{3(2z-3)}{(2z+3)(2z-3)}$

$=\frac{3}{2z+3}$

5. **(a)** $\frac{2}{a+3}+\frac{3}{a^2+4a+3}=\frac{2}{a+3}+\frac{3}{(a+3)(a+1)}$

$=\frac{2(a+1)}{(a+3)(a+1)}+\frac{3}{(a+3)(a+1)}$

$=\frac{2(a+1)+3}{(a+3)(a+1)}$

$=\frac{2a+2+3}{(a+3)(a+1)}$

$=\frac{2a+5}{(a+3)(a+1)}$

(b) $\frac{1}{b^2-5b-6}-\frac{b}{b-6}=\frac{1}{(b+1)(b-6)}-\frac{b}{b-6}$

$=\frac{1}{(b+1)(b-6)}-\frac{b(b+1)}{(b+1)(b-6)}$

$=\frac{1-b(b+1)}{(b+1)(b-6)}$

$=\frac{1-b^2-b}{(b+1)(b-6)}$

$=\frac{-b^2-b+1}{(b+1)(b-6)}$

(c) $\frac{1}{2p^2-8p-10}+\frac{2p}{p-5}$

$=\frac{1}{2(p+1)(p-5)}+\frac{2p}{p-5}$

$=\frac{1}{2(p+1)(p-5)}+\frac{2p[2(p+1)]}{2(p+1)(p-5)}$

$=\frac{1+2p[2(p+1)]}{2(p+1)(p-5)}$

$=\frac{1+2p(2p+2)}{2(p+1)(p-5)}$

$=\frac{1+4p^2+4p}{2(p+1)(p-5)}$

$=\frac{4p^2+4p+1}{2(p+1)(p-5)}$

(d) $\frac{x}{x+y}+\frac{4}{x^2+3xy+2y^2}-\frac{3x}{x+2y}$

$=\frac{x}{x+y}+\frac{4}{(x+y)(x+2y)}-\frac{3x}{x+2y}$

$=\frac{x(x+2y)}{(x+y)(x+2y)}+\frac{4}{(x+y)(x+2y)}-\frac{3x(x+y)}{(x+y)(x+2y)}$

$=\frac{x(x+2y)+4-3x(x+y)}{(x+y)(x+2y)}$

$=\frac{x^2+2xy+4-3x^2-3xy}{(x+y)(x+2y)}$

$=\frac{-2x^2-xy+4}{(x+y)(x+2y)}$

6. $\frac{\frac{1}{3x}+\frac{2}{y}}{\frac{2}{x}}=\frac{\frac{y}{3xy}+\frac{6x}{3xy}}{\frac{2}{x}}$

$=\frac{\frac{y+6x}{3xy}}{\frac{2}{x}}$

$=\frac{y+6x}{3xy}\div\frac{2}{x}$

$=\frac{y+6x}{3xy}\times\frac{x}{2}$

$=\frac{y+6x}{6y}$

Exercise 6C

1. **(a)** $ax+by=k$

$ax+by-ax=k-ax$

$by=k-ax$

$\frac{by}{b}=\frac{k-ax}{b}$

$\therefore y=\frac{k-ax}{b}$

(b) $PV=nRT$

$\frac{PV}{RT}=\frac{nRT}{RT}$

$\therefore n=\frac{PV}{RT}$

(c) $5b - 2d = 3c$

$$5b - 2d + 2d = 3c + 2d$$
$$5b = 3c + 2d$$
$$5b - 3c = 3c + 2d - 3c$$
$$5b - 3c = 2d$$
$$\frac{5b - 3c}{2} = \frac{2d}{2}$$
$$\therefore d = \frac{5b - 3c}{2}$$

(d) $R = m(a + g)$

$$\frac{R}{m} = \frac{m(a + g)}{m}$$
$$\frac{R}{m} = a + g$$
$$\frac{R}{m} - g = a + g - g$$
$$\therefore a = \frac{R}{m} - g$$

2. **(a)** $\frac{a}{m} = b + c$

$$m \times \frac{a}{m} = m \times (b + c)$$
$$\therefore a = m(b + c)$$

(b) $5q - r = \frac{2p}{3}$

$$\frac{3}{2} \times (5p - r) = \frac{3}{2} \times \frac{2p}{3}$$
$$\therefore p = \frac{3}{2}(5p - r)$$

(c) $\frac{k + a}{5} = 3k$

$$5 \times \frac{k + a}{5} = 5 \times 3k$$
$$k + a = 15k$$
$$k + a - k = 15k - k$$
$$a = 14k$$
$$\frac{a}{14} = \frac{14k}{14}$$
$$\therefore k = \frac{a}{14}$$

(d) $A = \frac{1}{2}(a + b)h$

$$2 \times A = 2 \times \frac{1}{2}(a + b)h$$
$$2A = (a + b)h$$
$$\frac{2A}{h} = \frac{(a + b)h}{h}$$
$$\frac{2A}{h} = a + b$$
$$\frac{2A}{h} - a = a + b - a$$
$$\therefore b = \frac{2A}{h} - a$$

3. **(a)** $\sqrt[3]{h - k} = m$

$$h - k = m^3$$
$$h - k + k = m^3 + k$$
$$\therefore h = m^3 + k$$

(b) $b = \sqrt{D + 4ac}$

$$b^2 = D + 4ac$$
$$b^2 - 4ac = D + 4ac - 4ac$$
$$\therefore D = b^2 - 4ac$$

(c) $P = \frac{V^2}{R}$

$$P \times R = \frac{V^2}{R} \times R$$
$$PR = V^2$$
$$\pm\sqrt{PR} = V$$
$$\therefore V = \pm\sqrt{PR}$$

(d) $A = \frac{\theta}{360} \times \pi r^2$

$$360 \times A = 360 \times \frac{\theta}{360} \times \pi r^2$$
$$360A = \theta \times \pi r^2$$
$$\frac{360A}{\pi r^2} = \frac{\theta \times \pi r^2}{\pi r^2}$$
$$\therefore \theta = \frac{360A}{\pi r^2}$$

4. When $a = 2$, $b = 7$ and $c = 5$,

$$\sqrt{2x^2 - 7} = 5$$
$$2x^2 - 7 = 25$$
$$2x^2 = 32$$
$$x^2 = 16$$
$$\therefore x = \pm\sqrt{16}$$
$$= \pm 4$$

5. **(a)** When $b = 7$ and $c = 2$,

$$a = \sqrt{\frac{3(7) + 2}{7 - 2}}$$
$$= \sqrt{\frac{21 + 2}{7 - 2}}$$
$$= \sqrt{\frac{23}{5}}$$

(b) When $a = 4$ and $b = 9$,

$$4 = \sqrt{\frac{3(9) + c}{9 - c}}$$
$$= \sqrt{\frac{27 + c}{9 - c}}$$
$$16 = \frac{27 + c}{9 - c}$$
$$16(9 - c) = 27 + c$$
$$144 - 16c = 27 + c$$
$$144 = 27 + 17c$$
$$117 = 17c$$
$$\therefore c = 6\frac{15}{17}$$

6. **(a)**
$$\frac{a}{a+2} = \frac{3}{5}$$
$$5a = 3(a+2)$$
$$5a = 3a+6$$
$$2a = 6$$
$$\therefore a = 3$$

(b)
$$\frac{1}{b-2} = \frac{2}{b-1}$$
$$b-1 = 2(b-2)$$
$$b-1 = 2b-4$$
$$b+3 = 2b$$
$$\therefore b = 3$$

(c)
$$\frac{4}{c+3} - \frac{3}{c+2} = 0$$
$$4(c+2) - 3(c+3) = 0$$
$$4c+8-3c-9 = 0$$
$$c-1 = 0$$
$$\therefore c = 1$$

(d)
$$\frac{5}{d+4} - \frac{2}{d-2} = 0$$
$$5(d-2) - 2(d+4) = 0$$
$$5d-10-2d-8 = 0$$
$$3d-18 = 0$$
$$3d = 18$$
$$\therefore d = 6$$

(e)
$$\frac{6}{f} - \frac{10}{3f} = 2$$
$$18-10 = 6f$$
$$8 = 6f$$
$$\therefore f = 1\frac{1}{3}$$

(f)
$$\frac{5}{6h} + \frac{6}{7h} - \frac{9}{14h} = 4$$
$$35+36-27 = 168h$$
$$44 = 168h$$
$$\therefore h = \frac{11}{42}$$

(g)
$$\frac{3}{k+1} - \frac{1}{2k+2} = 5$$
$$6-1 = 5(2k+2)$$
$$5 = 10k+10$$
$$-5 = 10k$$
$$\therefore k = -\frac{1}{2}$$

7. **(a)**
$$F = \frac{9}{5}C + 32$$
$$F - 32 = \frac{9}{5}C + 32 - 32$$
$$F - 32 = \frac{9}{5}C$$
$$\frac{5}{9} \times (F-32) = \frac{5}{9} \times \frac{9}{5}C$$
$$\therefore C = \frac{5}{9}(F-32)$$

(b)
$$A = 2\pi r^2 + \pi rl$$
$$A - 2\pi r^2 = 2\pi r^2 + \pi rl - 2\pi r^2$$
$$A - 2\pi r^2 = \pi rl$$
$$\frac{A-2\pi r^2}{\pi r} = \frac{\pi rl}{\pi r}$$
$$\therefore l = \frac{A-2\pi r^2}{\pi r}$$

(c)
$$s = ut + \frac{1}{2}at^2$$
$$s - \frac{1}{2}at^2 = ut + \frac{1}{2}at^2 - \frac{1}{2}at^2$$
$$s - \frac{1}{2}at^2 = ut$$
$$\frac{s-\frac{1}{2}at^2}{t} = \frac{ut}{t}$$
$$\therefore u = \frac{s}{t} - \frac{1}{2}at$$

(d)
$$S = \frac{n}{2}[2a + (n-1)d]$$
$$\frac{2}{n} \times S = \frac{2}{n} \times \frac{n}{2}[2a+(n-1)d]$$
$$\frac{2S}{n} = 2a + (n-1)d$$
$$\frac{2S}{n} - 2a = 2a + (n-1)d - 2a$$
$$\frac{2S-2an}{n} = (n-1)d$$
$$\frac{2S-2an}{n(n-1)} = \frac{(n-1)d}{n-1}$$
$$\therefore d = \frac{2S-2an}{n(n-1)}$$

8. **(a)**
$$\frac{1}{h+1} + 2 = k$$
$$\frac{1}{h+1} + 2 - 2 = k-2$$
$$\frac{1}{h+1} = k-2$$
$$(h+1) \times \frac{1}{h+1} = (h+1)(k-2)$$
$$1 = (h+1)(k-2)$$
$$\frac{1}{k-2} = h+1$$
$$\frac{1}{k-2} - 1 = h+1-1$$
$$\therefore h = \frac{1}{k-2} - 1$$
$$= \frac{1-(k-2)}{k-2}$$
$$= \frac{1-k+2}{k-2}$$
$$= \frac{3-k}{k-2}$$

(b)
$$z = \frac{y(z-y)}{x}$$
$$x \times z = x \times \frac{y(z-y)}{x}$$
$$xz = y(z-y)$$
$$xz = yz - y^2$$
$$xz - yz = yz - y^2 - yz$$
$$xz - yz = -y^2$$
$$z(x-y) = -y^2$$
$$\frac{z(x-y)}{x-y} = -\frac{y^2}{x-y}$$
$$\therefore z = -\frac{y^2}{x-y}$$
$$= \frac{y^2}{y-x}$$

(c)
$$\frac{px}{q} = p + q$$
$$q \times \left(\frac{px}{q}\right) = q \times (p+q)$$
$$px = pq + q^2$$
$$px - pq = pq + q^2 - pq$$
$$px - pq = q^2$$
$$\frac{p(x-q)}{x-q} = \frac{q^2}{x-q}$$
$$\therefore p = \frac{q^2}{x-q}$$

(d)
$$\frac{1}{a} + \frac{1}{b} = 1$$
$$ab \times \left(\frac{1}{a} + \frac{1}{b}\right) = ab$$
$$b + a = ab$$
$$b + a - b = ab - b$$
$$a = ab - b$$
$$a = b(a-1)$$
$$\frac{a}{a-1} = \frac{b(a-1)}{a-1}$$
$$\therefore b = \frac{a}{a-1}$$

9. (a)
$$V = \frac{4}{3}\pi r^3$$
$$\frac{3}{4\pi} \times V = \frac{3}{4\pi} \times \frac{4}{3}\pi r^3$$
$$\frac{3V}{4\pi} = r^3$$
$$\therefore r = \sqrt[3]{\frac{3V}{4\pi}}$$

(b)
$$v^2 = u^2 + 2as$$
$$v^2 - 2as = u^2 + 2as - 2as$$
$$v^2 - 2as = u^2$$
$$\therefore u = \pm\sqrt{v^2 - 2as}$$

(c)
$$y = (x-p)^2 + q$$
$$y - q = (x-p)^2 + q - q$$
$$y - q = (x-p)^2$$
$$\pm\sqrt{y-q} = x - p$$
$$p \pm \sqrt{y-q} = p + x - p$$
$$\therefore x = p \pm \sqrt{y-q}$$

(d)
$$t = \sqrt{\frac{4z}{m-3}}$$
$$t^2 = \frac{4z}{m-3}$$
$$t^2 \times (m-3) = \frac{4z}{m-3} \times (m-3)$$
$$t^2(m-3) = 4z$$
$$\frac{t^2(m-3)}{4} = \frac{4z}{4}$$
$$\therefore z = \frac{t^2(m-3)}{4}$$

10. (i)
$$V = \pi r^2 h + \frac{2}{3}\pi r^3$$
$$V - \frac{2}{3}\pi r^3 = \pi r^2 h + \frac{2}{3}\pi r^3 - \frac{2}{3}\pi r^3$$
$$V - \frac{2}{3}\pi r^3 = \pi r^2 h$$
$$\frac{V - \frac{2}{3}\pi r^3}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2}$$
$$\therefore h = \frac{V - \frac{2}{3}\pi r^3}{\pi r^2}$$
$$= \frac{V}{\pi r^2} - \frac{2}{3}r$$

(ii) When $V = 245$ and $r = 7$,
$$h = \frac{245}{\pi(7^2)} - \frac{2}{3}(7)$$
$$= -3.08 \text{ (to 3 s.f.)}$$

11. (i)
$$a = \sqrt{\frac{3b+c}{b-c}}$$
$$a^2 = \frac{3b+c}{b-c}$$
$$a^2 \times (b-c) = 3b + c$$
$$a^2b - a^2c = 3b + c$$
$$a^2b - a^2c + a^2c = 3b + c + a^2c$$
$$a^2b = 3b + c + a^2c$$
$$a^2b - 3b = 3b + c + a^2c - 3b$$
$$a^2b - 3b = c + a^2c$$
$$b(a^2 - 3) = c + a^2c$$
$$\frac{b(a^2-3)}{a^2-3} = \frac{c + a^2c}{a^2-3}$$
$$\therefore b = \frac{c + a^2c}{a^2-3}$$

(ii) When $a = 2$ and $c = 5$,

$$b = \frac{5 + 2^2(5)}{2^2 - 3}$$
$$= \frac{5 + 20}{4 - 3}$$
$$= 25$$

12. (a) When $m = 5$, $n = 7$, $x = 4$ and $y = -2$,

$$\frac{5[(7 \times 4) - (-2)^2]}{p} = 3 \times 7$$
$$\frac{5(28 - 4)}{p} = 21$$
$$\frac{5 \times 24}{p} = 21$$
$$\frac{120}{p} = 21$$
$$120 = 21p$$
$$\therefore p = 5\frac{5}{7}$$

(b) When $m = 14$, $p = 9$, $x = 2$ and $y = 3$,

$$\frac{14[n(2) - 3^2]}{9} = 3n$$
$$\frac{14(2n - 9)}{9} = 3n$$
$$14(2n - 9) = 27n$$
$$28n - 126 = 27n$$
$$n - 126 = 0$$
$$\therefore n = 126$$

(c) When $m = 5$, $n = 4$, $p = 15$ and $x = 42$,

$$\frac{5[4(42) - y^2]}{15} = 3(4)$$
$$\frac{5(168 - y^2)}{15} = 12$$
$$5(168 - y^2) = 180$$
$$168 - y^2 = 36$$
$$132 - y^2 = 0$$
$$y^2 = 132$$
$$\therefore y = \pm\sqrt{132}$$
$$= \pm 11.5 \text{ (to 3 s.f.)}$$

13. (a) When $\pi = 3.142$, $h = 15$ and $r = 7$,

$$A = \frac{1}{3}(3.142)(7^2)(15) + \frac{4}{3}(3.142)(7^3)$$
$$= 769.79 + 1436.94$$
$$= 2210 \text{ (to 3 s.f.)}$$

(b) When $\pi = 3.142$, $A = 15\,400$ and $r = 14$,

$$15\,400 = \frac{1}{3}(3.142)(14^2)h + \frac{4}{3}(3.142)(14^3)$$
$$15\,400 = 205.277h + 11\,495.531$$
$$3904.469 = 205.277h$$
$$\therefore h = 19.0 \text{ (to 3 s.f.)}$$

14. (a)

$$2 - \frac{5}{x + 2} = 1\frac{3}{5}$$
$$10(x + 2) - 25 = 8(x + 2)$$
$$10x + 20 - 25 = 8x + 16$$
$$10x - 5 = 8x + 16$$
$$10x = 8x + 21$$
$$2x = 21$$
$$\therefore x = 10\frac{1}{2}$$

(b)

$$\frac{7x - 4}{15} + \frac{x - 1}{3} = \frac{3x - 1}{5} - \frac{7 + x}{10}$$
$$2(7x - 4) + 10(x - 1) = 6(3x - 1) - 3(7 + x)$$
$$14x - 8 + 10x - 10 = 18x - 6 - 21 - 3x$$
$$24x - 18 = 15x - 27$$
$$24x = 15x - 9$$
$$9 = -9$$
$$\therefore x = -1$$

(c)

$$\frac{x + 1}{5x - 1} + \frac{1}{2(1 - 5x)} = \frac{1}{4}$$
$$4(x + 1) - 2 = 5x - 1$$
$$4x + 4 - 2 = 5x - 1$$
$$4x + 2 = 5x - 1$$
$$4x + 3 = 5x$$
$$\therefore x = 3$$

(d)

$$\frac{5}{2x - 1} - \frac{4}{4x - 2} - \frac{3}{6x - 3} = 1$$
$$\frac{5}{2x - 1} - \frac{4}{2(2x - 1)} - \frac{3}{3(2x - 1)} = 1$$
$$\frac{5}{2x - 1} - \frac{2}{2x - 1} - \frac{1}{2x - 1} = 1$$
$$\frac{5 - 2 - 1}{2x - 1} = 1$$
$$\frac{2}{2x - 1} = 1$$
$$2 = 2x - 1$$
$$3 = 2x$$
$$\therefore x = 1\frac{1}{2}$$

(e)

$$\frac{3}{2 - x} + \frac{5}{4 - 2x} - \frac{1}{x - 2} = 4$$
$$\frac{3}{2 - x} + \frac{5}{2(2 - x)} - \frac{1}{-(2 - x)} = 4$$
$$\frac{3}{2 - x} + \frac{5}{2(2 - x)} + \frac{1}{2 - x} = 4$$
$$\frac{6}{2(2 - x)} + \frac{5}{2(2 - x)} + \frac{2}{2(2 - x)} = 4$$
$$\frac{6 + 5 + 2}{2(2 - x)} = 4$$
$$\frac{13}{2(2 - x)} = 4$$
$$13 = 8(2 - x)$$
$$13 = 16 - 8x$$
$$-3 = -8x$$
$$\therefore x = \frac{3}{8}$$

15. (i)

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$
$$uv = fv + fu$$
$$uv - fv = fv + fu - fv$$
$$uv - fv = fu$$
$$v(u - f) = fu$$
$$\frac{v(u-f)}{u-f} = \frac{fu}{u-f}$$
$$\therefore v = \frac{fu}{u-f}$$

(ii) When $f = 20$ and $u = 30$,

$$v = \frac{20 \times 30}{30 - 20}$$
$$= \frac{600}{10}$$
$$= 60$$

$\therefore$ The image distance is 60 cm.

16. (i)

$$T = 2\pi\sqrt{\frac{l}{g}}$$
$$\frac{T}{2n} = \frac{2\pi\sqrt{\frac{l}{g}}}{2\pi}$$
$$\frac{T}{2n} = \sqrt{\frac{l}{g}}$$
$$\left(\frac{T}{2\pi}\right)^2 = \frac{l}{g}$$
$$g \times \left(\frac{T}{2\pi}\right)^2 = g \times \frac{l}{g}$$
$$\therefore l = g\left(\frac{T}{2\pi}\right)^2$$

(ii) When $T = \frac{12}{20}$ and $g = 10$,

$$l = 10 \times \left(\frac{\frac{12}{20}}{2\pi}\right)^2$$
$$= 10 \times \left(\frac{3}{10\pi}\right)^2$$
$$= 0.0912 \text{ (to 3 s.f.)}$$

$\therefore$ The length of the pendulum is 0.0912 m.

17. (i)

$$E = mgh + \frac{1}{2}mv^2$$
$$E - mgh = mgh + \frac{1}{2}mv^2 - mgh$$
$$E - mgh = \frac{1}{2}mv^2$$
$$\frac{2}{m} \times (E - mgh) = \frac{2}{m} \times \frac{1}{2}mv^2$$
$$\frac{2(E - mgh)}{m} = v^2$$
$$\therefore v = \sqrt{\frac{2(E - mgh)}{m}} \quad (v > 0)$$

(ii) When $m = 0.5$, $h = 2$, $E = 100$ and $g = 10$,

$$v = \sqrt{\frac{2[100 - 0.5(10)(2)]}{0.5}}$$
$$= \sqrt{\frac{2(100 - 10)}{0.5}}$$
$$= \sqrt{\frac{2(90)}{0.5}}$$
$$= \sqrt{\frac{180}{0.5}}$$
$$= \sqrt{360}$$
$$= 19.0 \text{ (to 3 s.f.)}$$

$\therefore$ The velocity of the objects is 19.0 m s^{-1}.

18.

$$\frac{\frac{1}{x} + \frac{1}{y}}{\frac{2}{x}} = \frac{4}{3}$$
$$3\left(\frac{1}{x} + \frac{1}{y}\right) = \frac{8}{x}$$
$$\frac{3}{x} + \frac{3}{y} = \frac{8}{x}$$
$$\frac{3}{y} = \frac{5}{x}$$
$$3x = 5y$$
$$\frac{5y}{x} = 3$$
$$\therefore \frac{y}{x} = \frac{3}{5}$$

19. (a) When $a = 13$, $b = 15$ and $x = 3.8$,

$$y = 3(3.8) + \sqrt[3]{13 + 15^2}$$
$$= 11.4 + \sqrt[3]{13 + 225}$$
$$= 11.4 + \sqrt[3]{238}$$
$$= 11.4 + 6.197$$
$$= 17.6 \text{ (to 3 s.f.)}$$

(b) When $b = 13$, $x = 8.5$ and $y = 35$,

$$35 = 3(8.5) + \sqrt[3]{a + 13^2}$$
$$35 = 25.5 + \sqrt[3]{a + 169}$$
$$9.5 = \sqrt[3]{a + 169}$$
$$857.375 = a + 169$$
$$\therefore a = 688 \text{ (to 3 s.f.)}$$

(c) When $a = 23$, $x = 15.6$ and $y = 56$,

$$56 = 3(15.6) + \sqrt[3]{23 + b^2}$$
$$56 = 46.8 + \sqrt[3]{23 + b^2}$$
$$9.2 = \sqrt[3]{23 + b^2}$$
$$778.688 = 23 + b^2$$
$$755.688 = b^2$$
$$\therefore b = \pm\sqrt{755.688}$$
$$= \pm 27.5 \text{ (to 3 s.f.)}$$

20. **(a)** $R = \frac{kl}{r^2}$

(b)
$$R = \frac{kl}{r^2}$$
$$R \times r^2 = \frac{kl}{r^2} \times r^2$$
$$Rr^2 = kl$$
$$\frac{Rr^2}{R} = \frac{kl}{R}$$
$$r^2 = \frac{kl}{R}$$
$$\therefore r = \sqrt{\frac{kl}{R}} \quad (r > 0)$$

(c) **(i)** When $l = 2$, $r = \frac{0.5}{100}$ and $R = 40$,
$$40 = \frac{2k}{\left(\frac{0.5}{100}\right)^2}$$
$$40 = \frac{2k}{0.000\,025}$$
$$0.001 = 2k$$
$$\therefore k = 0.0005$$

(ii) From **(i)**, when $k = 0.0005$, $l = 4$ and $R = 60$,
$$r = \sqrt{\frac{0.0005 \times 4}{60}}$$
$= 0.00577$ m (to 3 s.f.)
$= 0.577$ cm

Review Exercise 6

1. **(a)** $$\frac{\cancel{24}^{4}\,\cancel{x^2}^{1}\,\cancel{y^2}^{y}}{\cancel{6}_{1}\,\cancel{x^3}_{x}\,\cancel{y}_{1}} = \frac{4y}{x}$$

(b) $$\frac{\cancel{x^2}^{x}\,\cancel{(c-d)}^{1}\,(c+3d)}{5\cancel{x}\,\cancel{(c-d)^2}_{(c-d)}} = \frac{x(c+3d)}{5(c-d)}$$

(c) $$\frac{5a+10b}{10ac+20bc} = \frac{\cancel{5}^{1}\,\cancel{(a+2b)}^{1}}{\cancel{10}_{2}\,c\,\cancel{(a+2b)}_{1}}$$
$$= \frac{1}{2c}$$

(d) $$\frac{3f+4d}{(6f+8d)^2} = \frac{3f+4d}{2^2(3f+4d)^2}$$
$$= \frac{\cancel{3f+4d}^{1}}{4\cancel{(3f+4d)^2}_{(3f+4d)}}$$
$$= \frac{1}{4(3f+4d)}$$

(e) $$\frac{p^2-4p+4}{p^2-2p} = \frac{\cancel{(p-2)^2}^{(p-2)}}{p\,\cancel{(p-2)}_{1}}$$
$$= \frac{p-2}{p}$$

(f) $$\frac{3r-qs+3s-qr}{9+q^2-6q} = \frac{-qr+3r-qs+3s}{q^2-6q+9}$$
$$= \frac{-r(q-3)-s(q-3)}{(q-3)^2}$$
$$= \frac{-(r+s)\,\cancel{(q-3)}^{1}}{\cancel{(q-3)^2}_{(q-3)}}$$
$$= \frac{-(r+s)}{q-3}$$

(g) $$\frac{3w^2+8w+4}{3w^2-12} = \frac{(3w+2)\,\cancel{(w+2)}^{1}}{3\,\cancel{(w+2)}_{1}\,(w-2)}$$
$$= \frac{3w+2}{3(w-2)}$$

(h) $$\frac{z^2+7z+6}{3z^2+9z+6} = \frac{\cancel{(z+1)}^{1}\,(z+6)}{3\,\cancel{(z+1)}_{1}\,(z+2)}$$
$$= \frac{z+6}{2(z+2)}$$

2. **(a)** $$\frac{3\,\cancel{a^2}^{a}\,\cancel{b}}{4\cancel{c}} \times \frac{\cancel{c^3}^{c^2}}{\cancel{ab}} = \frac{3ac^2}{4}$$

(b) $$\frac{27}{d-3f} \div \frac{6}{9f-3d} = \frac{27}{d-3f} \div \frac{6}{-3(d-3f)}$$
$$= \frac{27}{d-3f} \times \frac{\cancel{-3}^{-1}\,(d-3f)}{\cancel{6}_{2}}$$
$$= -\frac{27}{2}$$
$$= -13\frac{1}{2}$$

(c) $$\frac{10k}{k-5} \times \frac{3k^2-15k}{5k^2} = \frac{10k}{k-5} \times \frac{3k(k-5)}{5k^2}$$
$$= \frac{\cancel{30}^{6}\,\cancel{k^2}\,\cancel{(k-5)}}{\cancel{5}\,\cancel{k^2}\,\cancel{(k-5)}}$$
$$= 6$$

(d) $$\frac{4x-1}{4x-4y} \div \frac{1}{9x-9y} = \frac{4x-1}{4(x-y)} \div \frac{1}{9(x-y)}$$
$$= \frac{4x-1}{4\,\cancel{(x-y)}} \times 9\cancel{(x-y)}$$
$$= \frac{9(4x-1)}{4}$$

(e) $$\frac{mn-4m+2n^2-8n}{m+2n} \div \frac{4-n}{n}$$
$$= \frac{m(n-4)+2n(n-4)}{(m+2n)} \div \frac{-(n-4)}{n}$$
$$= \frac{(m+2n)(n-4)}{m+2n} \div \frac{-(n-4)}{n}$$
$$= \frac{\cancel{(m+2n)}\,\cancel{(n-4)}}{\cancel{m+2n}} \times \frac{n}{-\cancel{(n-4)}}$$
$$= -n$$

(f) $\dfrac{7(p-q)^2}{\cancel{pq}} \times \dfrac{2\,\cancel{p^3}^{\,p^2}\,\cancel{q^2}^{\,q}}{(p-q^2)^2} = \dfrac{14p^2q(p-q)^2}{(p-q^2)^2}$

(g) $\dfrac{3x}{x^2-2x+1} \times \dfrac{2}{x^2} \div \dfrac{1}{x(x-1)} = \dfrac{3x}{(x-1)^2} \times \dfrac{2}{x^2} \div \dfrac{1}{x(x-1)}$

$= \dfrac{3x}{(x-1)^2} \times \dfrac{2}{x^2} \times x(x-1)$

$= \dfrac{6\cancel{x^2}\,\cancel{(x-1)}}{\cancel{x^2}\,\cancel{(x-1)^2}_{(x-1)}}$

$= \dfrac{6}{x-1}$

(h) $\dfrac{y^2-16}{2y^2-12y+16} \div \dfrac{2}{y-2} = \dfrac{(y+4)(y-4)}{2(y-2)(y-4)} \div \dfrac{2}{y-2}$

$= \dfrac{(y+4)\cancel{(y-4)}}{2\cancel{(y-2)}\,\cancel{(y-4)}} \times \dfrac{\cancel{(y-2)}}{2}$

$= \dfrac{y+4}{4}$

3. (a) $\dfrac{5}{4a} - \dfrac{3}{8a} = \dfrac{10}{8a} - \dfrac{3}{8a}$

$= \dfrac{10-3}{8a}$

$= \dfrac{7}{8a}$

(b) $\dfrac{5b}{3b+2c} + \dfrac{b}{9b+6c} = \dfrac{5b}{3b+2c} + \dfrac{b}{3(3b+2c)}$

$= \dfrac{15b}{3(3b+2c)} + \dfrac{b}{3(3b+2c)}$

$= \dfrac{15b+b}{3(3b+2c)}$

$= \dfrac{16b}{3(3b+2c)}$

(c) $\dfrac{9}{2d-4f} + \dfrac{7}{8f-4d} = \dfrac{9}{2(d-2f)} + \dfrac{7}{-4(d-2f)}$

$= \dfrac{9}{2(d-2f)} - \dfrac{7}{4(d-2f)}$

$= \dfrac{18}{4(d-2f)} - \dfrac{7}{4(d-2f)}$

$= \dfrac{18-7}{4(d-2f)}$

$= \dfrac{11}{4(d-2f)}$

(d) $\dfrac{h}{2h+5} + \dfrac{5}{6} = \dfrac{6h}{6(2h+5)} + \dfrac{5(2h+5)}{6(2h+5)}$

$= \dfrac{6h+5(2h+5)}{6(2h+5)}$

$= \dfrac{6h+10h+25}{6(2h+5)}$

$= \dfrac{16h+25}{6(2h+5)}$

(e) $\dfrac{3}{k-1} - \dfrac{2}{4k+5} = \dfrac{3(4k+5)}{(k-1)(4k+5)} - \dfrac{2(k-1)}{(k-1)(4k+5)}$

$= \dfrac{3(4k+5)-2(k-1)}{(k-1)(4k+5)}$

$= \dfrac{12k+15-2k+2}{(k-1)(4k+5)}$

$= \dfrac{10k+17}{(k-1)(4k+5)}$

(f) $\dfrac{m}{2m-5} + \dfrac{4}{8+3m}$

$= \dfrac{m(8+3m)}{(2m-5)(8+3m)} + \dfrac{4(2m-5)}{(2m-5)(8+3m)}$

$= \dfrac{m(8+3m)+4(2m-5)}{(2m-5)(8+3m)}$

$= \dfrac{8m+3m^2+8m-20}{(2m-5)(8+3m)}$

$= \dfrac{3m^2+16m-20}{(2m-5)(8+3m)}$

(g) $\dfrac{3n}{(2n-1)^2} - \dfrac{5}{2n-1} = \dfrac{3n}{(2n-1)^2} - \dfrac{5(2n-1)}{(2n-1)^2}$

$= \dfrac{3n-5(2n-1)}{(2n-1)^2}$

$= \dfrac{3n-10n+5}{(2n-1)^2}$

$= \dfrac{-7n+5}{(2n-1)^2}$

(h) $\dfrac{1}{p^2+3p-4} - \dfrac{1}{p+4} = \dfrac{1}{(p+4)(p-1)} - \dfrac{1}{p+4}$

$= \dfrac{1}{(p+4)(p-1)} - \dfrac{p-1}{(p+4)(p-1)}$

$= \dfrac{1-(p-1)}{(p+4)(p-1)}$

$= \dfrac{1-p+1}{(p+4)(p-1)}$

$= \dfrac{2-p}{(p+4)(p-1)}$

4. (a) $\dfrac{a}{2c} + \dfrac{b}{4} = 2$

$8c \times \left(\dfrac{a}{2c} + \dfrac{b}{4}\right) = 8c \times 2$

$4a + 2bc = 16c$

$4a + 2bc - 2bc = 16c - 2bc$

$4a = 16c - 2bc$

$\dfrac{4a}{4} = \dfrac{16c-2bc}{4}$

$\therefore a = \dfrac{2c(8-b)}{4}$

$= \dfrac{c(8-b)}{2}$

(b) $$A = P\left(1 + \frac{r}{100}\right)$$

$$\frac{A}{P} = \frac{P\left(1 + \frac{r}{100}\right)}{P}$$

$$\frac{A}{P} = 1 + \frac{r}{100}$$

$$\frac{A}{P} - 1 = 1 + \frac{r}{100} - 1$$

$$\frac{A - P}{P} = \frac{r}{100}$$

$$100 \times \frac{A - P}{P} = 100 \times \frac{r}{100}$$

$$\therefore r = \frac{100(A - P)}{P}$$

(c) $$a = \frac{1 - t}{1 + t}$$

$$a \times (1 + t) = \frac{1 - t}{1 + t} \times (1 \times t)$$

$$a(1 + t) = 1 - t$$

$$a + at = 1 - t$$

$$a + at - a = 1 - t - a$$

$$at = 1 - t - a$$

$$t + at = t + 1 - t - a$$

$$t + at = 1 - a$$

$$t(1 + a) = 1 - a$$

$$\frac{t(1 + a)}{1 + a} = \frac{1 - a}{1 + a}$$

$$\therefore t = \frac{1 - a}{1 + a}$$

(d) $$k = h + \frac{2hk}{5}$$

$$= \frac{5h}{5} + \frac{2hk}{5}$$

$$= \frac{5h + 2hk}{5}$$

$$= \frac{h(5 + 2k)}{5}$$

$$\frac{5}{5 + 2k} \times k = \frac{5}{5 + 2k} \times \frac{h(5 + 2k)}{5}$$

$$\therefore h = \frac{5k}{5 + 2k}$$

(e) $$\sqrt{3a - 2} = \sqrt{\frac{a}{b}}$$

$$3a - 2 = \frac{a}{b}$$

$$b \times (3a - 2) = b \times \frac{a}{b}$$

$$b(3a - 2) = a$$

$$\frac{b(3a - 2)}{3a - 2} = \frac{a}{3a - 2}$$

$$\therefore b = \frac{a}{3a - 2}$$

(f) $$x + \sqrt[3]{y^2 + z} = k$$

$$x + \sqrt[3]{y^2 + z} - x = k - x$$

$$\sqrt[3]{y^2 + z} = k - x$$

$$y^2 + z = (k - x)^3$$

$$y^2 + z - z = (k - x)^3 - z$$

$$\therefore y = \pm\sqrt{(k - x)^3 - z}$$

5. **(a)** When $a = 3$, $b = 6$ and $c = 20$,

$$3\sqrt{6^2 - 20} = 5k$$

$$3\sqrt{36 - 20} = 5k$$

$$3\sqrt{16} = 5k$$

$$3 \times 4 = 5k$$

$$12 = 5k$$

$$\therefore k = 2\frac{2}{5}$$

(b) When $a = 4$, $b = 7$ and $k = 11$,

$$4\sqrt{7^2 - c} = 5(11)$$

$$4\sqrt{49 - c} = 55$$

$$16(49 - c) = 3025$$

$$784 - 16c = 3025$$

$$-16c = 2241$$

$$\therefore c = -140\frac{1}{16}$$

6. **(a)** When $a = 8$,

$$8^2 + 8^2b = 320$$

$$64 + 64b = 320$$

$$64b = 256$$

$$\therefore b = 4$$

(b) When $b = 2\frac{1}{5}$,

$$a^2 + a^2\left(2\frac{1}{5}\right) = 320$$

$$\frac{16}{5}a^2 = 320$$

$$a^2 = 100$$

$$\therefore a = \pm\sqrt{100}$$

$$= \pm 10$$

7. **(a)** When $\pi = 3.142$, $R = 12$, $h = 14$ and $r = 9$,

$$V = (3.142)(12^2)(14) + \frac{2}{3}(3.142)(9^3)$$

$$= 6334.272 + 1572.012$$

$$= 7860 \text{ (to 3 s.f.)}$$

(b) When $\pi = 3.142$, $R = 8$, $V = 3800$ and $r = 6$,

$$3800 = (3.142)(8^2)h + \frac{2}{3}(3.142)(6^3)$$

$$3800 = 201.088h + 452.448$$

$$3347.552 = 201.088h$$

$$\therefore h = 16.6 \text{ (to 3 s.f.)}$$

(c) When $\pi = 3.142$, $V = 3500$, $h = 17$ and $r = 8.5$,

$$3500 = (3.142)R^2(17) + \frac{2}{3}(3.142)(8.5^3)$$
$$3500 = 53.414R^2 + 1286.387$$
$$2213.613 = 53.414R^2$$
$$R^2 = \frac{2213.613}{53.414}$$
$$\therefore R = \pm\sqrt{\frac{2213.613}{53.414}}$$
$$= \pm 6.44 \text{ (to 3 s.f.)}$$

(d) When $\pi = 3.142$, $R = 11$, $V = 4600$ and $h = 6.9$,

$$4600 = (3.142)(11^2)(6.9) + \frac{2}{3}(3.142)r^3$$
$$4600 = 2623.2558 + 2.0947r^3$$
$$1976.7442 = 2.0947r^3$$
$$r^3 = \frac{1976.7442}{2.0947}$$
$$\therefore r = \sqrt[3]{\frac{1976.7442}{2.0947}}$$
$$= 9.81 \text{ (to 3 s.f.)}$$

8. (a) $\dfrac{12}{5q} + 1 = \dfrac{7}{q}$

$$12 + 5q = 35$$
$$5q = 23$$
$$\therefore q = 4\frac{3}{5}$$

(b) $\dfrac{5}{y-3} - \dfrac{7}{3y-9} + \dfrac{10}{6-2y} = 3$

$$\frac{5}{y-3} - \frac{7}{3(y-3)} + \frac{10}{-2(y-3)} = 3$$
$$\frac{5}{y-3} - \frac{7}{3(y-3)} - \frac{5}{y-3} = 3$$
$$15 - 7 - 15 = 3[3(y-3)]$$
$$-7 = 3(3y-9)$$
$$-7 = 9y - 27$$
$$20 = 9y$$
$$\therefore y = 2\frac{2}{9}$$

9. (i) $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$

$$R_1R_2 = RR_2 + RR_1$$
$$R_1R_2 = R(R_2 + R_1)$$
$$\frac{R_1R_2}{R_1 + R_2} = \frac{R(R_2 + R_1)}{R_1 + R_2}$$
$$\therefore R = \frac{R_1R_2}{R_1 + R_2}$$

(ii) When $R_1 = 2$ and $R_2 = 3$,

$$R = \frac{2 \times 3}{2 + 3}$$
$$= \frac{6}{5}$$
$$= 1.2$$

$\therefore$ The effective resistance is 1.2 Ω.

10. (i) $(x-a)^2 + (y-b)^2 = r^2$

$$(x-a)^2 + (y-b)^2 - (x-a)^2 = r^2 - (x-a)^2$$
$$(y-b)^2 = r^2 - (x-a)^2$$
$$y - b = \pm\sqrt{r^2 - (x-a)^2}$$
$$b + y - b = b \pm \sqrt{r^2 - (x-a)^2}$$
$$\therefore y = b \pm \sqrt{r^2 - (x-a)^2}$$

(ii) When $a = 2$, $b = 3$, $r = 5$ and $x = 5$,

$$y = 3 \pm \sqrt{5^2 - (5-2)^2}$$
$$= 3 \pm \sqrt{5^2 - 3^2}$$
$$= 3 \pm \sqrt{25 - 9}$$
$$= 3 \pm \sqrt{16}$$
$$= 3 \pm 4$$
$$= -1 \text{ or } 7$$

Challenge Yourself

1. $$\frac{(a+b)^n}{bc^2} \div \frac{(a+b)^{n+3}}{abc} = \frac{(a+b)^n}{bc^2} \times \frac{abc}{(a+b)^{n+3}}$$
$$= \frac{\cancel{(a+b)^n}}{\cancel{b}\cancel{c^2}_{c}} \times \frac{a\cancel{b}\cancel{c}}{\cancel{(a+b)^n}(a+b)^3}$$
$$= \frac{a}{c(a+b)^3}$$

2. $$\frac{x^{n+2}y^{3-k}}{x^k y^{2n+5}} = \frac{x^{13}}{y^9}$$
$$\frac{x^{n+2}\cancel{y^3}y^{-k}}{x^k y^{2n}\cancel{y^5}_{y^2}} = \frac{x^{13}}{y^9}$$
$$\frac{x^{n+2-k}}{y^{2n+k+2}} = \frac{x^{13}}{y^9}$$

Comparing terms,

$n + 2 - k = 13$ — (1)

$2n + k + 2 = 9$ — (2)

From (1),

$n = k + 11$ — (3)

Substitute (3) into (2):

$$2(k + 11) + k + 2 = 9$$
$$2k + 22 + k + 2 = 9$$
$$3k + 24 = 9$$
$$3k = -15$$
$$k = -5$$

Substitute $k = -5$ into (3):

$$n = -5 + 11$$
$$= 6$$

$\therefore$ The values of n and k are 6 and –5 respectively.

3. $\dfrac{a+b-c}{a} = \dfrac{a-b+c}{c} = \dfrac{-a+b+c}{b}$

$\dfrac{a}{a} + \dfrac{b-c}{a} = \dfrac{a-b}{c} + \dfrac{c}{c} = \dfrac{c-a}{b} + \dfrac{b}{b}$

$1 + \dfrac{b-c}{a} = \dfrac{a-b}{c} + 1 = \dfrac{c-a}{b} + 1$

$\dfrac{b-c}{a} = \dfrac{a-b}{c} = \dfrac{c-a}{b}$

$\therefore p = \dfrac{(a-b)(b-c)(c-a)}{abc}$

$= \dfrac{a-b}{c} \times \dfrac{b-c}{a} \times \dfrac{c-a}{b}$

$= \dfrac{a-b}{c} \times \dfrac{a-b}{c} \times \dfrac{a-b}{c}$

$= \dfrac{(a-b)^3}{c^3}$ (shown)

Chapter 7 Relations and Functions

TEACHING NOTES

Suggested Approach

In this chapter, students learn the important concept of functions. This chapter takes the approach of progressing from relations to functions. As a start, teachers may use the example of a fruit juicer in the chapter opener to give an analogy of a function.

Section 7.1: Relations

The concept of relations is an introduction to the concept of functions, so teachers should spend some time to explain it. Teachers should use the arrow diagram, which consists of two sets connected by arrows, to explain this. Teachers are to explain the terms: domain, codomain and image, which the students would be required to know for this chapter.

Section 7.2: Functions

Teachers should highlight what makes a relation a function and emphasise that when a function is applied to any input x, it will produce exactly one output y. Once the students have understood the relationship between the input x and output y, they are then able to represent the function using an equation, a table and a graph.

Students are to learn to write equations using the notation of a function.

Challenge Yourself

Solving the problem involves the manipulation of the given equations of the function f.

WORKED SOLUTIONS

Thinking Time (Page 184)

The relation shown in Fig. 7.3 is a one-to-one function for which every image in the codomain corresponds to exactly one element of the domain. The relation in Fig. 7.4 is not a one-to-one function because A and B are images of more than one element in the domain.
The relation in Fig. 7.5 is not a one-to-one function because there are elements in the codomain that are not images of the elements in the domain.

Practise Now 1

(a) The relation is a function since every element in the domain 'Students' has a unique image in the codomain 'Scores'.
(b) The relation is not a function since the element d in the domain A has no image in the codomain B.

Practise Now 2

$f(x) = 10x + 4$
$g(x) = 4x - 6$

(i) $f(4) = 10(4) + 4$
$= 44$

(ii) $f(-7) = 10(-7) + 4$
$= -66$

(iii) $f\left(-\frac{2}{3}\right) = 10\left(-\frac{2}{3}\right) + 4$
$= -2\frac{2}{3}$

(iv) $g(2) = 4(2) - 6$
$= 2$

(v) $2g(6) = 2[4(6) - 6]$
$= 36$

(vi) $g\left(\frac{7}{8}\right) = 4\left(\frac{7}{8}\right) - 6$
$= -2\frac{1}{2}$

(vii) $f\left(\frac{1}{2}\right) = 10\left(\frac{1}{2}\right) + 4$
$= 5 + 4$
$= 9$
$g(1) = 4(1) - 6$
$= -2$
$f\left(\frac{1}{2}\right) + g(1) = 9 - 2$
$= 7$

(viii) $f(x) = g(x)$
$10x + 4 = 4x - 6$
$10x - 4x = -6 - 4$
$6x = -10$
$x = -1\frac{2}{3}$

(ix) $f(x) = 34$
$10x + 4 = 34$
$10x = 30$
$x = 3$

Practise Now 3

$f(x) = 2x - 5$
$F(x) = 7x + 12$

(i) $f(b) = 2b - 5$

(ii) $F(b - 1) = 7(b - 1) + 12$
$= 7b - 7 + 12$
$= 7b + 5$

(iii) $f(2b) = 2(2b) - 5$
$= 4b - 5$
$F(2b - 5) = 7(2b - 5) + 12$
$= 14b - 35 + 12$
$= 14b - 23$
$f(2b) + F(2b - 5) = (4b - 5) + (14b - 23)$
$= 18b - 28$

Exercise 7A

1. **(a)** Yes
(b) No, because the element 2 has two images.
(c) Yes
(d) Yes
(e) No, because the element 4 has no image.
(f) Yes

2. $f(x) = 6x - 4$
$f(2) = 6(2) - 4$
$= 8$
$f(-4) = 6(-4) - 4$
$= -28$
$f\left(\frac{1}{3}\right) = 6\left(\frac{1}{3}\right) - 4$
$= -2$
$f\left(-\frac{1}{2}\right) = 6\left(-\frac{1}{2}\right) - 4$
$= -7$

3. **(i)** $f(1) = 5 - 2(1)$
$= 3$

(ii) $f(-2) = 5 - 2(-2)$
$= 5 + 4$
$= 9$

(iii) $f(0) = 5 - 2(0)$
$= 5$

(iv) $f(3) = 5 - 2(3)$
$= 5 - 6$
$= -1$
$f(-3) = 5 - 2(-3)$
$= 5 + 6$
$= 11$
$f(3) + f(-3) = -1 + 11$
$= 10$

4. **(i)** $g(2) = 7(2) + 4$
$= 18$

(ii) $g(-3) = 7(-3) + 4$
$= -17$

(iii) $g\left(\frac{4}{7}\right) = 7\left(\frac{4}{7}\right) + 4$
$= 8$

(iv) $g(0) = 7(0) + 4$
$= 4$

$g(-1) = 7(-1) + 4$
$= -3$

$g(0) + g(-1) = 4 + (-3)$
$= 4 - 3$
$= 1$

(v) $g\left(\frac{1}{7}\right) = 7\left(\frac{1}{7}\right) + 4$
$= 5$

$g\left(-\frac{1}{7}\right) = 7\left(-\frac{1}{7}\right) + 4$
$= 3$

$g\left(\frac{1}{7}\right) - g\left(-\frac{1}{7}\right) = 5 - 3$
$= 2$

5. $f(x) = \frac{x}{2} + 3$

$g(x) = \frac{3}{4}x - 2$

(a) **(i)** $f(2) = \frac{2}{2} + 3$
$= 4$

$g(2) = \frac{3}{4}(2) - 2$
$= \frac{3}{2} - 2$
$= -\frac{1}{2}$

$f(2) + g(2) = 4 - \frac{1}{2}$
$= 3\frac{1}{2}$

(ii) $f(-1) = \frac{-1}{2} + 3$
$= 2\frac{1}{2}$

$g(-1) = \frac{3}{4}(-1) - 2$
$= -2\frac{3}{4}$

$f(-1) - g(-1) = 2\frac{1}{2} - \left(-2\frac{3}{4}\right)$
$= 5\frac{1}{4}$

(iii) $2f(4) = 2\left(\frac{4}{3} + 3\right)$
$= 2(5)$
$= 10$

$3g(6) = 3\left[\frac{3}{4}(6) - 2\right]$
$= 3\left[\frac{5}{2}\right]$
$= 7\frac{1}{2}$

$2f(4) - 3g(6) = 10 - 7\frac{1}{2}$
$= 2\frac{1}{2}$

(iv) $5f(-2) = 5\left(\frac{-2}{2} + 3\right)$
$= 5(2)$
$= 10$

$7g(-4) = 7\left[\frac{3}{4}(-4) - 2\right]$
$= 7[-5]$
$= -35$

$5f(-2) - 7g(-4) = 10 - (-35)$
$= 45$

(b) $f(x) = g(x)$

$\frac{x}{2} + 3 = \frac{3}{4}x - 2$

$\frac{1}{4}x = 5$

$x = 20$

For $f(x) = 17$,

$\frac{x}{2} + 3 = 17$

$\frac{x}{2} = 14$

$x = 28$

6. **(i)** $f(x) = 16$
$5x - 9 = 16$
$5x = 25$
$x = 5$

(ii) $g(x) = 14$
$2 - 6x = 14$
$-6x = 12$
$x = -2$

(iii) $g(x) = x$
$2 - 6x = x$
$2 = 7x$
$x = \frac{2}{7}$

(iv) $f(x) = 2x$
$5x - 9 = 2x$
$3x = 9$
$x = 3$

(v) $f(x) = g(x)$

$5x - 9 = 2 - 6x$

$11x = 11$

$x = 1$

(vi) $2f(x) = 3g(x)$

$2(5x - 9) = 3(2 - 6x)$

$10x - 18 = 6 - 18x$

$28x = 24$

$x = \frac{6}{7}$

7. $f(x) = 4x + 9$

$f(1) = 4(1) + 9$

$= 13$

$f(2) = 4(2) + 9$

$= 17$

$f(3) = 4(3) + 9$

$= 21$

(i) $f(1) + f(2) = 13 + 17$

$= 30$

$f(1 + 2) = f(3) = 21$

$\therefore f(1) + f(2) \neq f(1 + 2)$

(ii) $f(3) - f(2) = 21 - 17$

$= 4$

$f(3 - 2) = f(1) = 13$

$\therefore f(3) - f(2) \neq f(3 - 2)$

(iii) $f(1) \times f(2) = 13 \times 17$

$= 221$

$f(1 \times 2) = f(2) = 17$

$\therefore f(1) \times f(2) \neq f(1 \times 2)$

(iv) $f(2) \div f(1) = 17 \div 13$

$= 1\frac{4}{13}$

$f(2 \quad 1) = f(2) = 17$

$\therefore f(2) \div f(1) \neq f(2 \div 1)$

8. $f(x) = \frac{3}{4}x + \frac{1}{2}$

$f(2) = \frac{3}{4}(2) + \frac{1}{2}$

$= 2$

$f\left(-\frac{1}{2}\right) = \frac{3}{4}\left(-\frac{1}{2}\right) + \frac{1}{2}$

$= \frac{1}{8}$

$g(x) = 1\frac{1}{4} - \frac{2}{3}x$

$g(3) = 1\frac{1}{4} - \frac{2}{3}(3)$

$= -\frac{3}{4}$

$g(-6) = 1\frac{1}{4} - \frac{2}{3}(-6)$

$= 5\frac{1}{4}$

(i) $f(3) = \frac{3}{4}(3) + \frac{1}{2}$

$= 2\frac{3}{4}$

$f(2) + f(3) = 2 + 2\frac{3}{4}$

$= 4\frac{3}{4}$

$f(2 + 3) = f(5)$

$= \frac{3}{4}(5) + \frac{1}{2}$

$= 4\frac{1}{4}$

$\therefore f(2) + f(3) \neq f(2 + 3)$

(ii) $g(4) = 1\frac{1}{4} - \frac{2}{3}(4)$

$= -1\frac{5}{12}$

$g(2) = 1\frac{1}{4} - \frac{2}{3}(2)$

$= -\frac{1}{12}$

$g(4) - g(2) = -1\frac{5}{12} - \left(-\frac{1}{12}\right)$

$= -1\frac{5}{12} + \frac{1}{12}$

$= -1\frac{1}{3}$

$g(4 - 2) = g(2) = -\frac{1}{12}$

$\therefore g(4) - g(2) \neq g(4 - 2)$

(iii) $f(x) = g(x)$

$\frac{3}{4}x + \frac{1}{2} = 1\frac{1}{4} - \frac{2}{3}x$

$\frac{3}{4}x + \frac{2}{3}x = 1\frac{1}{4} - \frac{1}{2}$

$\frac{17}{12}x = \frac{3}{4}$

$x = \frac{9}{17}$

(iv) $f(a) = \frac{3}{4}a + \frac{1}{2}$

$f(2a) = \frac{3}{4}(2a) + \frac{1}{2}$

$= 1\frac{1}{2}a + \frac{1}{2}$

$g(3a) = 1\frac{1}{4} - \frac{2}{3}(3a)$

$= 1\frac{1}{4} - 2a$

(v) $f(a + 1) = \frac{3}{4}(a + 1) + \frac{1}{2}$

$= \frac{3}{4}a + \frac{3}{4} + \frac{1}{2}$

$= \frac{3}{4}a + 1\frac{1}{4}$

$g(a) = 1\frac{1}{4} - \frac{2}{3}a$

$f(a + 1) + g(a) = \frac{3}{4}a + 1\frac{1}{4} + 1\frac{1}{4} - \frac{2}{3}a$

$= \frac{1}{12}a + 2\frac{1}{2}$

For $f(a + 1) + g(a) = 5$,

$\frac{1}{12}a + 2\frac{1}{2} = 5$

$\frac{1}{12}a = 5 - 2\frac{1}{2}$

$= 2\frac{1}{2}$

$\therefore a = 30$

(vi) $f(2a) = \frac{3}{4}(2a) + \frac{1}{2}$

$= \frac{3}{2}a + \frac{1}{2}$

$g(6a) = 1\frac{1}{4} - \frac{2}{3}(6a)$

$= 1\frac{1}{4} - 4a$

For $f(2a) = g(6a)$,

$\frac{3}{2}a + \frac{1}{2} = 1\frac{1}{4} - 4a$

$\frac{11}{2}a = \frac{3}{4}$

$\therefore a = \frac{3}{22}$

Review Exercise 7

1. **(a)** No, because the element 1 has two images.

(b) Yes

(c) No, because the element 3 has no image.

(d) Yes

2. $f(x) = \frac{1}{2}x + 2$

$f(-20) = \frac{1}{2}(-20) + 2$

$= -8$

$f(6) = \frac{1}{2}(6) + 2$

$= 5$

$f\left(\frac{1}{8}\right) = \frac{1}{2}\left(\frac{1}{8}\right) + 2$

$= 2\frac{1}{16}$

$f\left(-\frac{2}{3}\right) = \frac{1}{2}\left(-\frac{2}{3}\right) + 2$

$= 1\frac{2}{3}$

3. **(i)** $f(2) = 12 - 5(2)$

$= 2$

(ii) $f(-3) = 12 - 5(-3)$

$= 12 + 15$

$= 27$

(iii) $f(0) = 12 - 5(0)$

$= 12$

(iv) $f(3) = 12 - 5(3)$

$= 12 - 15$

$= -3$

$f(-5) = 12 - 5(-5)$

$= 12 + 25$

$= 37$

$f(3) + f(-5) = -3 + 37$

$= 34$

4. **(i)** $g(2) = 5(2) - 9$

$= 1$

(ii) $g(-5) = 5(-5) - 9$

$= -25 - 9$

$= -34$

(iii) $g\left(\frac{3}{5}\right) = 5\left(\frac{3}{5}\right) - 9$

$= 3 - 9$

$= -6$

(iv) $g(0) = 5(0) - 9$

$= -9$

$g(-3) = 5(-3) - 9$

$= -24$

$g(0) + g(-3) = -9 - 24$

$= -33$

(v) $g\left(\frac{4}{5}\right) = 5\left(\frac{4}{5}\right) - 9$

$= 4 - 9$

$= -5$

$g\left(-\frac{1}{5}\right) = 5\left(-\frac{1}{5}\right) - 9$

$= -1 - 9$

$= -10$

$g\left(\frac{4}{5}\right) - g\left(-\frac{1}{5}\right) = -5 - (-10)$

$= -5 + 10$

$= 5$

5. $f(x) = \frac{x}{4} + 5$

$g(x) = \frac{2}{5}x - 1$

(a) **(i)** $f(8) = \frac{8}{4} + 5$

$= 7$

$g(5) = \frac{2}{5}(5) - 1$

$= 1$

$f(8) + g(5) = 7 + 1$

$= 8$

(ii) $f(-1) = \frac{-1}{4} + 5$

$= 4\frac{3}{4}$

$g(-10) = \frac{2}{5}(-10) - 1$

$= -5$

$f(-1) - g(-10) = 4\frac{3}{4} - (-5)$

$= 9\frac{3}{4}$

(iii) $2f(3) = 2\left(\frac{3}{4} + 5\right)$

$= 11\frac{1}{2}$

$3g(5) = 3\left[\frac{2}{5}(5) - 1\right]$

$= 3$

$2f(3) - 3g(5) = 11\frac{1}{2} - 3$

$= 8\frac{1}{2}$

(iv) $4f(-3) = 4\left(\frac{-3}{5} + 5\right)$

$= 17$

$7g(0) = 7\left[\frac{2}{5}(0) - 1\right]$

$= -7$

$4f(-3) - 7g(0) = 17 - (-7)$

$= 24$

(b) $f(x) = g(x)$

$\frac{x}{4} + 5 = \frac{2}{5}x - 1$

$\left(\frac{2}{5} - \frac{1}{4}\right)x = 5 + 1$

$\frac{3}{20}x = 6$

$\therefore x = 40$

For $f(x) = 6$,

$\frac{x}{4} + 5 = 6$

$\frac{x}{4} = 1$

$\therefore x = 4$

6. $f(x) = 4x - 3$

$g(x) = 8 - 3x$

(i) $f(x) = 13$

$4x - 3 = 13$

$4x = 16$

$x = 4$

(ii) $g(x) = 20$

$8 - 3x = 20$

$-3x = 12$

$3x = -12$

$x = -4$

(iii) $g(x) = x$

$8 - 3x = x$

$4x = 8$

$x = 2$

(iv) $f(x) = -2x$

$4x - 3 = -2x$

$6x = 3$

$x = \frac{1}{2}$

(v) $f(x) = g(x)$

$4x - 3 = 8 - 3x$

$7x = 11$

$x = \frac{11}{7}$

$= 1\frac{4}{7}$

7. $f(x) = 10x - 3$

$F(x) = \frac{3}{4}x + 2$

(i) $f(a) = 10a - 3$

(ii) $F(8a + 1) = \frac{3}{4}(8a + 1) + 2$

$= 6a + \frac{3}{4} + 2$

$= 6a + 2\frac{3}{4}$

(iii) $f\left(\frac{1}{2}a\right) = 10\left(\frac{1}{2}a\right) - 3$

$= 5a - 3$

$F(2a - 4) = \frac{3}{4}(2a - 4) + 2$

$= \frac{3}{2}a - 3 + 2$

$= 1\frac{1}{2}a - 1$

$f\left(\frac{1}{2}a\right) + F(2a - 4) = (5a - 3) + \left(1\frac{1}{2}a - 1\right)$

$= 6\frac{1}{2}a - 4$

Challenge Yourself

(i) $f(a + b)$

$$= \left(\frac{a+b+1}{a+1}\right)f(a) + \left(\frac{a+b+1}{b+1}\right)f(b) + a + 2 \text{ — (1)}$$

Substitute $b = 1$ into (1):

$$f(a+1) = \left(\frac{a+2}{a+1}\right)f(a) + \left(\frac{a+2}{2}\right)f(1) + a + 2$$

$$= (a+2)\left[\frac{1}{a+1}f(a) + \frac{1}{2}\left(\frac{5}{2}\right) + 1\right]$$

$$= (a+2)\left[\frac{1}{a+1}f(a) + \frac{5}{4} + 1\right]$$

$$= (a+2)\left[\frac{1}{a+1}f(a) + \frac{9}{4}\right] \text{ — (2)}$$

$$\frac{f(a+1)}{a+2} = \frac{f(a)}{a+1} + \frac{9}{4}$$

$$\therefore \frac{f(a+1)}{a+2} - \frac{f(a)}{a+1} = \frac{9}{4} \text{ (shown)}$$

(ii) Substitute $a = 1$ into (2):

$$f(2) = 3\left[\frac{1}{2}f(1) + \frac{9}{4}\right]$$

$$= 3\left[\frac{1}{2}\left(\frac{5}{2}\right) + \frac{9}{4}\right]$$

$$= 3\left[\frac{5}{4} + \frac{9}{4}\right]$$

$$= \frac{21}{2}$$

$$= 10\frac{1}{2}$$

Chapter 8 Congruence and Similarity

TEACHING NOTES

Suggested Approach

In this chapter, students will be introduced to the concepts of congruence and similarity which are properties of geometrical figures. The definitions of both terms must be clearly stated, with their similarities and differences explored and discussed to minimise any confusion. A recap on angle properties and geometrical construction may be required in this chapter.

Section 8.1: Congruent Figures

Teachers may wish to show the properties of congruent figures (see Investigation: Properties of Congruent Figures) before stating the definition. Students should list and state more examples of congruence in real-life to their class (see Class Discussion: Congruence in the Real World).

In stating the congruence relation, it is crucial that the order of vertices reflects the equal corresponding angles and sides in both congruent figures. A wrong order will indicate an incorrect relation.

The worked examples aim to allow students to understand and apply the properties of congruence, as well as test whether two figures are congruent. Teachers should provide guidance to students who require explanations and assistance.

Section 8.2: Similar Figures

Students, after knowing the definition of similarity, should be able to realise that congruence is a special case of similarity. The Class Discussion on page 203 tests students' understanding of similarity whereas the Investigation on page 204 allows students to derive the properties that corresponding angles are equal and the ratios of corresponding sides are equal.

Students should explore the concept of similarity for different figures (see Thinking Time on page 205) as the results imply that although both conditions are needed for polygons with four sides or more, for triangles, one condition is sufficient to show similarity.

Teachers should also go through the activity on page 206 (see Class Discussion: Identifying Similar Triangles). Students should discover that right-angled triangles and isosceles triangles need not be similar but all equilateral triangles are definitely similar.

Section 8.3: Similarity, Enlargement and Scale Drawings

From the previous section, when two figures are similar, one will be 'larger' than the other. The concept of a scale factor should then be a natural result. Teachers and students should note (see Information on page 213) that enlargement does not always mean the resultant figure is larger than the original figure. The resultant figure can be smaller than the original figure, and the scale factor will be less than 1, but it is still known as an enlargement. If the scale factor is 1, then the resultant figure is congruent to the original figure.

Students are required to recall their lessons on geometrical construction while learning about and making scale drawings. Observant students may note that scale drawing is actually an application of ratios, and the concepts of linear scales and area scales of maps/models further illustrate this.

WORKED SOLUTIONS

Investigation (Properties of Congruent Figures)

1. The shape and size of the pairs of scissors are the same, whereas the orientation and position of the pairs of scissors are different.
2. The pairs of scissors will stack on top of one another nicely.
3. The scissors in **(a)** can be moved to the scissors in **(c)** by a reflection of $A_1 \to A_3$ about a vertical line.
   The scissors in **(a)** can be moved to the scissors in **(d)** by a rotation of $A_1 \to A_4$ about 135° in an anticlockwise direction.
   The scissors in **(a)** has the same orientation and position as **(e)**.

Thinking Time (Page 194)

Yes, the two pairs of scissors are congruent as they have the same shape and size.

Class Discussion (Congruence in the Real World)

1. Examples of 3 different sets of congruent objects are the chairs, tables and projectors available in the classroom.
2. Examples of some other objects that exhibit tessellations are wallpapers, carpets, school fences, brick walls etc.
3. *Teachers may wish to have students create their own tessellations using unit shapes like squares, triangles, rectangles, circles and other regular polygons. Tessellations can include one or more unit shapes.*

Class Discussion (Similarity in the Real World)

1. Examples of 3 different sets of similar objects are rulers, beakers and balls used in sports (e.g. tennis ball, basketball, soccer ball etc).
2. *As an activity, teachers may want to ask students a pair of similar objects and explain how they are similar. Poll the class to determine the most popular pair of similar objects.*

Investigation (Properties of Similar Polygons)

1. **(a)** $\angle A = 100°, \angle A' = 100°$
   (b) $\angle B = 136°, \angle B' = 136°$
   (c) $\angle C = 110°, \angle C' = 110°$
   (d) $\angle D = 145°, \angle D' = 145°$
   (e) $\angle E = 108°, \angle E' = 108°$
   (f) $\angle F = 121°, \angle F' = 121°$
   The size of each pair of angles is the same.
2. **(a)** **(i)** $AB = 1$ cm, $A'B' = 2$ cm
   (ii) $BC = 0.9$ cm, $B'C = 1.8$ cm
   (iii) $CD = 1.1$ cm, $C'D' = 2.2$ cm
   (iv) $EF = 1.2$ cm, $E'F' = 2.4$ cm
   (vi) $FA = 1$ cm $F'A' = 2$cm
   (b) **(i)** $\frac{A'B'}{AB} = \frac{2}{1} = 2$
   (ii) $\frac{B'C'}{BC} = \frac{1.8}{0.9} = 2$
   (iii) $\frac{C'D'}{CD} = \frac{2.2}{1.1} = 2$
   (iv) $\frac{D'E'}{DE} = \frac{1.2}{0.6} = 2$
   (v) $\frac{E'F'}{EF} = \frac{2.4}{1.2} = 2$
   (vi) $\frac{F'A'}{FA} = \frac{2}{1} = 2$
   The values of the ratios are equal to 2.

Thinking Time (Page 205)

1. **(i)** $\angle A = 90°, \angle A' = 90°$
   $\angle B = 90°, \angle B' = 90°$
   $\angle C = 90°, \angle C' = 90°$
   $\angle D = 90°, \angle D' = 90°$
   The corresponding angles are equal.
   (ii) $AB = 3.3$ cm, $A'B' = 5$ cm
   $\frac{A'B'}{AB} = \frac{5}{3.3} = \frac{25}{18}$
   $BC = 1.2$ cm, $B'C' = 1.2$ cm
   $\frac{B'C'}{BC} = \frac{1.2}{1.2} = 1$
   $CD = 3.3$ cm, $C'D' = 5$ cm
   $\frac{C'D'}{CD} = \frac{5}{3.3} = \frac{25}{18}$
   $DA = 1.2$ cm, $D'A' = 1.2$ cm
   $\frac{D'A'}{DA} = \frac{1.2}{1.2} = 1$
   The ratios of the corresponding sides are not equal.
   (iii) The two rectangles are not similar.
2. **(i)** $PQ = 1.05$ cm, $P'Q = 3.1$ cm
   $\frac{P'Q'}{PQ} = \frac{3.1}{1.05} = 2\frac{20}{21}$
   Since $PQRS$ is a square and $P'Q'R'S$ is a rhombus, the ratios of the corresponding sides are equal.
   (ii) $\angle P = 90°, \angle P = 120°$
   $\angle Q = 90°, Q' = 60°$
   $\angle R = 90°, \angle R' = 120°$
   $\angle S = 90°, \angle S' = 60°$
   The corresponding angles are not equal.
   (iii) The two quadrilaterals are not similar.
3. **(i)** $\angle X = 60°, \angle X' = 60°$
   $\angle Y = 67°, \angle Y' = 67°$
   $\angle Z = 53°, \angle Z' = 53°$
   The corresponding angles are equal.

(ii) $XY = 2.1$ cm, $X'Y' = 3.15$ cm

$$\frac{X'Y'}{XY} = \frac{3.15}{2.1} = 1.5$$

$YZ = 2.25$ cm, $Y'Z' = 3.4$ cm

$$\frac{Y'Z'}{YZ} \approx 1.5$$

$ZX = 2.4$ cm, $Z'X' = 3.6$ cm

$$\frac{Z'X'}{ZX} = \frac{3.6}{2.4} = 1.5$$

The ratios of the corresponding sides are equal.

(iii) The two triangles are similar.

(iv) No

(v) No

Class Discussion (Identifying Similar Triangles)

2. $\triangle A$ is similar to $\triangle B$ as $\triangle A$ can fit inside $\triangle B$ with an equivalent width around it.

$\triangle D$ is similar to $\triangle F$ as $\triangle D$ can fit inside $\triangle F$ with an equivalent width around it.

$\triangle G$ is similar to $\triangle H$ as $\triangle G$ can fit inside $\triangle H$ with an equivalent width around it.

$\triangle I$ is similar to $\triangle J$ as $\triangle I$ overlaps with $\triangle J$. $\triangle I \equiv \triangle J$.

$\triangle K$ is similar to $\triangle L$ and $\triangle M$ as $\triangle L$ can fit inside $\triangle K$, which can fit inside $\triangle M$ with an equivalent width around it.

3. **(a)** No. The corresponding angles may not be equal, e.g. $\triangle D$ and $\triangle E$.

(b) No. The corresponding angles may not be equal, e.g. $\triangle G$ and $\triangle I$.

(e) Yes. All angles are equal to 60° so the corresponding angles are the same. Since the sides of an equilateral triangle are the same, the ratios of the corresponding sides of two equilateral triangles are the same.

Performance Task (Page 224)

Students should be in groups of 2 to 4. It is important that students discuss what they are going to make a scale drawing of. For weaker students, an existing classroom would be an easier option. For better students, allow them to design their dream classroom, or bedroom, measure the things that occupy the place and come up with a scale drawing of it.

Students would need to use a measuring tape. Provide one to the groups who do not have it.

Assume that the students are making a scale drawing of an existing classroom. The dimensions that will need to be measured first are the length and width of the classroom, followed by the things inside the classroom, such as the tables (which includes the teacher's desk), the chairs, cupboard, cabinets etc.

The entire scale drawing should fit onto one A4-sized paper, with the scale used noted down on it.

After students have made a scale drawing, have some groups present their findings to the class and explain the choice of scale used. Here are some questions that can be asked during the presentation:

- Are the dimensions in the scale drawing correct and realistic? Are there any dimensions that are incorrect or unrealistic?
- From the drawing made, assess whether some objects are too close to or too far apart from each other.
- Determine whether the occupants (students in the classroom, or user of the bedroom) find the arrangement comfortable.
- What are the improvements that can be made to the current scale drawing, or arrangement in the classroom or bedroom?

Teachers may want to expand the project further, such as determining the occupied/unoccupied area and by using percentages, whether the figure is suitable. Teachers may even want to consider having students making three-dimensional scale models of the existing or dream classroom.

Practise Now 1

A is congruent to H.
B is congruent to E.
C is congruent to F.
D is congruent to G and I.

Practise Now 2

Since $ABCD \equiv PQRS$, then the corresponding vertices match:

$A \leftrightarrow P$
$B \leftrightarrow Q$
$C \leftrightarrow R$
$D \leftrightarrow S$

(i) $PQ = AB = \underline{5}$ cm

(ii) $SR = \underline{DC} = 6$ cm

(iii) $PS = \underline{AD} = \underline{2}$ cm

(iv) $QR = \underline{BC} = \underline{5.3}$ cm

(v) $\angle PQR = \underline{\angle ABC} = \underline{90}°$

Practise Now 3

(a) In $\triangle ABC$,

$AB = 4$ cm, $BC = 5.4$ cm, $AC = 6.1$ cm

In $\triangle PQR$,

$PQ = 8$ cm, $QR = 12.2$ cm, $PR = 10.8$ cm

$\therefore$ $\triangle PQR$ does not have any length corresponding to that in $\triangle ABC$.

$\therefore$ $\triangle PQR$ does not have the same size as $\triangle ABC$ and so it is not congruent to $\triangle ABC$.

(b) $D \leftrightarrow T$ (since $\angle D = \angle T = 80°$)

$E \leftrightarrow S$ (since $\angle E = \angle S = 60°$)

$F \leftrightarrow U$ (since $\angle F = \angle U = 40°$)

$\angle EDF = \angle STU = 80°$

$\angle DEF = \angle TSU = 60°$

$\angle DFE = \angle TUS = 40°$

$DE = TS = 4$ cm

$EF = SU = 6.13$ cm

$DF = TU = 5.29$ cm

$\therefore$ The two triangles have the same shape and size and so $\triangle DEF \equiv \triangle TSU$.

(c) In $\triangle LMN$,

$\angle N = 180° - 70° - 60°$ ($\angle$ sum of $\triangle LMN$)

$= 50°$

In $\triangle XYZ$,

$\angle X = 180° - 70° - 40°$ ($\angle$ sum of $\triangle XYZ$)

$= 70°$

$\therefore$ $\triangle XYZ$ does not have any angle of 50° or 60° that corresponds to that in $\triangle LMN$.

$\therefore$ $\triangle XYZ$ does not have the same shape as $\triangle LMN$ and so it is not congruent to $\triangle LMN$.

Practise Now 4

Since $\triangle ABC \equiv \triangle CDE$, then the corresponding vertices match:

$A \leftrightarrow C$

$B \leftrightarrow D$

$C \leftrightarrow E$

(a) **(i)** $\angle CDE = \angle ABC$

$= 38°$

(ii) $\angle CED = 180° - 114° - 38°$ ($\angle$ sum of $\triangle CDE$)

$= 28°$

(iii) $\angle ACB = \angle CED$

$= 28°$

(iv) Length of BC = length of DE

= 27 cm

(v) Length of CE = length of AC

= 18 cm

$\therefore$ Length of BE = length of BC – length of CE

= 27 – 18

= 9 cm

(b) $\angle ACB + \angle DCE + \angle CDE$

$= 28° + 114° + 38°$

$= 180°$

By converse of int. $\angle$s, AC // ED.

Practise Now 5

(a) $\angle C = 180° - 90° - 58°$ ($\angle$ sum of $\triangle ABC$)

$= 32°$

$\angle P = 180° - 90° - 35°$ ($\angle$ sum of $\triangle ABC$)

$= 55°$

$\angle A = 58° \neq 55° = \angle P$

$\angle B = \angle Q = 90°$

$\angle C = 32° \neq 35° = \angle R$

Since not all the corresponding angles are equal, then $\triangle ABC$ is not similar to $\triangle PQR$.

(b) $\frac{ST}{DE} = \frac{10}{12} = \frac{5}{6}$

$\frac{TU}{EF} = \frac{7}{8.4} = \frac{5}{6}$

$\frac{SU}{DF} = \frac{6}{7.2} = \frac{5}{6}$

Since all the ratios of the corresponding sides are equal, $\triangle DEF$ is similar to $\triangle STU$.

Practise Now 6

1. Since $\triangle ABC$ is similar to $\triangle PRQ$, then the corresponding vertices match:

$A \leftrightarrow P$

$B \leftrightarrow R$

$C \leftrightarrow Q$

Since $\triangle ABC$ is similar to $\triangle PRQ$, then all the corresponding angles are equal.

$\therefore x° = \angle QPR$

$= \angle BAC$

$= 30°$

$\therefore x = 30$

Since $\triangle ABC$ is similar to $\triangle PRQ$, then all the ratios of the corresponding sides are equal.

$\therefore \frac{BC}{RQ} = \frac{AC}{PQ}$

i.e. $\frac{y}{2.8} = \frac{6}{4}$

$\therefore y = \frac{6}{4} \times 2.8$

$= 4.2$

2. Since $ABCD$ is similar to $PQRS$, then the corresponding vertices match:

$A \leftrightarrow P$

$B \leftrightarrow Q$

$C \leftrightarrow R$

$D \leftrightarrow S$

Since $ABCD$ is similar to $PQRS$, then all the corresponding angles are equal.

$\therefore w° = \angle BCD$

$= \angle QRS$

$= 60°$

$\therefore w = 60$

$\therefore x° = \angle QPS$
$= \angle BAD$
$= 100°$

$\therefore x = 100$

Since $ABCD$ is similar to $PQRS$, then all the ratios of the corresponding sides are equal.

$\therefore \frac{BC}{QR} = \frac{AB}{PQ}$

i.e. $\frac{y}{5.4} = \frac{4}{3}$

$\therefore y = \frac{4}{3} \times 5.4$
$= 7.2$

$\therefore \frac{PS}{AD} = \frac{PQ}{AB}$

i.e. $\frac{z}{6} = \frac{3}{4}$

$\therefore z = \frac{3}{4} \times 6$
$= 4.5$

Practise Now 7

Let the height of the lamp post be x cm.
Let the point on the line BD, 180 cm vertically above C be E.
We observe that $\triangle ABD$ and $\triangle CED$ are right-angled triangles with one common angle D. Hence the two triangles are similar.
Since $\triangle ABD$ and $\triangle CED$ are similar, then all the ratios of the corresponding sides are equal.

$\frac{AB}{CE} = \frac{AD}{CD}$

$\frac{x}{180} = \frac{256 + 144}{144}$

$144x = 180(256 + 144)$

$144x = 180(400)$

$\therefore x = 500$

Height of lamp post $= 500$ cm
$= 5$ m

The height AB, of the lamp post is 5 m.

Practise Now 8

1. Since $\triangle XYZ$ is similar to $\triangle XRS$, then the corresponding vertices match:
$X \leftrightarrow X$
$Y \leftrightarrow R$
$Z \leftrightarrow S$
Since $\triangle XYZ$ is similar to $\triangle XRS$, then all the corresponding angles are equal.
$\therefore a° = \angle XSR$
$= \angle XZY$
$= 30°$
$\therefore a = 30$
Since $\triangle XYZ$ is similar to $\triangle XRS$, then all the ratios of the corresponding sides are equal.
$\therefore \frac{XS}{XZ} = \frac{XR}{XY}$
i.e. $\frac{XS}{5} = \frac{4 + 6}{4}$
$\therefore XS = \frac{10}{4} \times 5$
$= 12.5$
$\therefore b = 12.5 - 5$
$= 7.5$

2. Since $\triangle ABC$ is similar to $\triangle DEC$, then the corresponding vertices match:
$A \leftrightarrow D$
$B \leftrightarrow E$
$C \leftrightarrow C$
Since $\triangle ABC$ is similar to $\triangle DEC$, then all the corresponding angles are equal.
$\angle ACB = \angle DCE$ (vert. opp. $\angle$s)
$= 60°$
$\angle CAB = 180° - 60° - 48°$ ($\angle$ sum of $\triangle ABC$)
$= 72°$
$\therefore x° = \angle CDE$
$= \angle CAB$
$= 72°$
$\therefore x = 72$
Since $\triangle ABC$ is similar to $\triangle DEC$, then all the ratios of the corresponding sides are equal.
$\therefore \frac{DE}{AB} = \frac{CE}{CB}$
i.e. $\frac{y}{7.3} = \frac{10}{8}$
$\therefore y = \frac{10}{8} \times 7.3$
$= 9.125$

Practise Now 9

1. $\triangle ABC$ is similar to $\triangle A'B'C'$ under enlargement.
$\therefore \frac{A'B'}{AB} = \frac{A'C'}{AC} = 3$
i.e. $\frac{A'B'}{6} = 3$ and $\frac{A'C'}{10} = 3$
$\therefore A'B' = 18$ cm and $A'C = 30$ cm
2. $\triangle XYZ$ is similar to $\triangle XY'Z'$ under enlargement.
$\therefore \frac{XY'}{XY} = \frac{Y'Z'}{YZ} = 1.5$
i.e. $\frac{XY'}{5} = 1.5$ and $\frac{12}{YZ} = 1.5$
$\therefore XY' = 7.5$ cm and $YZ = 8$ cm

3. Let the actual height of the house be x m or $100x$ cm.

$$\therefore \frac{100x}{180} = \frac{22.5}{9}$$

$$x = \frac{22.5}{9} \times \frac{180}{100}$$

$$= 4.5$$

The actual height of the house is 4.5 m.

Practise Now 10

(i) By measuring the vertical width, which represents 750 cm, we get 5 cm. So the scale is 5 : 750, which is 1 : 150.

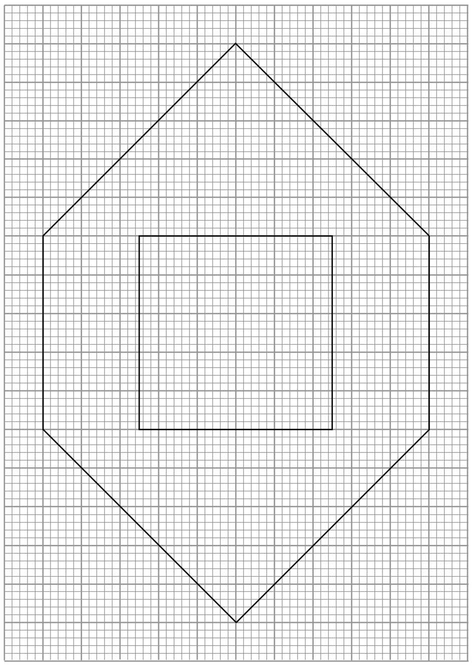
(ii)

Plan		**Actual**
1 cm	represents	150 cm
7 cm	represents	(7 × 150) cm
		= 1050 cm
		= 10.5 m

$\therefore$ The actual length L of the apartment is 10.5 m.

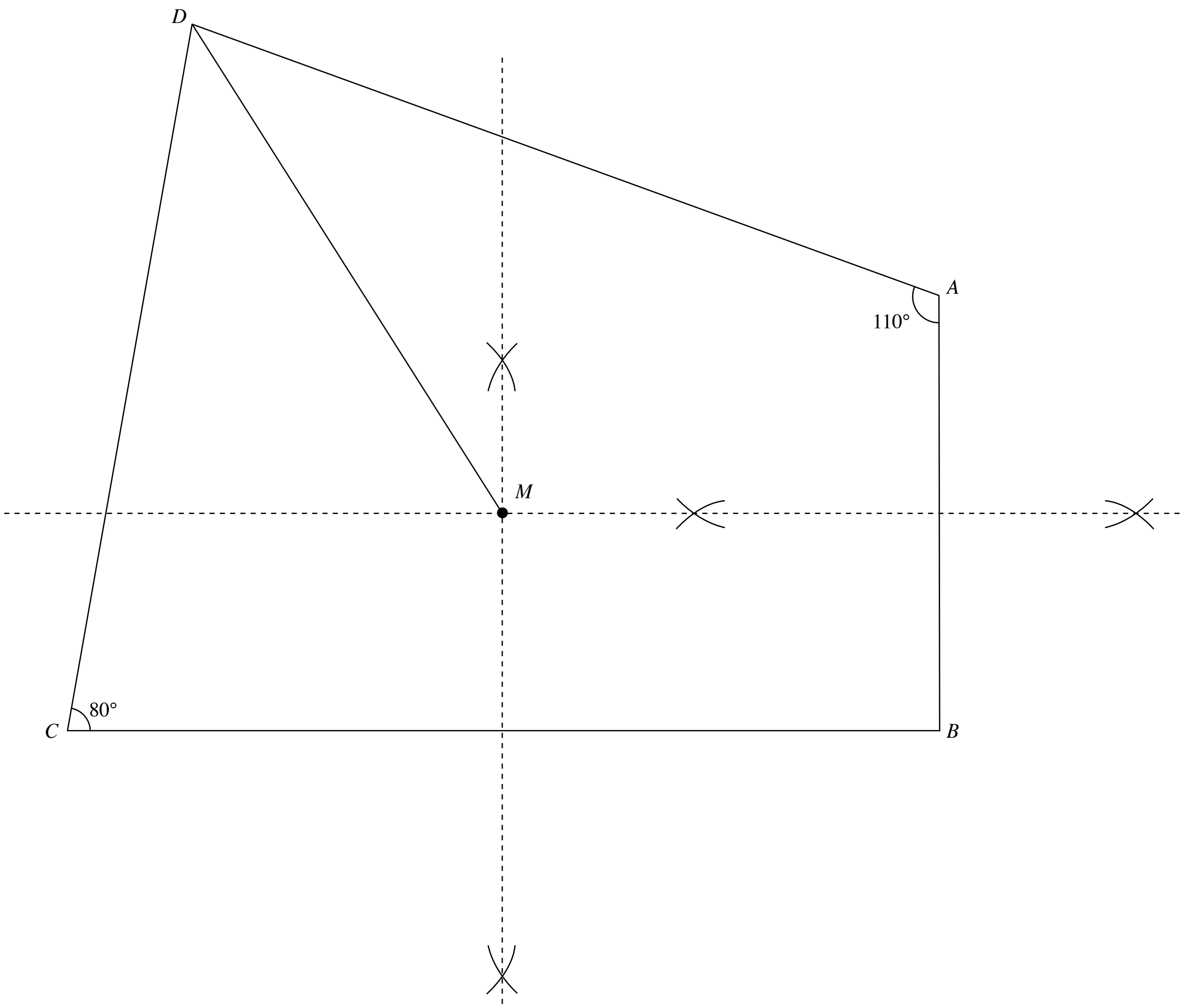
Practise Now 11

Since the scale is 1 : 2.5, we use the grid and enlarge each part of the figure by multiplying the length by 2.5. For example, the length of the square changes from 2 cm to 5 cm. The scale drawing is shown below.



Practise Now 12

(i) & **(ii):**



(iii) By measurement, $AB = 6$ cm on the scale drawing.
Hence, the scale is 6 cm : 48 km, i.e. 1 cm to 8 km.

(iv) From part **(ii)**, the distance of D to the mast $M = 7.9$ cm
The actual distance from D to the mast $M = 7.9 \times 8$
$= 63.2$ km

Practise Now 13

1. (i)

Plan		**Actual**
1 cm	represents	2.5 m
1.25 cm	represents	(1.25×2.5) m
		$= 3.125$ m

$\therefore$ The actual length of the dining room is 3.125 m.

(ii)

Actual		**Plan**
2.5 m	is represented by	1 cm
1 m	is represented by	$\frac{1}{2.5} = 0.4$ cm
3.4 m	is represented by	(3.4×0.4) cm
		$= 1.36$ cm

$\therefore$ The length on the plan is 1.36 cm.

2. **(i)** **Plan** | | **Actual**

Plan		**Actual**
1 cm	represents	4 m
67 cm	represents	(67×4) m
		$= 268$ m

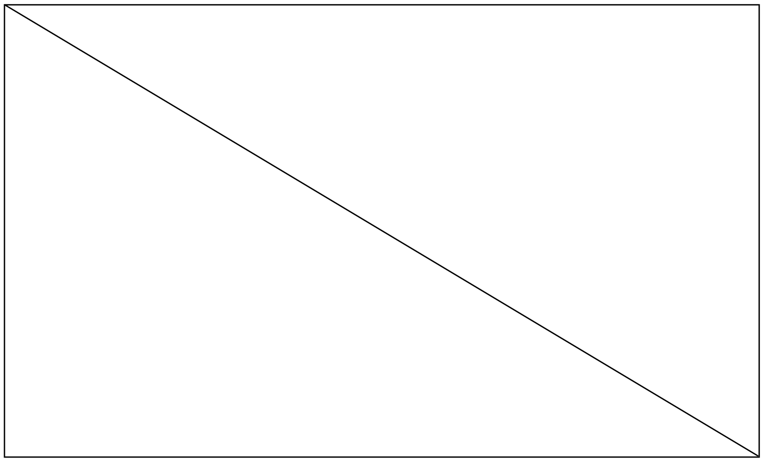
$\therefore$ The actual length of the cruise liner is 268 m.

(ii)

Actual		**Plan**
10 m	is represented by	1 cm
1 m	is represented by	$\frac{1}{10} = 0.1$ cm
268 m	is represented by	(268×0.1) cm
		$= 26.8$ cm

$\therefore$ The length of the model cruise liner is 26.8 cm.

3. **(i)** Scale: 1 cm to 5 m



(ii) From **(i)**,

Distance between opposite corners on scale drawing = 11.7 cm

Actual distance between opposite corners $= 11.7 \times 5$

$= 58.5$ m

Practise Now 14

1. **(i)**

Map		**Actual**
1 cm	represents	5 km
6.5 cm	represents	(6.5×5) km
		$= 32.5$ km

$\therefore$ The actual length of the road is 32.5 km.

(ii)

Actual		**Map**
5 km	is represented by	1 cm
1 km	is represented by	$\frac{1}{5} = 0.2$ cm
25 km	is represented by	(0.2×25) cm
		$= 5$ cm

$\therefore$ The corresponding distance on the map is 5 cm.

(iii) 5 km = 500 000 cm

i.e. the scale of the map is $\frac{1}{500\,000}$.

2. **(i)**

Map		**Actual**
1 cm	represents	50 000 cm = 0.5 km
2 cm	represents	(2×0.5) km
		= 1 km

$\therefore$ The actual length is 1 km.

(ii)

Actual		**Map**
0.5 km	is represented by	1 cm
1 km	is represented by	$\frac{1}{0.5} = 2$ cm
14.5 km	is represented by	(14.5×2) cm
		= 29 cm

$\therefore$ The length on the map is 29 cm.

Practise Now 15

1. **(i)**

Map		**Actual**
1 cm	represents	2 km
1 cm^2	represents	$(2 \text{ km})^2 = 4 \text{ km}^2$
3 cm^2	represents	(3×4) km^2
		= 12 km^2

$\therefore$ The actual area of the plot of land is 12 km^2.

(ii) $18\,000\,000 \text{ m}^2 = \left(\frac{18\,000\,000}{1\,000\,000}\right) \text{km}^2$

$= 18 \text{ km}^2$

Actual		**Map**
2 km	is represented by	1 cm
1 km	is represented by	$\frac{1}{2}$ cm
1 km^2	is represented by	$\left(\frac{1}{2}\text{ cm}\right)^2 = \frac{1}{4}$ cm^2
18 km^2	is represented by	$\left(18 \times \frac{1}{4}\right)$ cm^2
		= 4.5 cm^2

$\therefore$ The area on the map is 4.5 cm^2.

2. Area of $\triangle ABC$ on the scale drawing

$= \frac{1}{2} \times 7 \times 4$

$= 14 \text{ cm}^2$

Map		**Actual**
1 cm	represents	3 km
1 cm^2	represents	$(3 \text{ km})^2 = 9 \text{ km}^2$
14 cm^2	represents	(14×9) km^2
		= 126 km^2

$\therefore$ The actual area of the plot of land is 126 km^2.

Exercise 8A

1. A is congruent to F.
B is congruent to J.
C is congruent to E.
D is congruent to G.
I is congruent to K.

2. **(i)** $PQ = VW = \underline{3.5}$ cm
(ii) $PT = \underline{VZ} = 2$ cm
(iii) $QR = \underline{WX} = \underline{3.5}$ cm
(iv) $TS = \underline{ZY} = \underline{2.1}$ cm
(v) $SR = \underline{YX} = \underline{2}$ cm
(vi) $\angle PQR = \underline{\angle VWX} = \underline{90}°$

3. Since $EFGH \equiv LMNO$, then the corresponding vertices match.
$EF = LM$
$= 3.4$ cm
$GH = NO$
$= 2.4$ cm
$\angle FEH = \angle MLO$
$= 100°$
$\angle FGH = \angle MNO$
$= 75°$
$MN = FG$
$= 5$ cm
$OL = HE$
$= 3$ cm
$\angle LMN = \angle EFG$
$= 65°$
$\angle NOL = \angle GHE$
$= 120°$

4. **(a)** $\angle ACB = 180° - 90° - 36.9°$ ($\angle$ sum of $\triangle ABC$)
$= 53.1°$
$\angle PRQ = 180° - 90° - 36.9°$ ($\angle$ sum of $\triangle PQR$)
$= 53.1°$
$A \leftrightarrow P$ (since $\angle A = \angle P = 36.9°$)
$B \leftrightarrow Q$ (since $\angle B = \angle Q = 90°$)
$C \leftrightarrow R$ (since $\angle C = \angle R = 53.1°$)
$\angle BAC = \angle QPR = 36.9°$
$\angle ABC = \angle PQR = 90°$
$\angle ACB = \angle PRQ = 53.1°$
$AB = PQ = 4$ cm
$BC = QR = 3$ cm
$AC = PR = 5$ cm
$\therefore$ The two triangles have the same shape and size and so $\triangle ABC \equiv \triangle PQR$.

(b) $\angle EDF = 180° - 80° - 70°$ ($\angle$ sum of $\triangle DEF$)
$= 30°$
$\angle SUT = 180° - 80° - 30°$ ($\angle$ sum of $\triangle STU$)
$= 70°$
$D \leftrightarrow T$ (since $\angle D = \angle T = 30°$)
$E \leftrightarrow S$ (since $\angle E = \angle B = 80°$)
$F \leftrightarrow U$ (since $\angle F = \angle U = 70°$)
$\angle EDF = \angle STU = 30°$
$\angle DEF = \angle TSU = 80°$
$\angle DFE = \angle SUT = 70°$
$DE = TS = 18.8$ cm
$EF = QR = 3$ cm
$DF = TU = 19.7$ cm
$\therefore$ The two triangles have the same shape and size and so $\triangle DEF \equiv \triangle TSU$.

(c) $\angle LNM = 180° - 65° - 70°$ ($\angle$ sum of $\triangle LMN$)
$= 45°$
$\angle XZY = 180° - 65° - 70°$ ($\angle$ sum of $\triangle XYZ$)
$= 45°$
$L \leftrightarrow X$ (since $\angle L = \angle X = 65°$)
$M \leftrightarrow Y$ (since $\angle M = \angle Y = 70°$)
$N \leftrightarrow Z$ (since $\angle N = \angle Z = 45°$)
$MN = 4 \neq 5.13 = YZ$
$\therefore$ Since the corresponding sides are not equal, $\triangle LMN$ is not congruent to $\triangle XYZ$.

5. (i) Since $\triangle ABK \equiv \triangle ACK$, then the corresponding vertices match.
$\angle ABK = \angle ACK$
$= 62°$
$\angle BAK = 180° - 90° - 62°$ ($\angle$ sum of $\triangle ABK$)
$= 28°$
$\angle CAK = \angle BAK$
$= 28°$
$\therefore \angle BAC = \angle BAK + \angle CAK$
$= 28° + 28°$
$= 56°$

(ii) Length of CK = length of BK
$= 8$ cm
$\therefore$ Length of BC = length of BK + length of CK
$= 8 + 8$
$= 16$ cm

6. (i) Since $\triangle ABC \equiv \triangle DEC$, then the corresponding vertices match.
$\angle BAC = \angle EDC$
$= 34°$
$\therefore \angle ABC = 180° - 71° - 34°$
$= 75°$

(ii) Length of CD = length of CA
$= 6.9$ cm
$\therefore$ Length of BD = length of BC + length of CD
$= 4 + 6.9$
$= 10.9$ cm

7. (i) Since $\triangle ABK \equiv \triangle ACH$, then the corresponding vertices match.
$\angle AHC = \angle AKB$
$= 180° - 90°$ (adj. $\angle$s on a str. line)
$= 90°$
Length of AH = length of AK
$\therefore \triangle AHK$ is an isosceles triangle.
Let $\angle AHK$ be $x°$.
$\angle AKH = \angle AHK$ (base $\angle$s of isos. $\triangle AHK$)
$= x$
$\angle CHK = 90° - x°$
$\angle CKH = 90° - x°$
$\therefore \triangle CHK$ is an isosceles triangle.
Let the length of CH be n cm.
Length of $CK = n$ cm (isos. $\triangle$)
Length of BK = length of CH
$= n$ cm
$n + n = 12$
$2n = 12$
$n = 6$
$\therefore$ The length of CH is 6 cm.

(ii) $\angle BAC$
$= 180° - 58° - 58°$ ($\angle$ sum of $\triangle ABC$) (base $\angle$s of isos. $\triangle ABC$)
$= 64°$
$\triangle ACH = \angle ABK$
$= 58°$
$\angle CAH = 180° - 90° - 58°$ ($\angle$ sum of $\triangle ACH$)
$= 32°$
$\therefore \angle BAH = \angle BAC + \angle CAH$
$= 64° + 32°$
$= 96°$

Exercise 8B

1. (a) Since $\triangle ABC$ is similar to $\triangle PQR$, then all the corresponding angles are equal.
$x° = \angle PQR$
$= \angle ABC$
$= 90°$
$y° = \angle ACB$
$= \angle PRQ$
$= 35°$
$z° = \angle QPR$
$= 180° - 90° - 35°$ ($\angle$ sum of $\triangle PQR$)
$= 55°$
$\therefore x = 90, y = 35, z = 55$

(b) Since $\triangle ABC$ is similar to $\triangle PQR$, then all the corresponding angles are equal.
$x° = \angle PRQ$
$= \angle ACB$
$= 28°$
$y° = \angle BAC$
$= \angle QPR$
$= 180° - 118° - 28°$ ($\angle$ sum of $\triangle PQR$)
$= 34°$
$\therefore x = 28, y = 34$

(c) Since $\triangle ABC$ is similar to $\triangle PQR$, then all the ratios of the corresponding sides are equal.

$$\frac{QR}{BC} = \frac{PQ}{AB}$$
$$\frac{x}{12} = \frac{6}{10}$$
$$x = \frac{6}{10} \times 12$$
$$= 7.2$$
$$\frac{PR}{AC} = \frac{PQ}{AB}$$
$$\frac{y}{18} = \frac{6}{10}$$
$$y = \frac{6}{10} \times 18$$
$$= 10.8$$
$\therefore x = 7.2, y = 10.8$

(d) Since $\triangle ABC$ is similar to $\triangle PQR$, then all the ratios of the corresponding sides are equal.

$$\frac{AC}{PR} = \frac{AB}{PQ}$$
$$\frac{x}{8} = \frac{12}{10}$$
$$x = \frac{12}{10} \times 8$$
$$= 9.6$$
$$\frac{QR}{BC} = \frac{PQ}{AB}$$
$$\frac{y}{7} = \frac{10}{12}$$
$$y = \frac{10}{12} \times 7$$
$$= 5\frac{5}{6}$$
$\therefore x = 9.6, y = 5\frac{5}{6}$

2. (a) $\angle B = \frac{180° - 40°}{2}$ ($\angle$ sum of $\triangle ABC$)(base $\angle$s of isos. $\triangle ABC$)

$= 70°$

$\angle C = 70°$ (base $\angle$s of isos. $\triangle ABC$)

$\triangle R = 50°$ (base $\angle$s of isos. $\triangle PQR$)

$\triangle P = 180° - 50° - 50°$ ($\angle$ sum of $\triangle PQR$)

$= 80°$

$\angle A = 40° \neq 80° = \angle P$

$\angle B = 70° \neq 50° = \angle Q$

$\angle C = 70° \neq 50° = \angle R$

Since all the corresponding angles are not equal, then $\triangle ABC$ is not similar to $\triangle PQR$.

(b) $\frac{DE}{ST} = \frac{3.3}{2.4} = 1.375$

$\frac{EF}{TU} = \frac{5.7}{3.8} = 1.5$

$\frac{DF}{SU} = \frac{5.4}{3.6} = 1.5$

Since not all the ratios of the corresponding sides are equal, $\triangle DEF$ is not similar to $\triangle STU$.

3. (a) Since $ABCD$ is similar to $PQRS$, then all the corresponding angles are equal.

$x° = \angle QPS$

$= \angle BAD$

$= 95°$

$y° = \angle QRS$

$= \angle BCD$

$= 360° - 95° - 105° - 108°$ ($\angle$ sum of quad.)

$= 52°$

Since $ABCD$ is similar to $PQRS$, then all the ratios of the corresponding sides are equal.

$$\frac{PQ}{AB} = \frac{QR}{BC}$$
$$\frac{z}{8} = \frac{7.2}{12}$$
$$z = \frac{7.2}{12} \times 8$$
$$= 4.8$$
$\therefore x = 95, y = 52, z = 4.8$

(b) Since $ABCD$ is similar to $PQRS$, then all the corresponding angles are equal.

$x° = \angle ADC$

$= \angle PSR$

$= 180° - 100°$ (int. $\angle$s, PQ // SR)

$= 80°$

Since $ABCD$ is similar to $PQRS$, then all the ratios of the corresponding sides are equal.

$$\frac{PS}{AD} = \frac{RS}{CD}$$
$$\frac{y}{14} = \frac{9}{12}$$
$$y = \frac{9}{12} \times 14$$
$$= 10.5$$
$\therefore x = 80, y = 10.5$

4. Since the two water bottles are similar, then all the ratios of the corresponding sides are equal.

$$\frac{x}{10} = \frac{8}{5}$$
$$x = \frac{8}{5} \times 10$$
$$= 16$$
$$\frac{y}{3} = \frac{5}{8}$$
$$y = \frac{5}{8} \times 3$$
$$= 1.875$$
$\therefore x = 16, y = 1.875$

5. Since the two toy houses are similar, then all the corresponding angles are equal and all the ratios of the corresponding sides are equal.

$x° = 100°$

$\frac{y}{180} = \frac{180}{120}$

$y = \frac{180}{120} \times 180$

$= 270$

$\frac{z}{150} = \frac{120}{180}$

$z = \frac{120}{180} \times 150$

$= 100$

$\therefore x = 100, y = 270, z = 100$

6. Let the height of the lamp be x m.

$\therefore \frac{x}{3} = \frac{10 + 6}{6}$

$x = \frac{16}{6} \times 3$

$= 8$

The height of the lamp is 8 m.

7. Since $\triangle ABC$ is similar to $\triangle ADE$, then all the corresponding angles are equal.

$x° = \angle ADE$

$= \angle ABC$

$= 56°$

Since $\triangle ABC$ is similar to $\triangle ADE$, then all the ratios of the corresponding sides are equal.

$\frac{AD}{AB} = \frac{AE}{AC}$

$\frac{y + 4}{4} = \frac{6 + 9}{6}$

$y + 4 = \frac{15}{6} \times 4$

$= 10$

$y = 6$

$\therefore x = 56, y = 6$

8. Since $\triangle PQR$ is similar to $\triangle BAR$, then all the corresponding angles are equal.

$\angle ABR = \angle QPR$

$= 60°$

$x° = \angle BAR$

$= 180° - 60° - 52°$ ($\angle$ sum of $\triangle BAR$)

$= 68°$

Since $\triangle PQR$ is similar to $\triangle BAR$, then all the ratios of the corresponding sides are equal.

$\frac{BR}{PR} = \frac{AB}{QP}$

$\frac{y}{14} = \frac{9}{12}$

$y = \frac{9}{12} \times 14$

$= 10.5$

$\therefore x = 60, y = 10.5$

9. **(i)** Since $\triangle TBP$ is similar to $\triangle TAQ$, then all the ratios of the corresponding sides are equal.

$\frac{x}{y} = \frac{AQ}{BP}$

$\frac{x}{y} = \frac{6}{2}$

$x = 3y$

$\therefore$ Length of $PA = x + y$

$= 3y + y$

$= 4y$ m

(ii) Since $\triangle PTM$ is similar to $\triangle PQA$, then all the ratios of the corresponding sides are equal.

$\frac{TM}{QA} = \frac{PM}{PA}$

$\frac{TM}{6} = \frac{y}{4y}$

$\therefore TM = \frac{1}{4} \times 6$ (From **(i)**, $PA = 4y$)

$= 1.5$ m

Exercise 8C

1. $\triangle XYZ$ is similar to $\triangle X'Y'Z'$ under enlargement.

$\frac{X'Y'}{XY} = \frac{Y'Z'}{YZ} = 2.5$

$\frac{X'Y'}{4} = 2.5$ and $\frac{8.75}{YZ} = 2.5$

$\therefore X'Y' = 10$ cm and $YZ = 3.5$ cm

2. **(i)** $PQRS$ is similar to $P'Q'R'S'$ under enlargement.

$k = \frac{P'Q'}{PQ}$

$= \frac{16}{8}$

$= 2$

$\therefore k = 2$

(ii) $\frac{Q'R'}{QR} = \frac{S'R'}{SR} = 2$

$\frac{Q'R'}{4} = 2$ and $\frac{14}{SR} = 2$

$\therefore Q'R' = 8$ cm and $SR = 7$ cm

3. **(i)** By measuring the vertical width, which represents 28 km, we get 3.5 cm.

Hence, the scale is 3.5 cm : 28 km, which is 1 cm : 8 km.

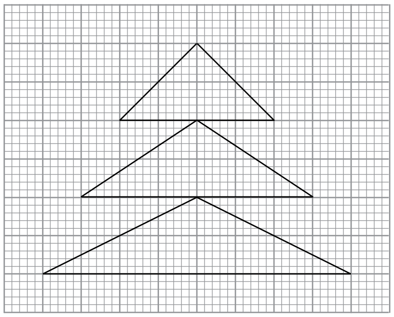
(ii) By measuring x, we get 7 cm.

Actual distance between the East and West of Singapore

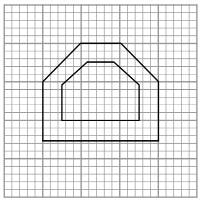
$= 7 \times 8$

$= 56$ km

4. **(a)** Scale = 1 : 2

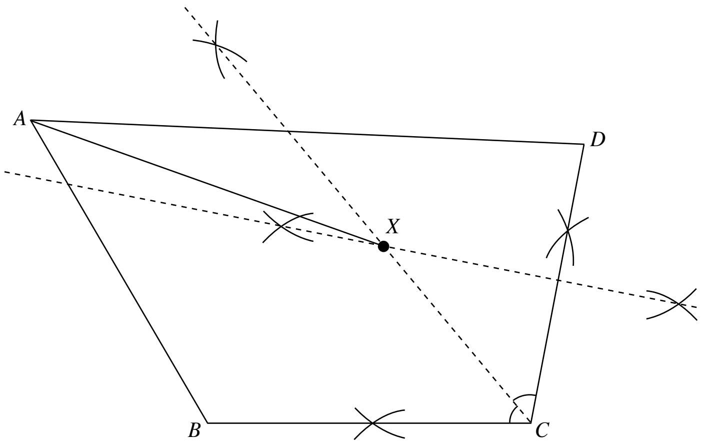


(b) Scale = 1 : 0.5



5. **(a)** **(i)** & **(ii)**

(b) **(i)**



(ii) From **(i)**, the shortest distance of the treasure from the corner A = 5 cm

The actual shortest distance of the treasure from the corner $A = 5 \times 25$

$= 125$ m

6. **(i)**

Map		**Actual**
1 cm	represents	250 m
7.68 cm	represents	(7.68×250) m
		$= 1920$ m

$\therefore$ The actual length of the Tuas Second Link is 1920 m.

(ii)

Actual		**Map**
250 m	is represented by	1 cm
1 m	is represented by	$\frac{1}{250} = 0.004$ cm
25 m	is represented by	(25×0.004) cm
		$= 0.1$ cm

$\therefore$ The corresponding width on the map is 0.1 cm.

(iii) 250 m = 25 000 cm

i.e. the scale of the map is $\frac{1}{25\,000}$.

7. **(i)**

Map		**Actual**
1 cm	represents	20 000 cm = 0.2 km
$5\frac{1}{2}$ cm	represents	$\left(5\frac{1}{2} \times 0.2\right)$ km
		$= 1.1$ km

$\therefore$ The actual length is 1.1 km.

(ii)

Actual		**Map**
0.2 km = 200 m	is represented by	1 cm
1 m	is represented by	$\frac{1}{200} = 0.005$ cm
100 m	is represented by	(100×0.005) cm
		$= 0.5$ cm

$\therefore$ The length on the map is 0.5 cm.

8. **(i)**

Map		**Actual**
1 cm	represents	8 km
1 cm^2	represents	$(8 \text{ km})^2 = 64 \text{ km}^2$
5 cm^2	represents	$(5 \times 64) \text{ km}^2$
		$= 320 \text{ km}^2$

$\therefore$ The actual area of the forest is 320 km^2.

(ii)

Actual		**Map**
8 km	is represented by	1 cm
1 km	is represented by	$\frac{1}{8}$ cm
1 km^2	is represented by	$\left(\frac{1}{8} \text{ cm}\right)^2 = \frac{1}{64} \text{ cm}^2$
128 km^2	is represented by	$\left(128 \times \frac{1}{64}\right) \text{ cm}^2$
		$= 2 \text{ cm}^2$

$\therefore$ The area of the park on the map is 2 cm^2.

9. $\triangle ABC$ is similar to $\triangle AB'C'$ under enlargement.

$$\frac{B'C'}{BC} = \frac{AB'}{AB} = 3$$

$$\frac{12}{BC} = 3 \quad \text{and} \quad \frac{AB + 6}{AB} = 3$$

$\therefore BC = 4$ cm

$6 + AB = 3AB$

$2AB = 6$

$AB = 3$ cm

$\therefore AB' = 3 + 6$

$= 9$ cm

10. Let the height of the tin of milk on the screen be h cm.

$$\frac{h}{24} = \frac{25}{75}$$

$$\therefore h = \frac{25}{75} \times 24$$

$$= 8$$

The height of the tin of milk on the screen is 8 cm.

11. **(i)**

Plan		**Actual**
1 cm	represents	1.5 m
2.5 cm	represents	(2.5×1.5) m
		$= 3.75$ m
3 cm	represents	(3×1.5) m
		$= 4.5$ m

$\therefore$ The actual dimensions of Bedroom 1 are 4.5 m by 3.75 m.

(ii) Area of the kitchen on the plan $= 2 \times 1.5$
$= 3\ \text{cm}^2$

Plan		**Actual**
1 cm	represents	1.5 m
$1\ \text{cm}^2$	represents	$(1.5\ \text{m})^2 = 2.25\ \text{m}^2$
$3\ \text{cm}^2$	represents	$(3 \times 2.25)\ \text{m}^2$
		$= 6.75\ \text{m}^2$

$\therefore$ The actual area of the kitchen is $6.75\ \text{m}^2$.

(iii) Area of the apartment on the plan
$= (3 + 2) \times (3 + 1.5 + 2.5)$
$= 5 \times 7$
$= 35\ \text{cm}^2$

Plan		**Actual**
1 cm	represents	1.5 m
$1\ \text{cm}^2$	represents	$(1.5\ \text{m})^2 = 2.25\ \text{m}^2$
$35\ \text{cm}^2$	represents	$(35 \times 2.25)\ \text{m}^2$
		$= 78.75\ \text{m}^2$

$\therefore$ The actual total area of the apartment is $78.75\ \text{m}^2$.

12. (i) The scale used is 12 cm : 3 m, which is 1 cm : 0.25 m.

(ii)

Actual		**Plan**
0.25 m = 2.5 cm	is represented by	1 cm
1 cm	is represented by	$\frac{1}{25} = 0.04$ cm
425 cm	is represented by	(425×0.04) cm
		= 17 cm

$\therefore$ The width of the living room on the floor plan is 17 cm.

13. (i)

Model		**Actual**
1 cm	represents	15 m
42.4 cm	represents	(42.4×15) m
		= 636 m

$\therefore$ The actual height of the tower is 636 m.

(ii)

Actual		**Model**
12 m	is represented by	1 cm
1 m	is represented by	$\frac{1}{12}$ cm
636 m	is represented by	$\left(636 \times \frac{1}{12}\right)$ cm
		= 53 cm

$\therefore$ The height of the model tower is 53 cm.

14. (i)

Map		**Actual**
4 cm	represents	5 km
1 cm	represents	$\frac{5}{4}$ km = 1.25 km
1.12 cm	represents	(1.12×1.25) km
		= 1.4 km

$\therefore$ The actual distance between the two shopping centres is 1.4 km.

(ii)

Actual		**Map**
175 000 cm = 1.75 km	is represented by	1 cm
1 km	is represented by	$\frac{1}{1.75}$ cm
1.4 km	is represented by	$\left(1.4 \times \frac{1}{1.75}\right)$ cm
		= 0.8 cm

$\therefore$ The distance between the two shopping centres on another map is 0.8 cm.

15. (i) By measuring the bar on the map, which represents 300 m, we get 2.4 cm.
Hence, the scale of the map is 2.4 cm : 300 m, which is 1 cm : 125 m.
125 m = 12 500 cm
i.e. the scale of the map is 1 : 12 500.

(ii) By measuring *XY* on the map, we get 2.5 cm.

Map		**Actual**
1 cm	represents	125 m
2.5 cm	represents	(2.5×125) m
		= 312.5 m

$\therefore$ The actual distance *XY* of the biking trail is 312.5 m.

(iii) The actual trail *XY* is not a fully straight trail.

16. (i) 500 m = 50 000 cm
i.e. the scale of the map is 1 : 50 000.

(ii)

Actual		**Map**
500 m = 0.5 km	is represented by	1 cm
1 km	is represented by	$\frac{1}{0.5}$ cm = 2 cm
28 km	is represented by	(28×2) cm
		= 56 cm

$\therefore$ The corresponding distance on the map is 56 cm.

(iii)

Map		**Actual**
1 cm	represents	0.5 km
$1\ \text{cm}^2$	represents	$(0.5\ \text{km})^2 = 0.25\ \text{km}^2$
$12\ \text{cm}^2$	represents	$(12 \times 0.25)\ \text{km}^2$
		$= 3\ \text{km}^2$

$\therefore$ The actual area of the jungle is $3\ \text{km}^2$.

17. (i)

Map		**Actual**
1 cm	represents	240 000 cm = 2.4 km
$1\ \text{cm}^2$	represents	$(2.4\ \text{km})^2 = 5.76\ \text{km}^2$
$3.8\ \text{cm}^2$	represents	$(3.8 \times 5.76)\ \text{km}^2$
		$= 21.888\ \text{km}^2$
		$= 21.9\ \text{km}^2$ (to 3 s.f.)

$\therefore$ The actual area of the lake is $21.9\ \text{km}^2$.

(ii)

Actual		**Map**
2.4 km = 2400 m	is represented by	1 cm
1 m	is represented by	$\frac{1}{2400}$ cm
$1\ \text{m}^2$	is represented by	$\left(\frac{1}{2400}\text{cm}\right)^2$
$2\ 908\ 800\ \text{m}^2$	is represented by	$\left(2\ 908\ 800 \times \frac{1}{2400^2}\right)\text{cm}^2$
		$= 0.505\ \text{cm}^2$

$\therefore$ The area on the map is $0.505\ \text{cm}^2$.

18. Area of the field on the scale drawing
$= 21 \times 13.6$
$= 285.6 \text{ cm}^2$

Plan		**Actual**
1 cm	represents	5 m
1 cm^2	represents	$(5 \text{ m})^2 = 25 \text{ m}^2$
285.6 cm^2	represents	$(285.6 \times 25) \text{ m}^2$
		$= 7140 \text{ m}^2$

∴ The actual area of the field is 7140 m^2.

19. (i) By measuring 60 km on the scale, we get 1 cm.
60 km = 6 000 000 cm
i.e. the scale of the map is $\frac{1}{6\,000\,000}$.

(ii) By measuring the distance between Singapore and Kuantan on the map, we get 5.2 cm.

Map		**Actual**
1 cm	represents	60 km
5.2 cm	represents	(5.2×60) km
		= 312 km

∴ The actual distance between Singapore and Kuantan is 312 km.

(iii) By measuring the distance between Melaka and Kuala Lumpur on the map, we get 2.3 cm.

Map		**Actual**
1 cm	represents	60 km
2.3 cm	represents	(2.3×60) km
		= 138 km

Actual distance between Melaka and Kuala Lumpur = 138 km
Taxi fare $= 13.8 \times \$0.60$
$= \$82.80$
∴ The taxi fare is $82.80.

(iv) By measuring the distance between Batu Pahat and Port Dickson on the map, we get 2.7 cm.

Map		**Actual**
1 cm	represents	60 km
2.7 cm	represents	(2.7×60) km
		= 162 km

Time to travel $= \frac{162}{60}$
= 2.7 hours
= 2 hours 42 minutes
∴ The time taken to travel is 2 hours 42 minutes.

(v) By measuring the distance between Johor Bahru and Segamat on the map, we get 2.8 cm.

Map		**Actual**
1 cm	represents	60 km
2.8 cm	represents	(2.8×60) km
		= 168 km

Actual distance between Johor Bahru and Segamat = 168 km
Average speed $= \frac{168}{4}$
= 42 km/h
∴ The average speed is 42 km/h.

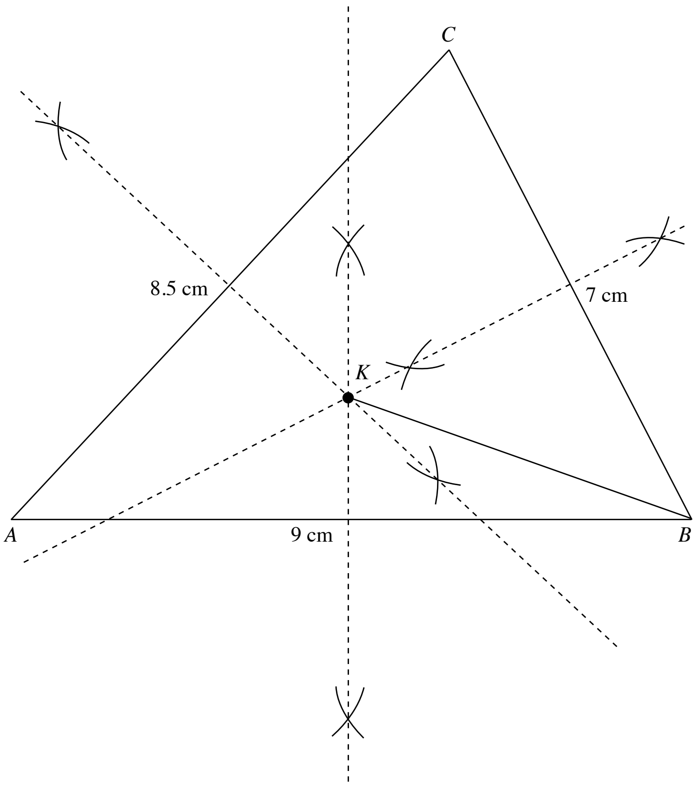
20. Length of plot of land on the map
$= 8 \times 175\%$
$= 8 \times \frac{175}{100}$
= 14 cm
Area of plot of land on the map
$= 14 \times 8$
$= 112 \text{ cm}^2$

Map		**Actual**
1 cm	represents	500 m
1 cm^2	represents	$(500 \text{ m})^2 = 250\,000 \text{ m}^2$
112 cm^2	represents	$(112 \times 250\,000) \text{ m}^2$
		$= 28\,000\,000 \text{ m}^2$

Actual area in hectares $= \frac{28\,000\,000}{10\,000}$
= 2800 hectares
∴ The actual area of the plot of land is 2800 hectares.

21. (i)

Actual		Plan
10 m	is represented by	1 cm
1 m	is represented by	$\frac{1}{10}$ = 0.1 cm
90 m	is represented by	(90×0.1) cm = 9 cm
70 m	is represented by	(70×0.1) cm = 7 cm
85 m	is represented by	(85×0.1) cm = 8.5 cm



(ii) From scale drawing, length of $BK = 4.9$ cm.

Plan		Actual
1 cm	represents	10 m
4.9 cm	represents	(4.9×10) m = 49 m

$\therefore$ The actual distance from B to the point K is 49 m.

Review Exercise 8

1. (a) In $\triangle ABC$,

$\angle C = 180° - 90° - 67°$ ($\angle$ sum of $\triangle ABC$)
$= 23°$

$\therefore$ $\triangle ABC$ does not have an angle of 18° that is in $\triangle PQR$.

$\therefore$ $\triangle ABC$ does not have the same shape as $\triangle PQR$ and so it is not congruent to $\triangle PQR$.

(b) $\angle EDF = 180° - 70° - 62°$ ($\angle$ sum of $\triangle DEF$)
$= 48°$

$\angle TSU = 180° - 70° - 48°$ ($\angle$ sum of $\triangle STU$)
$= 62°$

$D \leftrightarrow T$ (since $\angle D = \angle T = 48°$)
$E \leftrightarrow U$ (since $\angle E = \angle U = 70°$)
$F \leftrightarrow S$ (since $\angle F = \angle S = 62°$)
$\angle EDF = \angle UTS = 48°$
$\angle DEF = \angle TUS = 70°$
$\angle DEF = \angle TSU = 62°$
$DE = TU = 6$ cm
$EF = US = 5$ cm
$DF = TS = 6.4$ cm

$\therefore$ The two triangles have the same shape and size and so $\triangle DEF \equiv \triangle TUS$.

2. (i) Since $ABCD \equiv PQRS$, then the corresponding vertices match.

$\therefore$ Length of AB = length of PQ
$= 6$ cm

(ii) $\angle S = \angle D$
$= 360° - \angle A - \angle B - \angle C - \angle D$ ($\angle$ sum of quad.)
$= 360° - 100° - 70° - 95°$
$= 95°$

3. (i) Since $\triangle ABC \equiv \triangle AKH$, then the corresponding vertices match.

$\angle HAK = \angle CAB$
$= 52°$

$\therefore \angle AKH = 180° - 52° - 36°$ ($\angle$ sum of $\triangle AKH$)
$= 92°$

(ii) Length of AC = length of AH
$= 10.2$ cm

Length of AK = length of AB
$= 6$ cm

$\therefore$ Length of KC = length of AC – length of AK
$= 10.2 - 6$
$= 4.2$ cm

4. (i) Since $\triangle ABC \equiv \triangle AHK$, then the corresponding vertices match.

$\therefore \angle ABC = \angle AHK$
$= 90°$

(ii) Length of AB = length of AH
$= 6$ cm

Area of $\triangle ABC = \frac{1}{2} \times AB \times BC$

$24 = \frac{1}{2} \times 6 \times BC$

$24 = 3BC$

$\therefore$ Length of $BC = 8$ cm

(iii) In $\triangle ABC$,

$\angle ACB = 180° - 90° - 53$ ($\angle$ sum of $\triangle ABC$)
$= 37°$

In $\triangle CHX$,

$\angle CHX = 180° - 90°$ (adj. $\angle$s on a str. line)
$= 90°$

$\angle CXH = 180° - 90° - 37°$ ($\angle$ sum of $\triangle CHX$)
$= 53°$

$\therefore \angle BXK = \angle CXH$ (vert. opp. $\angle$s)
$= 53°$

5. **(a)** $\angle A = 180° - 40° - 60°$ ($\angle$ sum of $\triangle ABC$)
$= 80°$
$\angle R = 180° - 40° - 80°$ ($\angle$ sum of $\triangle PQR$)
$= 60°$
$\angle A = \angle P = 80°$
$\angle B = \angle Q = 40°$
$\angle C = \angle R = 60°$
Since all the corresponding angles are equal, then $\triangle ABC$ is similar to $\triangle PQR$.

(b) $\frac{DE}{ST} = \frac{3}{5.4} = \frac{5}{9}$
$\frac{EF}{TU} = \frac{3.8}{6.84} = \frac{5}{9}$
$\frac{DF}{SU} = \frac{6}{10.8} = \frac{5}{9}$
Since all the ratios of the corresponding sides are equal, then $\triangle DEF$ is similar to $\triangle STU$.

6. Since $\triangle LMN$ is similar to $\triangle ZXY$, then all the corresponding angles are equal.
$a° = \angle MLN$
$= \angle XZY$
$= 180° - 90° - 37°$ ($\angle$ sum of $\triangle ZXY$)
$= 53°$
Since $\triangle LMN$ is similar to $\triangle ZXY$, then all the ratios of the corresponding sides are equal.
$\frac{YZ}{NL} = \frac{XZ}{ML}$
$\frac{b}{10} = \frac{9}{6}$
$b = \frac{9}{6} \times 10$
$= 15$
$\therefore a = 53, b = 15$

7. Since $ABCD$ is similar to $PQRS$, then all the corresponding angles are equal.
$x° = \angle BCD$
$= \angle QRS$
$= 85°$
Since $ABCD$ is similar to $PQRS$, then all the ratios of the corresponding sides are equal.
$\frac{PS}{AD} = \frac{PQ}{AB}$
$\frac{y}{12} = \frac{6}{8}$
$y = \frac{6}{8} \times 12$
$= 9$
$\therefore x = 85, y = 9$

8. **(i)** Since $\triangle ABC$ is similar to $\triangle PQR$, then all the corresponding angles are equal.
$\therefore \angle P = \angle A$
$= 60°$

(ii) Since $\triangle ABC$ is similar to $\triangle PQR$, then all the ratios of the corresponding sides are equal.
$\frac{PQ}{AB} = \frac{PR}{AC}$
$\frac{PQ}{6} = \frac{10}{8}$
$\therefore$ Length of $PQ = \frac{10}{8} \times 6$
$= 7.5$ cm

9. **(i)**

Model		Actual
1 cm	represents	80 cm
25 cm	represents	(25×80) cm
		= 2000 cm
		= 20 m

$\therefore$ The wingspan of the actual aircraft is 20 m.

(ii)

Actual		Model
80 cm = 0.8 m	is represented by	1 cm
1 m	is represented by	$\frac{1}{0.8}$ cm = 1.25 cm
40 m	is represented by	(40×1.25) cm
		= 50 cm

$\therefore$ The length of the model aircraft is 50 cm.

10. **(i)**

Model		Actual
2 cm	represents	7.5 m
1 cm	represents	$\frac{7.5}{2}$ m = 3.75 m
41.6 cm	represents	(41.6×3.75) m
		= 156 m

$\therefore$ The actual height of the block of flats is 156 m.

(ii)

Actual		Model
12 m	is represented by	5 cm
1 m	is represented by	$\frac{5}{12}$ cm
156 m	is represented by	$\left(156 \times \frac{5}{12}\right)$ cm
		= 65 cm

$\therefore$ The height of the model block of flats is 65 cm.

11. **(i)** 1 km = 100 000 cm
i.e. the scale of the map is $\frac{4}{100\,000} = \frac{1}{25\,000}$.

(ii)

Map		Actual
4 cm	represents	1 km
1 cm	represents	$\frac{1}{4}$ km = 0.25 km
3 cm	represents	(0.25×3) km
		= 0.75 km

$\therefore$ The actual length of the river is 0.75 km.

(iii)

Actual		Map
1 km	is represented by	4 cm
8 km	is represented by	(8×4) cm
		= 32 cm

$\therefore$ The distance on the map is 32 cm.

12. (i) **Actual** | | **Map**

Actual		Map
180 000 cm = 1.8 km	is represented is	1 cm
1 km	is represented by	$\frac{1}{1.8}$ cm = $\frac{5}{9}$ cm
35.7 km	is represented by	$\left(35.7 \times \frac{5}{9}\right)$ cm
		= 19.83 cm (to 2 d.p.)

∴ The corresponding length on the map is 19.83 cm.

(ii)

Map		Actual
1 cm	represents	1.8 km
13.5 cm	represents	(13.5×1.8) km
		= 24.3 km

∴ The actual distance between Sentosa and Changi Ferry Terminal is 24.3 km.

(iii)

Actual		Map
1 km	is represented by	$\frac{5}{9}$ cm
1 km^2	is represented by	$\left(\frac{5}{9}\text{ cm}\right)^2 = \frac{25}{81}$ cm^2
5 km^2	is represented by	$\left(5 \times \frac{25}{81}\right)$ cm^2
		= 1.54 cm^2 (to 2 d.p.)

∴ The area of the map is 1.54 cm^2.

13. (i) 3 km = 300 000 cm

i.e. the scale of the map is 2 cm : 300 000 cm, which is 1 : 150 000.

(ii)

Map		Actual
1 cm	represents	150 000 cm = 1.5 km
7 cm	represents	(7×1.5) km
		= 10.5 km

∴ The actual distance between the two towns is 10.5 km.

(iii)

Actual		Map
3 km	is represented by	2 cm
1 km	is represented by	$\frac{2}{3}$ cm
1 km^2	is represented by	$\left(\frac{2}{3}\text{ cm}\right)^2 = \frac{4}{9}$ cm^2
81 km^2	is represented by	$\left(81 \times \frac{4}{9}\right)$ cm^2
		= 36 cm^2

∴ The area of the map is 36 cm^2.

14. (i)

Actual		Map
25 000 cm = 0.25 cm	is represented by	1 cm
1 km	is represented by	$\left(\frac{1}{0.25}\right)$ cm = 4 cm
3.5 km	is represented by	(3.5×4)
		= 14 cm

∴ The corresponding distance on the map is 14 cm.

(ii)

Map		Actual
1 cm	represents	0.25 km
1 cm^2	represents	$(0.25\text{ km})^2 = 0.0625$ km^2
16 cm^2	represents	(16×0.0625) km^2
		= 1 km^2

∴ The actual area is 1 km^2.

15. (i)

Plan		Actual
1 cm	represents	400 cm = 4 m
24.5 cm	represents	(24.5×4) m
		= 98 m

∴ The actual length of the corridor is 98 m.

(ii)

Actual		Plan
4 m	is represented by	1 cm
1 m	is represented by	$\frac{1}{4}$ cm
1 m^2	is represented by	$\left(\frac{1}{4}\text{ cm}\right)^2 = \frac{1}{16}$ cm^2
400 m^2	is represented by	$\left(400 \times \frac{1}{16}\right)$ cm^2
		= 25 cm^2

∴ The area on the plan is 25 cm^2.

(iii)

Plan		Actual
1 m	represents	400 m
1 m^2	represents	$(400\text{ m})^2 = 160\,000$ m^2
0.25 m^2	represents	$(0.25 \times 160\,000)$ m^2
		= 40 000 m^2
		= (40 000 ÷ 10 000) hectares
		= 4 hectares

∴ The actual area in hectares is 4 hectares.

Challenge Yourself

1. (i) $\angle DCP = \angle DBA = 90°$

∴ PC // AB

$\angle DPC = \angle DAB$ (corr. ∠s, PC // AB)

$\angle CDP = \angle BDA$ (common ∠)

Since all the corresponding angles are equal, then $\triangle DPC$ is similar to $\triangle DAB$.

$\angle BCP = 180° - 90°$ (adj. ∠s on a str. line)

$= 90°$

$\angle BCP = \angle BDQ = 90°$

∴ PC // QD

$\angle BPC = \angle BQD$ (corr. ∠s, PC // QD)

$\angle CBP = \angle DBQ$ (common ∠)

Since all the corresponding angles are equal, then $\triangle BPC$ is similar to $\triangle BQD$.

Since PC // AB and PC // QD, AB // QD.

$\angle BAP = \angle QDP$ (alt. ∠s, AB // QD)

$\angle ABP = \angle DQP$ (alt. ∠s, AB // QD)

$\angle APB = \angle DPQ$ (vert. opp. ∠s)

Since all the corresponding angles are equal, then $\triangle ABP$ is similar to $\triangle DQP$.

(ii) In $\triangle ABD$,

$$\frac{CD}{BD} = \frac{PC}{AB}$$

$$PC = \frac{CD}{BD} \times 4$$

In $\triangle QDB$,

$$\frac{BC}{BD} = \frac{PC}{QD}$$

$$PC = \frac{BC}{BD} \times 6$$

$$\therefore \frac{CD}{BD} \times 4 = \frac{BC}{BD} \times 6$$

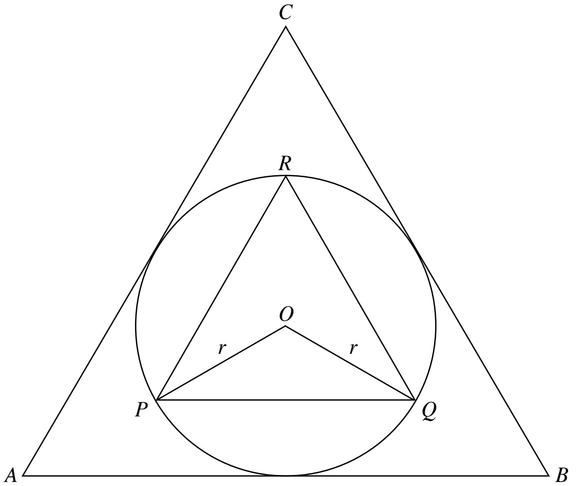
$$4CD = 6BC$$

$$\frac{BC}{CD} = \frac{4}{6}$$

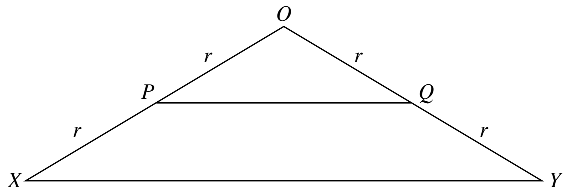
$$= \frac{2}{3}$$

$\therefore$ The ratio of the length of BC to the length of CD is 2 : 3.

2. Let the radius of the circle be r.



Consider $\triangle OPQ$. Extend OP to X and OQ to Y such that $PX = r$ and $QY = r$.



By symmetry, observe that OP bisects $\angle QPR$. $\triangle PQR$ is equilateral.

Hence, $\angle OPQ = \frac{60°}{2} = 30°$.

$\angle OQP = 30°$ (base $\angle$s of isos. $\triangle OPQ$)

$\angle POQ = 180° - 30° - 30°$ ($\angle$ sum of $\triangle OPQ$)

$= 120°$

In $\triangle OXY$,

$\angle OXY = \frac{180° - 120°}{2}$ ($\angle$sum of $\triangle OXY$) (base $\angle$s of isos. $\triangle OXY$)

$= 30°$

$\angle OYX = 30°$ (base $\angle$s of isos. $\triangle OXY$)

$\angle O = 120°$

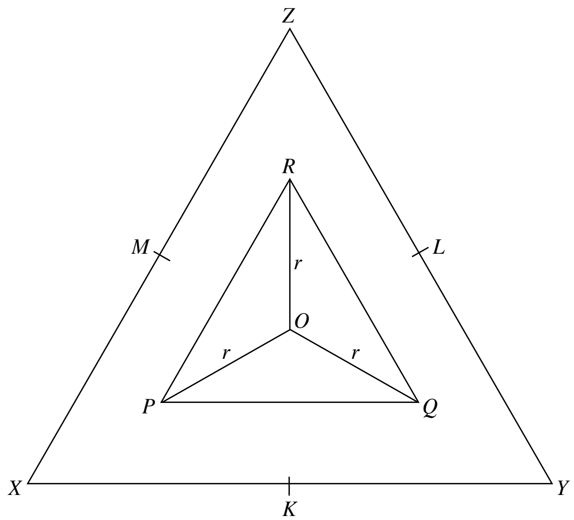
$\angle P = \angle X = 30°$

$\angle Q = \angle Y = 30°$

Since all the corresponding angles are equal, $\triangle OPQ$ is similar to $\triangle OXY$.

$\therefore \frac{XY}{PQ} = \frac{2r}{r} = 2$

By rotational symmetry, $\triangle OPR$ is similar to $\triangle OXZ$ and $\triangle OQR$ is similar to $\triangle OYZ$, where Z is OR extended.



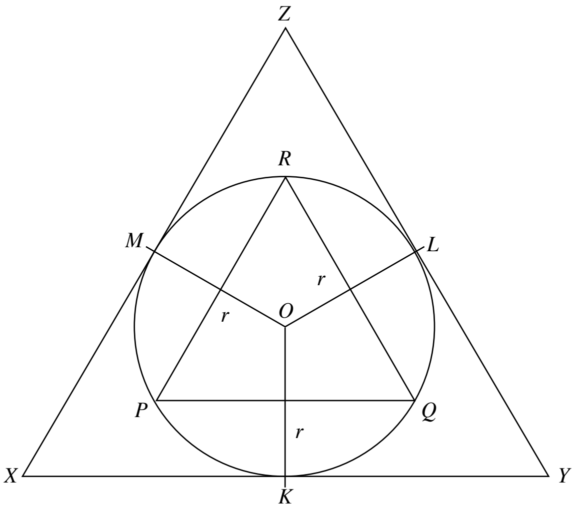
Since $\triangle PQR$ is a equilateral triangle, $\triangle XYZ$ is also an equilateral triangle.

$\therefore \triangle XYZ$ is similar to $\triangle ABC$.

Let K, L and M be the midpoints of XY, YZ and XZ.

By construction of $\triangle OXY$, $OK = r$. Similarly, $OL = OM = r$.

Since $OK = OL = OM = r$, a circle of radius r can be drawn passing through K, L and M.



∴ △*XYZ* is congruent to △*ABC*.

Let *AB* be *s* and *PQ* be *t*.

∴ $\frac{s}{t} = 2$

Let the height of △*ABC* be h_1 and the height of △*PQR* be h_2.

∴ $\frac{h_1}{h_2} = 2$

$$\frac{\text{Area of }\triangle ABC}{\text{Area of }\triangle PQR} = \frac{\frac{1}{2} \times s \times h_1}{\frac{1}{2} \times t \times h_2}$$

$$= \frac{s}{t} \times \frac{h_1}{h_2}$$

$$= 2 \times 2$$

$$= 4$$

∴ The ratio of the area of the bigger triangle to that of the smaller triangle is 4 : 1.

Chapter 9 Geometrical Transformation

TEACHING NOTES

Suggested Approach:

This topic deals with spatial visualisation and teachers would be able to find many examples in the surroundings. Teachers should make use of these everyday examples to help students understand transformations from a three-dimensional point of view, in order for them to apply the concepts to the drawing of graphs.

Section 9.1: Reflection

Students who study Physics would be familiar with the idea of reflection in terms of light. Teachers may further explore their understanding of it and introduce mathematical terms pertaining to motion geometry.

An interesting method to initiate the notion of an invariant point would be to ask students to picture themselves standing in front of the mirror. Teachers can prompt them by asking them to think about how far their images behind the mirror is compared to how far they are from the mirror; and what happens if they move closer to the mirror. Teachers can further suggest that students hold their palms in front of their faces and simulate it as a mirror, and ask what would happen when their noses touch the mirror i.e. pulling their palms to touch the tip of their noses. Students should make the deduction that the image of the nose tip is the nose tip itself.

Section 9.2: Rotation

Teachers should highlight the importance of providing exact specifications for transformations. In the case of a rotation, the centre of rotation needs to be specified. The importance of specifying the centre can be illustrated by calling up students to stretch out an arm each, and rotate it 90 clockwise. A few possibilities would arise as some might rotate their arms such that the centre of rotation is the shoulder, or at the elbow joints, or with their wrists at the centre of rotation.

Section 9.3: Translation

Teachers can arouse curiosity of students by pointing out the utilisation of translation in art, which in turn links back to Mathematics (see Journal Writing on page 248 of the textbook). With the introduction of common encounters with translation in everyday situations, teachers can encourage students to contribute more ideas of such instances, and share their insights with their peers.

WORKED SOLUTIONS

Thinking Time (Page 237)

The two points $M_mM_l(A)$ and $M_lM_m(A)$ are different.

Thinking Time (Page 242)

No. Reflection does not preserve orientation, and in this case, the rotation preserved orientation. A reflection of R in the line $x = 0$ gives the same R', but the results for Q and S are incorrect.

Thinking Time (Page 243)

The distance between A and A' and that between C and C' from the centre of rotation will be the same. Points along the perpendicular bisectors of AA' and CC' indicate where the respective distances between A and A', and C and C' are equal. Hence, the point of intersection of the perpendicular bisectors would mean that this point is equidistant from both A to A' and C to C', thus the centre of rotation lies here.

Thinking Time (Page 246)

No, there would be no invariant points as each point is translated in the same distance.

Thinking Time (Page 248)

(a) Yes, the result would be the same.

(b) The single vector that would produce the same result would be the equivalent of the addition of the two vectors $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 6 \end{pmatrix}$ i.e. $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$.

Journal Writing (Page 248)

Teachers may show students some art pieces of Escher's tessellations and illustrate how transformation was used. Students can also find out more on their own through books in the library or the Internet, where there are many documented records on his impressive tessellations.

Practise Now (Page 237)

Midpoint of CC', $\left(\frac{2+3}{2}, \frac{3+2}{2}\right) = \left(2\frac{1}{2}, 2\frac{1}{2}\right)$

Midpoint of DD', $\left(\frac{3+7}{2}, \frac{7+3}{2}\right) = (5, 5)$

Rise $= 5 - 2\frac{1}{2}$

$= 2\frac{1}{2}$

Run $= 5 - 2\frac{1}{2}$

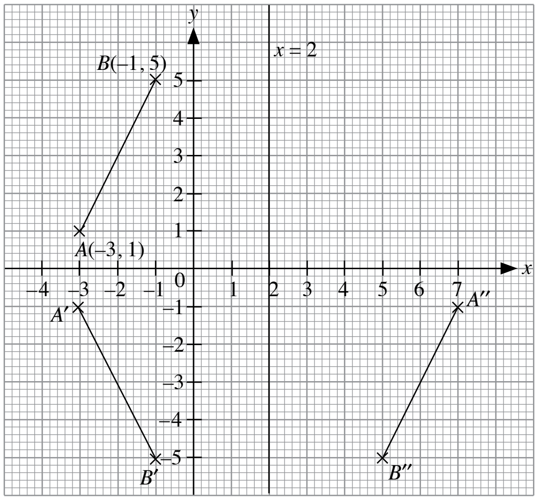
$= 2\frac{1}{2}$

Gradient of line $= \frac{\text{rise}}{\text{run}} = 1$

y-intercept $= 0$

$\therefore$ Equation of line of reflection is $y = x$.

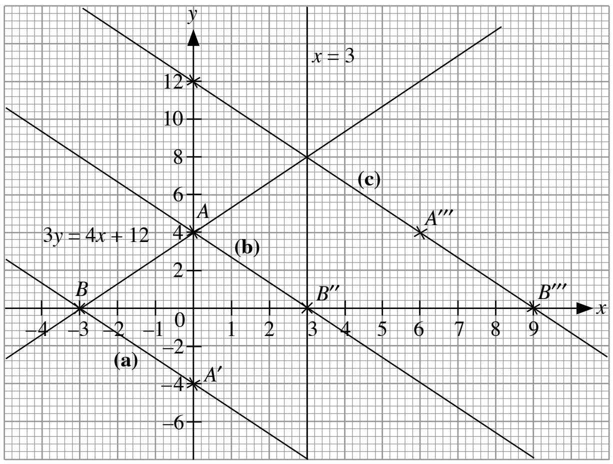
Practise Now 1



(i) From the graph, the coordinates of A' are $(-3, -1)$ and those of B' are $(-1, -5)$.

(ii) From the graph, the coordinates of A'' are $(7, -1)$ and those of B'' are $(5, -5)$.

Practise Now 2



Let AB represent the line $3y = 4x + 12$.

(a) Under reflection in the x-axis, the point A is mapped onto A' $(0, -4)$ and the point B is invariant.

Gradient of $A'B = \frac{\text{rise}}{\text{run}} = -\frac{4}{3}$

y-intercept $= -4$

$\therefore$ The equation of $A'B'$ is $y = -\frac{4}{3}x - 4$, i.e. $3y = -4x - 12$ or $3y + 4x = -12$.

(b) Under reflection in the y-axis, the point A is invariant and the point B is mapped onto $B''(3, 0)$.

Gradient of $AB'' = \frac{\text{rise}}{\text{run}} = -\frac{4}{3}$

y-intercept = 4

$\therefore$ The equation of AB'' is $y = -\frac{4}{3}x + 4$, i.e. $3y = -4x + 12$ or $3y + 4x = 12$.

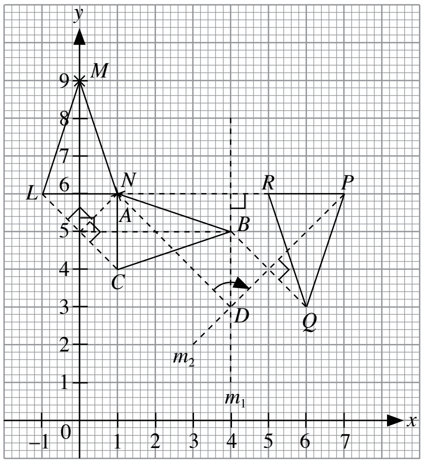
(c) The images of A and B under reflection in the line $x = 3$ are $A'''(6, 4)$ and $B'''(9, 0)$ respectively.

Gradient of $A'''B''' = \frac{\text{rise}}{\text{run}} = -\frac{4}{3}$

y-intercept = 12

$\therefore$ The equation of $A'''B'''$ is $y = -\frac{4}{3}x + 12$, i.e. $3y = -4x + 36$ or $3y + 4x = 36$.

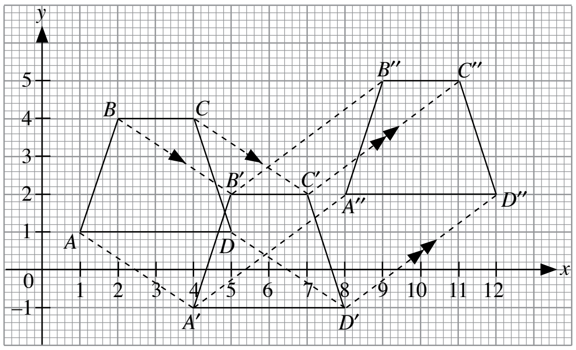
Practise Now 3



(a) From graph, vertices of $\triangle LMN$ are $L(-1, 6)$, $M(0, 9)$ and $N(1, 6)$.

(b) From graph, line m_1 is the perpendicular bisector of AP while line m_2 is the perpendicular bisector of BQ. The point of intersection of these two perpendicular bisectors $D(4, 3)$ gives the centre of rotation. Joining AD and PD gives the angle of rotation which is 90 clockwise.

Practise Now 4



$$T_1(A) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

$$T_1(B) = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

$$T_1(C) = \begin{pmatrix} 4 \\ 4 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$$

$$T_1(D) = \begin{pmatrix} 5 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 8 \\ -1 \end{pmatrix}$$

The vertices of the image are $A'(4, -1)$, $B'(5, 2)$, $C'(7, 2)$ and $D'(8, -1)$. Image of the quadrilateral $A'B'C'D'$ is plotted in the graph above. For the second translation T_2,

$$T_2(A') = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$$

$$T_2(B') = \begin{pmatrix} 5 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \end{pmatrix}$$

$$T_2(C') = \begin{pmatrix} 7 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 11 \\ 5 \end{pmatrix}$$

$$T_2(D') = \begin{pmatrix} 8 \\ -1 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 12 \\ 2 \end{pmatrix}$$

Vertices of the image of this new quadrilateral are $A''(8, 2)$, $B''(9, 5)$, $C''(11, 5)$ and $D''(12, 2)$.

Practise Now 5

(i) $A' = T_1(A)$

$$= \begin{pmatrix} 4 \\ 5 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ 9 \end{pmatrix}$$

(ii) Let the translation vector of T_2 be $\begin{pmatrix} a \\ b \end{pmatrix}$.

$$\begin{pmatrix} 5 \\ 7 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

(iii) $B' = T_2(B)$

$$= \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} 6 \\ -3 \end{pmatrix}$$

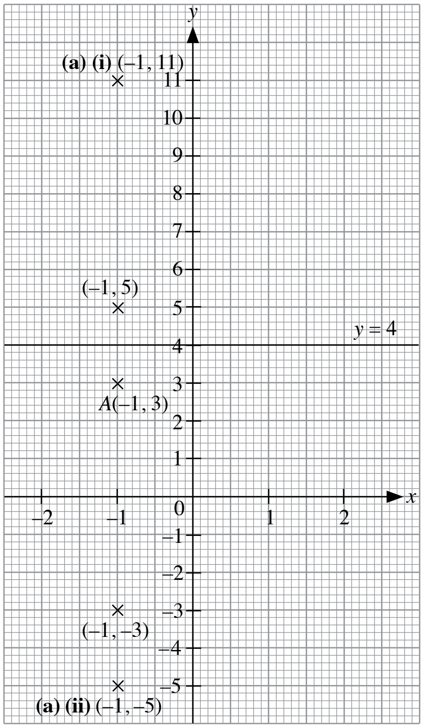
$$= \begin{pmatrix} 8 \\ 2 \end{pmatrix}$$

Exercise 9A

1. A reflection of a point in the x-axis brings about the negative of the original y-coordinate. A reflection in the y-axis results in the negative of the original x-coordinate. A reflection in the line $y = x$ results in the value of the x-coordinate becoming the y-coordinate and vice versa.

(i) **(a)** (3, –4) **(b)** (–3, 4) **(c)** (4, 3)
(ii) **(a)** (–1, –3) **(b)** (1, 3) **(c)** (3, –1)
(iii) **(a)** (3, –3) **(b)** (–3, 3) **(c)** (3, 3)
(iv) **(a)** (–3, 4) **(b)** (3, –4) **(c)** (–4, –3)
(v) **(a)** (3, 2) **(b)** (–3, –2) **(c)** (–2, 3)
(vi) **(a)** $(p, -q)$ **(b)** $(-p, q)$ **(c)** (q, p)

2.



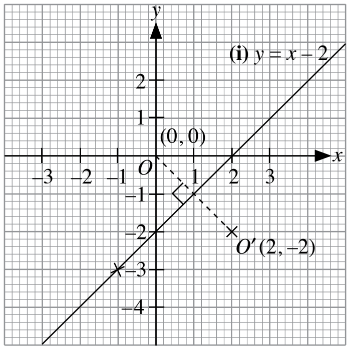
(a) **(i)** From graph, coordinates are (–1, 11)
(ii) From graph, coordinates are (–1, –5)

(b) No, the resultant points are different as the combination of reflections is not commutative.

3. Reflection in the line $x = 2$ results in the x-coordinate changing by $2 - 3 = -1$.
∴ Coordinates of the reflection of the point are (1, 2).

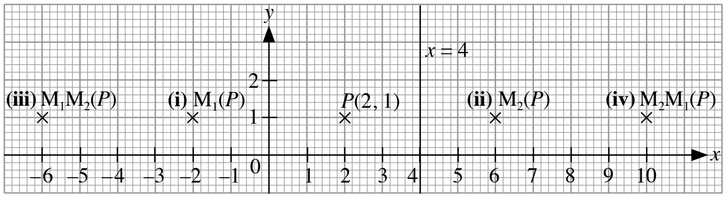
4. **(i)** $y = x - 2$

x	–1	0	2	3
y	–3	–2	0	1



(ii) From graph, the coordinates of O' are (2, –2).

5.



(i) $M_1(P)$ is obtained through a reflection in the y-axis. From graph, the coordinates are (–2, 1)

(ii) $M_2(P)$ is obtained through a reflection in the line $x = 4$. From graph, the coordinates are (6, 1).

(iii) $M_1M_2(P)$ is obtained through a reflection in the line $x = 4$ followed by a reflection in the y-axis. From graph, the coordinates are (–6, 1).

(iv) $M_2M_1(P)$ is obtained through a reflection in the y-axis followed by a reflection in the line $x = 4$. From graph, the coordinates are (10, 1).

6. **(i)** Let (3, 5) be the point A, and (5, 3) be the point A'.

Midpoint of AA', $\left(\frac{3+5}{2}, \frac{5+3}{2}\right) = (4, 4)$

Gradient of line $= \frac{\text{rise}}{\text{run}} = 1$

y-intercept = 0

∴ Equation of line of reflection is $y = x$.

(ii) Let the reflected point (–5, 3) be B.

Midpoint of AB, $\left(\frac{-5+5}{2}, \frac{3+3}{2}\right) = (0, 3)$

Rise = 3

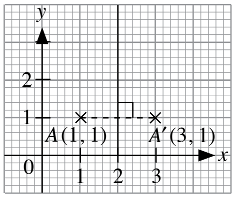
Run = 0

Gradient of line $= \frac{\text{rise}}{\text{run}}$ = undefined, line of reflection parallel to y-axis.

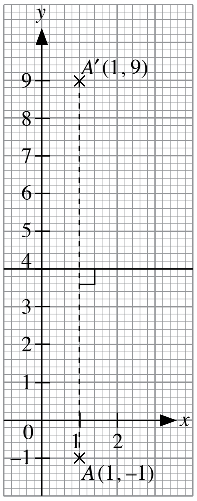
Since x-coordinate of midpoint is 0,

∴ Equation of line of reflection is $x = 0$.

7. **(a)** Equation of line of reflection is $x = 2$.



(b) Equation of line of reflection is $y = 4$.

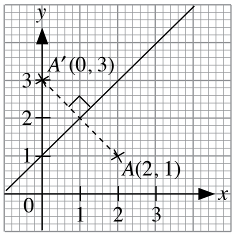


(c) From graph, midpoint of $AA' = (1, 2)$

Gradient of line = $\dfrac{\text{rise}}{\text{run}} = 1$

y-intercept = 1

$\therefore$ Equation of line of reflection is $y = x + 1$.

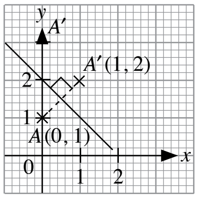


(d) From graph, midpoint of $AA' = \left(\dfrac{1}{2}, 1\dfrac{1}{2}\right)$

Gradient of line = $\dfrac{\text{rise}}{\text{run}} = -1$

y-intercept = 2

$\therefore$ Equation of line of reflection is $y = -x + 2$ or $y + x = 2$.

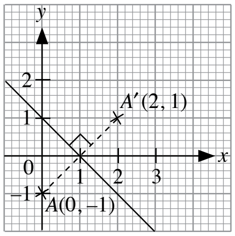


(e) From graph, midpoint of $AA' = (1, 0)$

Gradient of line = $\dfrac{\text{rise}}{\text{run}} = -1$

y-intercept = 1

$\therefore$ Equation of line of reflection is $y = -x + 1$ or $y + x = 1$.

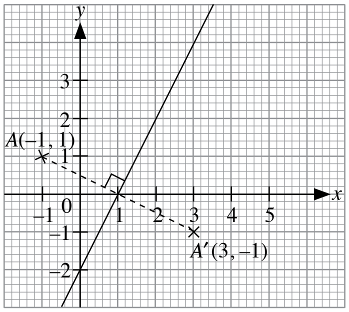


(f) From graph, midpoint of $AA' = (1, 0)$

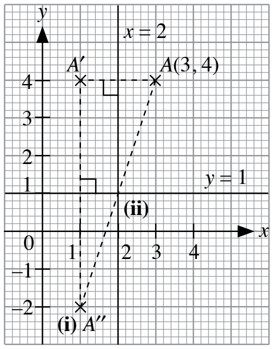
Gradient of line = $\dfrac{\text{rise}}{\text{run}} = 2$

y-intercept = –2

$\therefore$ Equation of line of reflection is $y = 2x - 2$.



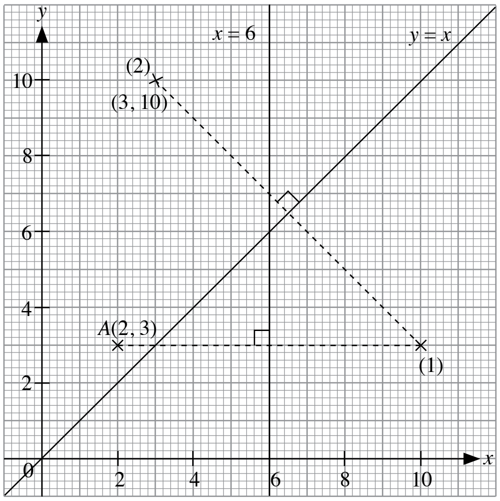
8.



(i) From graph, A' is obtained upon reflection of A in the line $x = 2$. A'' is obtained from reflecting A' in the line $y = 1$. Coordinates are $(1, -2)$.

(ii) Point which remains invariant lies on the line of reflection between A and A'', which can be obtained from the midpoint of line AA''. From graph, the coordinates of the invariant point are $(2, 1)$.

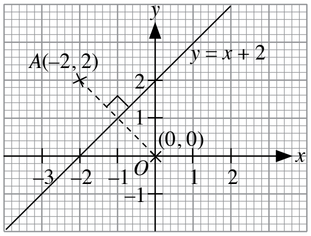
9.



From graph, coordinates of the image of the point under those two reflections are (3, 10).

10. (i) $y = x + 2$

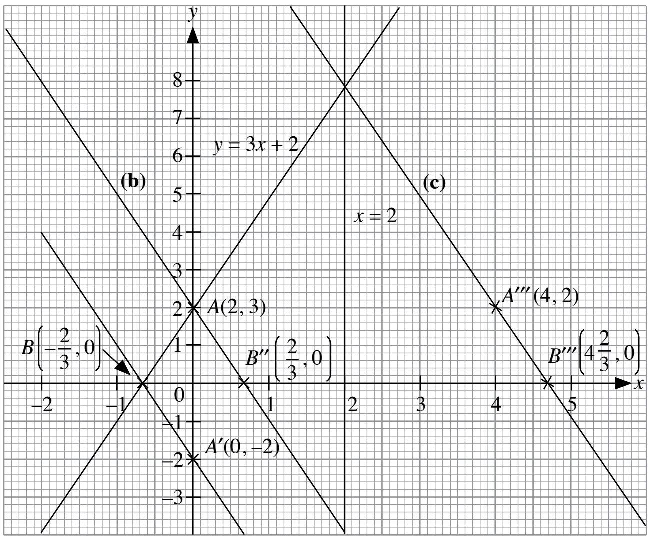
x	–2	–1	0	1
y	0	1	2	3



(ii) From graph, the coordinates of A are (–2, 2).

11. $y = 3x + 2$

x	–1	0	1
y	–1	2	5



(a) Under reflection in the x-axis, the point A is mapped onto $A'(0, –2)$ and the point B is invariant.

Gradient of $A'B = \frac{\text{rise}}{\text{run}} = -\frac{2}{\frac{2}{3}} = -3$

y-intercept = –2

$\therefore$ The equation of $A'B$ is $y = -3x - 2$ or $y + 3x = -2$.

(b) Under reflection in the y-axis, the point A is invariant and the point B is mapped onto $B''\left(\frac{2}{3}, 0\right)$.

Gradient of $AB'' = \frac{\text{rise}}{\text{run}} = -\frac{2}{\frac{2}{3}} = -3$

y-intercept = 2

$\therefore$ The equation of AB'' is $y = -3x + 2$ or $y + 3x = 2$.

(c) The images of A and B under reflection in the line $x = 2$ are $A'''(4, 2)$ and $B'''\left(4\frac{2}{3}, 0\right)$ respectively.

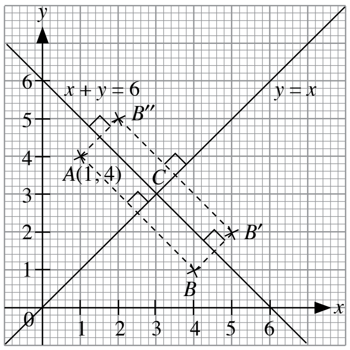
Gradient of $A'''B''' = \frac{\text{rise}}{\text{run}} = -\frac{2}{\frac{2}{3}} = -3$

y-intercept = 14

$\therefore$ The equation of $A'''B'''$ is $y = -3x + 14$ or $y + 3x = 14$.

12. $x + y = 6 \Rightarrow y = -x + 6$

(a)



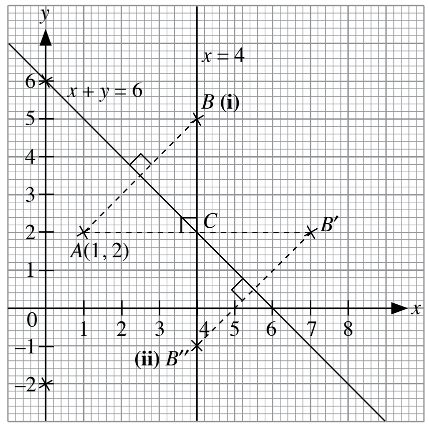
(i) From graph, reflection of A in the line $y = x$ gives B, and reflection of B in the line $x + y = 6$ gives B' with coordinates (5, 2).

(ii) From graph, reflection of A in the line $x + y = 6$ gives B'', and reflection of B'' in the line $y = x$ gives B' with coordinates (5, 2).

(b) Yes, they are the same. The gradients of the lines of reflection are the direct negative of each other.

(c) From graph, invariant point is given by C with coordinates (3, 3).

13. **(a)**



(i) From graph, reflection of A in the line $x + y = 6$ gives B, and reflection of B in the line $x = 4$ remains as B since B lies on the line $x = 4$. Coordinates of point B are (4, 5).

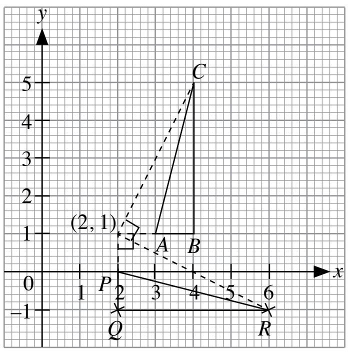
(ii) From graph, reflection of A in the line $x = 4$ gives B', and reflection of B' in the line $x + y = 6$ gives B'' with coordinates (4, –1).

(b) No, reflection is not commutative.

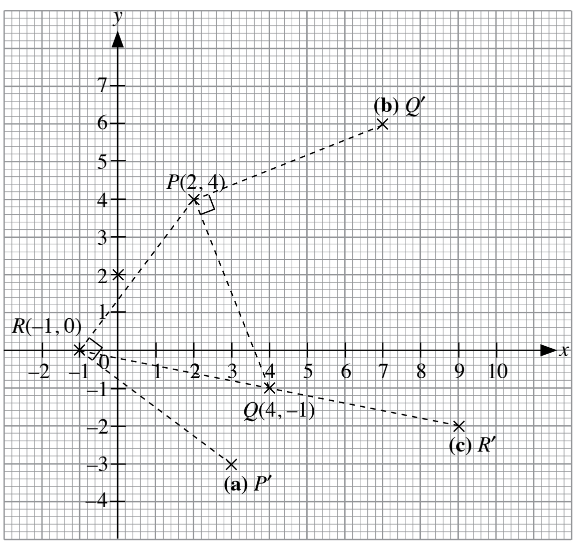
(c) From graph, invariant point is given by C with coordinates (4, 2).

Exercise 9B

1.



2.



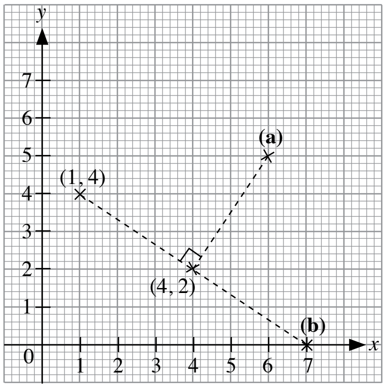
From graph,

(a) Image of P under clockwise rotation of 90 about R is $P'(3, -3)$

(b) Image of Q under anticlockwise rotation of 90 about P is $Q'(7, 6)$

(c) Image of R under 180 rotation about Q is $R'(9, -2)$

3.

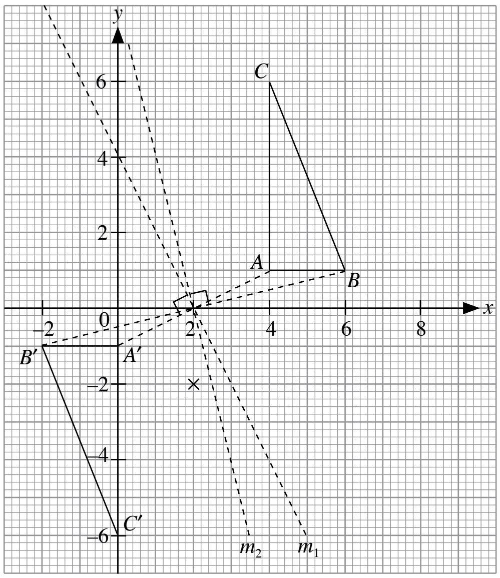


(a) From graph, coordinates of the image are (6, 5).

(b) From graph, coordinates of the image are (7, 0).

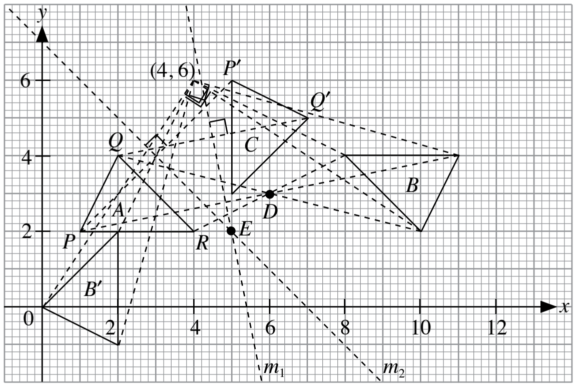
4. Since R represents an anticlockwise rotation of 240° about the origin, R^2 will be $(240° \times 2) - 360° = 480° - 360° = 120°$ anticlockwise rotation about the origin. R^4 will then be $120° \times 2 = 240°$ anticlockwise rotation about the origin.

5. **(i)**



(ii) From graph, line m_1 is the perpendicular bisector of AA' while line m_2 is the perpendicular bisector of BB'. The point of intersection of these two perpendicular bisectors (2, 0) gives the centre of rotation. The angle of rotation is 180°.

6.



(a) From graph, centre of rotation is given by point of intersection D.

(i) Coordinates are (6, 3)

(ii) Angle of rotation = 180°

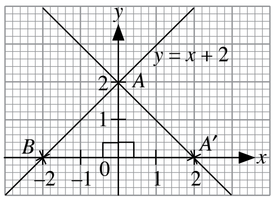
(b) From graph, line m_1 is the perpendicular bisector of QQ' while line m_2 is the perpendicular bisector of PP'.

(i) The point of intersection of these two perpendicular bisectors at E gives the centre of rotation, with coordinates (5, 2).

(ii) The angle of rotation obtained by joining PE and PE' is 90°.

(c) From graph, the coordinates of the vertices are (2, 2), (0, 0) and (2, –1).

7.



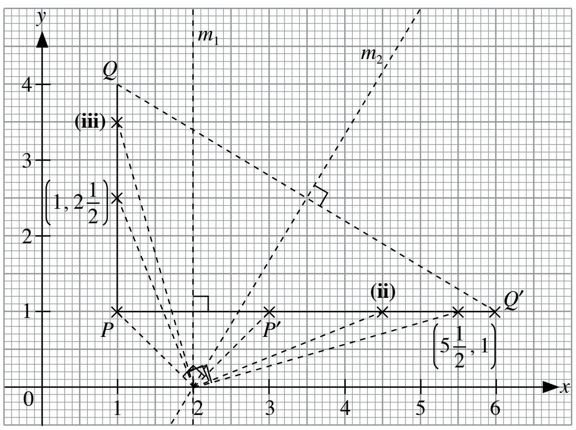
Points $A(0, 2)$ and $B(-2, 0)$ lie on the line $y = x + 2$. Under a clockwise rotation of 90 about the origin, point A becomes A', and B becomes A.

Gradient = $\dfrac{\text{rise}}{\text{run}} = -1$

y-intercept = 2

$\therefore$ Equation of the line is $y = -x + 2$ i.e. $x + y = 2$.

8.



(i) From graph, coordinates of Q' are (6, 1). $\therefore$ $k = 6$.

(ii) From graph, line m_1 is the perpendicular bisector of PP' while line m_2 is the perpendicular bisector of QQ'. The point of intersection of these two perpendicular bisectors (2, 0) gives the centre of rotation. The angle of rotation is 90° clockwise.

$\therefore$ The image of the point $\left(1, 2\frac{1}{2}\right)$ on line PQ is $\left(4\frac{1}{2}, 1\right)$ on line $P'Q'$.

(iii) The coordinates of the point on line PQ whose image is $\left(5\frac{1}{2}, 1\right)$ on line $P'Q'$ are $\left(1, 3\frac{1}{2}\right)$.

9. **(a)** Length of $AC = A'C$

$\therefore$ $\triangle CAA'$ is an isosceles triangle.

$\angle A'CA = 25°$

$\angle CAA' = \dfrac{180° - 25°}{2}$ (base $\angle$s of isos. $\triangle$)

$= 77.5°$

(b) $\angle ACB' = \angle ACB$

$= \tan^{-1} \dfrac{60}{40}$

$= 56.310°$ (to 3 d.p.)

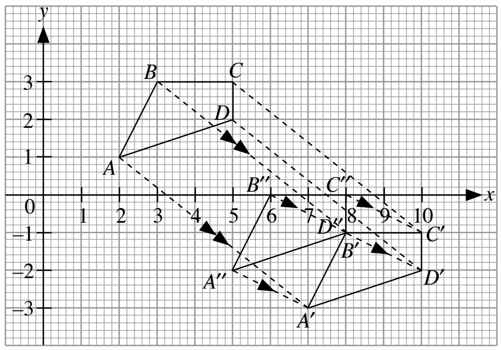
$\therefore$ $\angle ACB' = 56.310° - 25°$

$= 31.310°$

$= 31.5°$ (nearest 0.5)

Exercise 9C

1.



$$T_1(A) = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 5 \\ -4 \end{pmatrix} = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$$

$$T_1(B) = \begin{pmatrix} 3 \\ 3 \end{pmatrix} + \begin{pmatrix} 5 \\ -4 \end{pmatrix} = \begin{pmatrix} 8 \\ -1 \end{pmatrix}$$

$$T_1(C) = \begin{pmatrix} 5 \\ 3 \end{pmatrix} + \begin{pmatrix} 5 \\ -4 \end{pmatrix} = \begin{pmatrix} 10 \\ -1 \end{pmatrix}$$

$$T_1(D) = \begin{pmatrix} 5 \\ 2 \end{pmatrix} + \begin{pmatrix} 5 \\ -4 \end{pmatrix} = \begin{pmatrix} 10 \\ -2 \end{pmatrix}$$

The vertices of the image are $A'(7, -3)$, $B'(8, -1)$, $C'(10, -1)$ and $D'(10, -2)$. Image of the quadrilateral $A'B'C'D'$ is plotted in the graph above. For the second translation T_2,

$T_2(A') = \begin{pmatrix} 7 \\ -3 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$

$T_2(B') = \begin{pmatrix} 8 \\ -1 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$

$T_2(C') = \begin{pmatrix} 10 \\ -1 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$

$T_2(D') = \begin{pmatrix} 10 \\ -2 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 8 \\ -1 \end{pmatrix}$

Vertices of the image of this new quadrilateral are $A''(5, -2)$, $B''(6, 0)$, $C''(8, 0)$ and $D''(8, -1)$.

2. $P' = T(P)$

$= \begin{pmatrix} 1 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

$Q' = T(Q)$

$= \begin{pmatrix} 7 \\ 5 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} 10 \\ 3 \end{pmatrix}$

$R' = T(R)$

$= \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ -2 \end{pmatrix}$

$\therefore$ The coordinates of the vertices of the image $\triangle PQR$ are $P'(4, 1)$, $Q'(10, 3)$ and $R'(5, -2)$.

3. Let the translation vector of T be $\begin{pmatrix} a \\ b \end{pmatrix}$.

$\begin{pmatrix} 2 \\ 7 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} 6 \\ 2 \end{pmatrix}$

$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2 \\ 7 \end{pmatrix} - \begin{pmatrix} 6 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} -4 \\ 5 \end{pmatrix}$

$P = \begin{pmatrix} -1 \\ -5 \end{pmatrix} + \begin{pmatrix} -4 \\ 5 \end{pmatrix}$

$= \begin{pmatrix} -5 \\ 0 \end{pmatrix}$

$\therefore$ Coordinates of P are $(-5, 0)$.

4. **(i)** $A' = T(A)$

$= \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ 8 \end{pmatrix}$

$\therefore$ Coordinates of A' are $(4, 8)$.

(ii) $T(B) = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} p \\ q \end{pmatrix}$

$= \begin{pmatrix} 2+p \\ 4+q \end{pmatrix}$

Since $T(B) = A$,

$2 + p = 2$

$\therefore p = 0$

$4 + q = 4$

$\therefore q = 0$

(iii) $T_2(A) = T(A')$

$= \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ 8 \end{pmatrix}$

$\begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ 8 \end{pmatrix} = \begin{pmatrix} 6 \\ 12 \end{pmatrix} = \begin{pmatrix} h \\ k \end{pmatrix}$

$\therefore h = 6, k = 12$.

(iv) Let point D be $\begin{pmatrix} a \\ b \end{pmatrix}$.

For $\begin{pmatrix} 4 \\ 8 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$,

$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 8 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ -4 \end{pmatrix}$

$\therefore$ Coordinates of point D are $(-2, -4)$.

5. Let the translation vector of T_1 be $\begin{pmatrix} a \\ b \end{pmatrix}$.

$\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} 5 \\ -1 \end{pmatrix}$

$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

Let the translation vector of T_2 be $\begin{pmatrix} c \\ d \end{pmatrix}$.

$\begin{pmatrix} 4 \\ -5 \end{pmatrix} = \begin{pmatrix} c \\ d \end{pmatrix} + \begin{pmatrix} -2 \\ 5 \end{pmatrix}$

$\begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \end{pmatrix} - \begin{pmatrix} -2 \\ 5 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ -10 \end{pmatrix}$

(a) Image under $T_1 = \begin{pmatrix} 7 \\ 6 \end{pmatrix} + \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ 10 \end{pmatrix}$

(b) Image under $T_2 = \begin{pmatrix} 7 \\ 6 \end{pmatrix} + \begin{pmatrix} 6 \\ -10 \end{pmatrix}$

$= \begin{pmatrix} 13 \\ -4 \end{pmatrix}$

(c) Image under T_1T_2 = Image under $T_2 + \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 13 \\ -4 \end{pmatrix} + \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 10 \\ 0 \end{pmatrix}$

(d) Image under T_2T_1 = Image under $T_1 + \begin{pmatrix} 6 \\ -10 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ 10 \end{pmatrix} + \begin{pmatrix} 6 \\ -10 \end{pmatrix}$

$= \begin{pmatrix} 10 \\ 0 \end{pmatrix}$

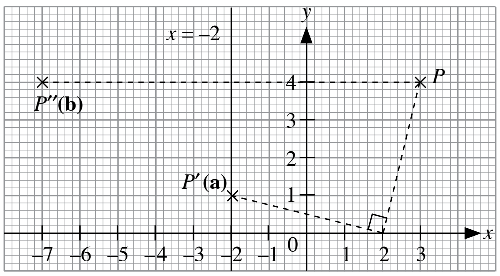
(e) $T_1^{\,2} = \begin{pmatrix} 7 \\ 6 \end{pmatrix} + 2\begin{pmatrix} -3 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 7 \\ 6 \end{pmatrix} + \begin{pmatrix} -3 \\ 4 \end{pmatrix} + \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 14 \end{pmatrix}$

Review Exercise 9

1.



(a) From graph, an anticlockwise rotation of 90° would produce P' with coordinates (–2, 1).

(b) $x + 2 = 0 \Rightarrow x = -2$

From graph, reflection of P in the line $x = -2$ will result in P'' with coordinates (–7, 4).

2. Let the translation vector be $\begin{pmatrix} a \\ b \end{pmatrix}$.

$\begin{pmatrix} 2 \\ 9 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} 5 \\ 7 \end{pmatrix}$

$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2 \\ 9 \end{pmatrix} - \begin{pmatrix} 5 \\ 7 \end{pmatrix}$

$= \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

Let point A be $\begin{pmatrix} x \\ y \end{pmatrix}$.

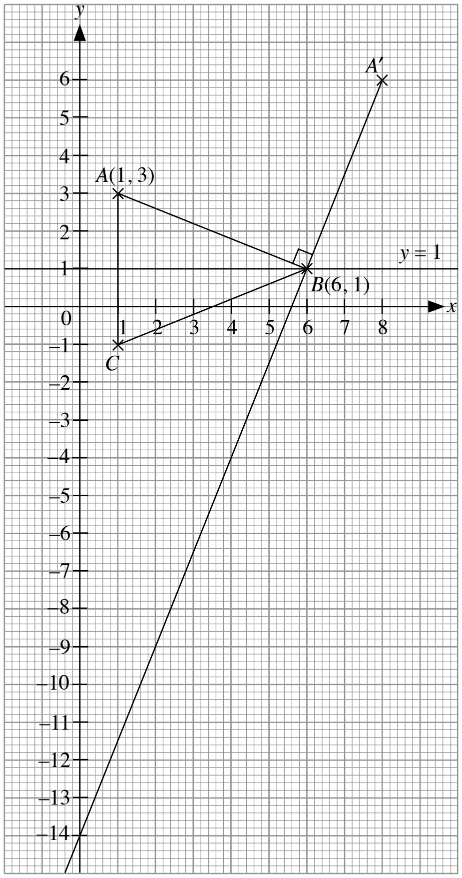
$\begin{pmatrix} -3 \\ -5 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} - \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 0 \\ -7 \end{pmatrix}$

∴ Coordinates of A are (0, –7).

3.



(a) From graph, 90° clockwise rotation of line AB about B produces line $A'B$, where $A'(8, 6)$.

Gradient of $A'B = \dfrac{\text{rise}}{\text{run}} = \dfrac{5}{2}$

y-intercept = –14

∴ The equation of line $A'B$ is $y = \dfrac{5}{2}x - 14$ i.e. $2y = 5x - 28$.

(b) From graph, coordinates of point C are (1, –1).

4. **(a)** To map $PQRS$ onto $ABCD$, a translation is done. Let the translation vector be $\begin{pmatrix} a \\ b \end{pmatrix}$.

Comparing points A and P,

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} 7 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 7 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -6 \\ 0 \end{pmatrix}$$

$\therefore$ A translation of $\begin{pmatrix} -6 \\ 0 \end{pmatrix}$ of square $PQRS$ will map it onto $ABCD$.

(b) To map $ABCD$ onto $QPSR$, a reflection is done in the line equidistant between the two squares. $ABCD$ is reflected in the line $x = 6$ to be mapped onto $QPSR$.

(c) To map $PQRS$ onto $CDAB$, $PQRS$ undergoes a 180° rotation about the point the perpendicular bisectors of CP and DQ intersect i.e. (6, 3).

(d) To map $ABCD$ onto $SPQR$, $ABCD$ is rotated 90 clockwise about the point the perpendicular bisectors of AS and BP intersect i.e. (6, 0).

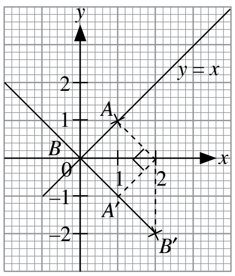
(e) To map $PQRS$ onto $DABC$, $PQRS$ is rotated 90° clockwise about the point the perpendicular bisectors of PD and QA intersect i.e. (6, 6).

5. **(a)** For a reflection in the line $y = -x$, the value of the x- and y-coordinates interchange and become negative of the original value. Coordinates of the image of the point (5, 2) under M are (–2, –5).

(b) Under M^2, the image obtained under M is reflected again in the line $y = -x$. Coordinates of the resultant point are (5, 2).

(c) Under M^3, the image obtained under M^2 is reflected again in the line $y = -x$. Coordinates of the resultant point are (–2, –5).

6.



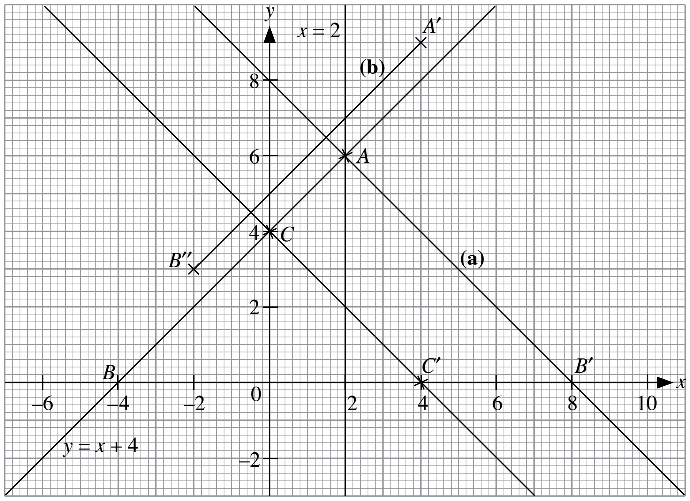
From graph, points $A(1, 1)$ and $B(0, 0)$ lie on the line $y = x$. An anticlockwise rotation of 90 about the point (2, 0) gives $A'(1, -1)$ and $B'(2, -2)$ respectively.

Gradient $= \dfrac{\text{rise}}{\text{run}} = -1$

y-intercept = 0

$\therefore$ Equation of the line is $y = -x$ i.e. $y + x = 0$.

7.



(a) Under reflection in the line $x = 2$, the point B is mapped onto $B'(8, 0)$ and the point A is invariant.

Gradient of $AB' = \dfrac{\text{rise}}{\text{run}} = -1$

y-intercept = 8

The equation of AB' is $y = -x + 8$, i.e. $y + x = 8$.

(b) Point B has coordinates (–4, 0)

$B'' = \text{T}(B)$

$$= \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -4 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

Point A has coordinates (2, 6)

$A' = \text{T}(A)$

$$= \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ 6 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ 9 \end{pmatrix}$$

After translation, the image is given by line $A'B''$.

Gradient of $A'B'' = \dfrac{\text{rise}}{\text{run}} = 1$

y-intercept = 5

$\therefore$ The equation of $A'B''$ is $y = x + 5$.

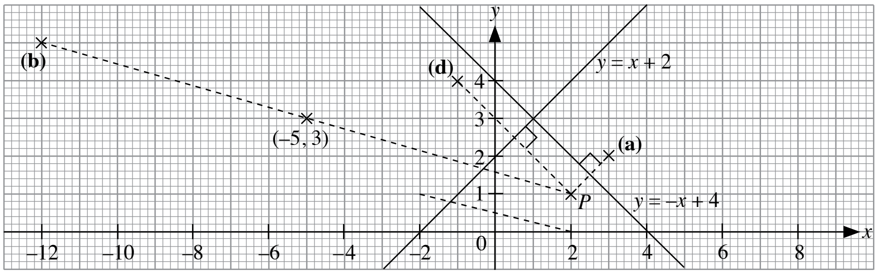
(c) Point C has coordinates (0, 4). Under a 90° clockwise rotation about the origin O, its image C' has coordinates (4, 0). For $B(-4, 0)$, a 90° clockwise rotation about the origin O will result in it being mapped onto point C with coordinates (4, 0).

Gradient of $CC' = \dfrac{\text{rise}}{\text{run}} = -1$

y-intercept = 4

$\therefore$ The equation of CC' is $y = -x + 4$ i.e. $y + x = 4$.

8.



(a) $x + y = 4$

$y = -x + 4$

From graph, the image of $P(2, 1)$ under a reflection in the line $y = -x + 4$ is a point with coordinates $(3, 2)$.

(b) From graph, the image of $P(2,1)$ under 180° rotation about the point $(-5, 3)$ is a point with coordinates $(-12, 5)$.

(c) $T(P) = \begin{pmatrix} -5 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

$\therefore$ The image of $P(2, 1)$ under a translation of $\begin{pmatrix} -5 \\ 3 \end{pmatrix}$ is a point with coordinates $(-3, 4)$.

(d) From graph, the image of $P(2, 1)$ under a reflection in the line $y = x + 2$ is a point with coordinates $(-1, 4)$.

9. $y = 2x + 3$

x	–1	1
y	1	5

Two points on the line $y = 2x + 3$ are $A(-1, 1)$ and $B(1, 5)$. After translation of $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$,

$A' = T(A)$

$= \begin{pmatrix} 3 \\ -2 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$B' = T(B)$

$= \begin{pmatrix} 3 \\ -2 \end{pmatrix} + \begin{pmatrix} 1 \\ 5 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

Gradient of line $A'B' = \frac{\text{rise}}{\text{run}} = \frac{-1 - 3}{2 - 4} = 2$

y-intercept = –5

$\therefore$ The equation of $A'B'$ is $y = 2x - 5$.

Thus, $m = 2$ and $c = -5$

10. **(a)** Under M, the image of the point $(1, 3)$ is $(3, 1)$.

(b) Under M^{-1}, the point will be reflected in the line $x = y$, which will produce an image $(5, 2)$.

(c) Under M^5, the number of reflections about the line is an odd number, hence it will be the same as the image of M, which is (7, 3).

11. **(a)** Under R, the points would rotate such that the values of the x- and y-coordinates would interchange, and the x-coordinate would be the negative of the original y value.

(i) The image of the point $(3, 5)$ is $(-5, 3)$.

(ii) The image of the point $(7, -4)$ is $(4, 7)$.

(b) Under R^{-1}, the rotation would be 90° clockwise. The points would rotate such that the values of the x- and y-coordinates would interchange, and the y-coordinate would be the negative of the original x value.

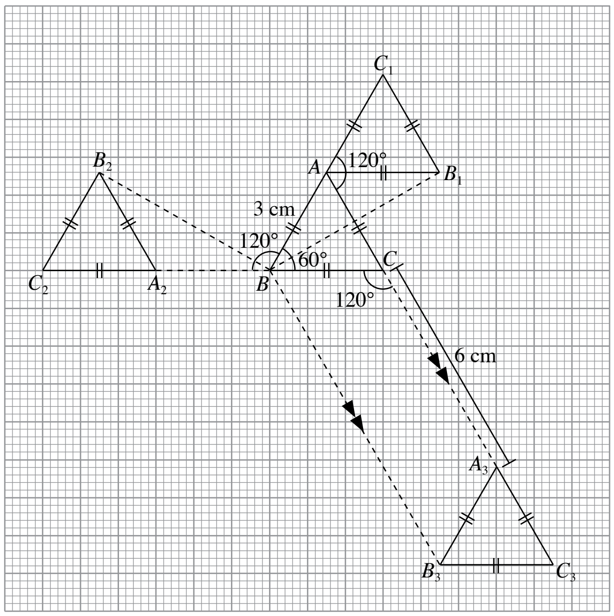
(i) The image of the point $(3, 4)$ is $(4, -3)$.

(ii) The image of the point $(-2, -3)$ is $(-3, 2)$.

(c) **(i)** Under R^5, the number of reflections about the line is an odd number, hence it will be the same as the image of R. The image of the point $(2, 5)$ is $(-5, 2)$.

(ii) Under R^8, the number of reflections about the line is an even number, hence it will be go back to the original point. The image of the point $(2, 5)$ is $(2, 5)$.

Challenge Yourself



From graph, $\triangle ABC$ can be mapped onto $\triangle A_3B_3C_3$ through a translation of 9 cm along line AC.

Revision Exercise B1

1. **(a)** $16a - a^2 - 64 = 0$

$a^2 - 16a + 64 = 0$

$(a - 8)^2 = 0$

$a - 8 = 0$

$\therefore a = 8$

(b) $b^2 - \frac{16}{25} = 0$

$b^2 = \frac{16}{25}$

$= \left(\frac{4}{5}\right)^2$

$\therefore b = \pm\sqrt{\left(\frac{4}{5}\right)^2}$

$= \pm\frac{4}{5}$

(c) $1 + 3c = 10c^2$

$10c^2 - 3c - 1 = 0$

$(2d + 1)(5c - 1) = 0$

$2d + 1 = 0$ or $5c - 1 = 0$

$\therefore c = -\frac{1}{2}$ or $c = \frac{1}{5}$

(d) $d^3 + d^2 - 132d = 0$

$d(d^2 + d - 132) = 0$

$d(d - 11)(d + 12) = 0$

$\therefore d = 0$ or $d - 11 = 0$ or $d + 12 = 0$

$d = 11$ or $d = -12$

2. Let the breadth of the rectangle be x m,
then the length of the rectangle is $(x + 5)$ m.

$\therefore x(x + 5) = 66$

$x^2 + 5x = 66$

$x^2 + 5x - 66 = 0$

$(x - 6)(x + 11) = 0$

$x - 6 = 0$ or $x + 11 = 0$

$\therefore x = 6$ or $x = -11$ (rejected, since $x > 0$)

Breadth = 6 m

Length = 6 + 5

= 11 m

$\therefore$ Perimeter of the rectangle = 2(6 + 11)

$= 2 \times 17$

= 34 m

3. **(a)** When $x = -2$, $y = p$,

$p = 3(-2) - 2(-2)^2$

$= -6 - 8$

$= -14$

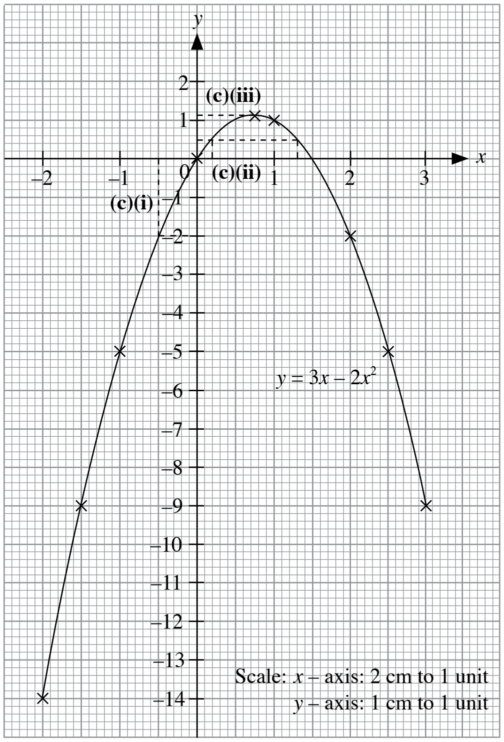
When $x = 2.5$, $y = q$,

$q = 3(2.5) - 2(2.5^2)$

$= 7.5 - 12.5$

$= -5$

(b)



(c) **(i)** When $x = -0.5$

$y = -2$

(ii) When $y = 0.5$,

$x = 1.3, 0.2$

(iii) Maximum value of $y = 1.1$

(d) The equation of the line of symmetry is $x = 0.75$.

4. **(a)** $\frac{x - 1}{4x - 2} - \frac{5 - 2x}{3 - 6x} = \frac{x - 1}{2(2x - 1)} - \frac{5 - 2x}{-3(2x - 1)}$

$= \frac{x - 1}{2(2x - 1)} + \frac{5 - 2x}{3(2x - 1)}$

$= \frac{3(x - 1)}{6(2x - 1)} + \frac{2(5 - 2x)}{6(2x - 1)}$

$= \frac{3(x - 1) + 2(5 - 2x)}{6(2x - 1)}$

$= \frac{3x - 3 + 10 - 4x}{6(2x - 1)}$

$= \frac{-x + 7}{6(2x - 1)}$

(b) $\frac{2x + 3}{9x^2 - 1} - \frac{7}{3x - 1} = \frac{2x + 3}{(3x + 1)(3x - 1)} - \frac{7}{3x - 1}$

$= \frac{2x + 3}{(3x + 1)(3x - 1)} - \frac{7(3x + 1)}{(3x + 1)(3x - 1)}$

$= \frac{2x + 3 - 7(3x + 1)}{(3x + 1)(3x - 1)}$

$= \frac{2x + 3 - 21x - 7}{(3x + 1)(3x - 1)}$

$= \frac{-19x - 4}{(3x + 1)(3x - 1)}$

5. **(i)** $5a = \dfrac{3x - 4}{2y - 3x}$

$5a(2y - 3x) = 3x - 4$

$10ay - 15ax = 3x - 4$

$15ax + 3x = 10ay + 4$

$x(15a + 3) = 10ay + 4$

$\therefore x = \dfrac{10ay + 4}{15a + 3}$

(ii) When $a = 1, y = 5$,

$x = \dfrac{10(1)(5) + 4}{15(1) + 3}$

$= \dfrac{54}{18}$

$= 3$

6. $f(x) = 7x - 2$

$F(x) = \dfrac{5}{8}x + 6$

(i) $f(a) = 7a - 2$

(ii) $F(16a - 8) = \dfrac{5}{8}(16a - 8) + 6$

$= 10a - 5 + 6$

$= 10a + 1$

(iii) $f\left(\dfrac{1}{7}a + 1\right) = 7\left(\dfrac{1}{7}a + 1\right) - 2$

$= a + 5$

$F(4a - 3) = \dfrac{5}{8}(4a - 3) + 6$

$= 2\dfrac{1}{2}a - \dfrac{15}{8} + 6$

$= 2\dfrac{1}{2}a + 4\dfrac{1}{8}$

$f\left(\dfrac{1}{7}a + 1\right) + F(4a - 3) = (a + 5) + \left(2\dfrac{1}{2}a + 4\dfrac{1}{8}\right)$

$= 3\dfrac{1}{2}a + 9\dfrac{1}{8}$

7. **(i)** Since $\triangle ABC \equiv \triangle PQR$, then the corresponding vertices match.

Length of PQ = length of AB

= 8 cm

(ii) $\angle R = \angle C$

$= 180° - 70° - 60°$ ($\angle$ sum of $\triangle ABC$)

$= 50°$

8. **(i)** 8 km = 800 000 cm

i.e. the scale of the map is $\dfrac{1}{800\,000}$.

(ii)

Actual		**Map**
8 km	is represented by	1 cm
1 km	is represented by	$\dfrac{1}{8}$ cm
72 cm	is represented by	$\left(72 \times \dfrac{1}{8}\right)$ cm
		= 9 cm

$\therefore$ The distance on the map is 9 cm.

(iii)

Actual		**Map**
8 km	is represented by	1 cm
1 km	is represented by	$\dfrac{1}{8}$ cm
1 km^2	is represented by	$\left(\dfrac{1}{8}\text{ cm}\right)^2$
496 km^2	is represented by	$\left(496 \times \dfrac{1}{8^2}\right)$ cm^2
		= 7.75 cm^2

$\therefore$ The area on the map is 7.75 cm^2.

9. For a reflection about the line $x = 2$, the image would be equidistant from the line along the x-axis. Coordinates of the reflection the point (4, 5) is (0, 5).

10. R^2 would be $(2 \times 187°) - 360° = 14°$ clockwise rotation about the origin. R^{-1} would be a 187° anticlockwise rotation about the origin.

Revision Exercise B2

1. $(2x-1)(3-4x) = 2(1-2x)$

$$6x - 8x^2 - 3 + 4x = 2 - 4x$$
$$8x^2 - 14x + 5 = 0$$
$$(4x-5)(2x-1) = 0$$
$$4x - 5 = 0 \quad \text{or} \quad 2x - 1 = 0$$
$$\therefore x = 1\frac{1}{4} \quad \text{or} \quad x = \frac{1}{2}$$

2. **(i)** When $x = 3$,

$$3^2 + p(3) + 15 = 0$$
$$9 + 3p + 15 = 0$$
$$3p + 24 = 0$$
$$3p = -24$$
$$\therefore p = -8$$

(ii) When $p = -8$,

$$x^2 - 8x + 15 = 0$$
$$(x-3)(x-5) = 0$$
$$x - 3 = 0 \quad \text{or} \quad x - 5 = 0$$
$$\therefore x = 3 \quad \text{or} \quad x = 5$$

$\therefore$ The other solution is $x = 5$.

3. Let $y = 0$.

$$x^2 + 5x + 4 = 0$$
$$(x+4)(x+1) = 0$$
$$x + 4 = 0 \quad \text{or} \quad x + 1 = 0$$
$$\therefore x = -4 \quad \text{or} \quad x = -1$$

$\therefore$ The coordinates of A and B are $(-4, 0)$ and $(-1, 0)$ respectively.

When $x = 0$,

$$y = 0^2 + 5(0) + 4$$
$$= 4$$

$\therefore$ The coordinates of C are $(0, 4)$.

4. **(a)** $$\frac{4a^2b^2 - 8a^3b - 14ab^3}{4ab} = \frac{2ab(2ab - 4a^2 - 7b^2)}{4ab}$$
$$= \frac{2ab - 4a^2 - 7b^2}{2}$$

(b) $$\frac{6c^3d^4 + 12c^4d^2 - 9c^2d^3}{-3c^2d^2} = \frac{3c^2d^2(2cd^2 + 4c^2 - 3d)}{-3c^2d^2}$$
$$= -(2cd^2 + 4c^2 - 3d)$$
$$= -2cd^2 - 4c^2 + 3d$$

5. **(i)** $$F = \frac{mv^2}{r}$$
$$Fr = mv^2$$
$$v^2 = \frac{Fr}{m}$$
$$\therefore v = \sqrt{\frac{Fr}{m}} \quad (\text{since } v > 0)$$

(ii) When $m = 225$, $r = 5$ and $F = 4500$,

$$v = \sqrt{\frac{4500 \times 5}{225}}$$
$$= \sqrt{100}$$
$$= 10 \text{ m s}^{-1}$$

6. $f(x) = 13 - 9x$

(i) $$f(2) = 13 - 9(2)$$
$$= 13 - 18$$
$$= -5$$

(ii) $$f\left(-\frac{4}{9}\right) = 13 - 9\left(-\frac{4}{9}\right)$$
$$= 13 + 4$$
$$= 17$$

(iii) $$2f(0) = 2[13 - 9(0)]$$
$$= 2(13)$$
$$= 26$$
$$f(1) = 13 - 9(1)$$
$$= 4$$
$$2f(0) + f(1) = 26 + 4$$
$$= 30$$

(iv) $$f(3) = 13 - 9(3)$$
$$= 13 - 27$$
$$= -14$$
$$f\left(\frac{2}{9}\right) = 13 - 9\left(\frac{2}{9}\right)$$
$$= 13 - 2$$
$$= 11$$
$$f(3) - f\left(\frac{2}{9}\right) = -14 - 11$$
$$= -25$$

7. **(i)** Since $\triangle ABC$ is similar to $\triangle APQ$, then all the ratios of the corresponding sides are equal.

$$\frac{QP}{CB} = \frac{AP}{AB}$$
$$\frac{QP}{5} = \frac{6+5}{6}$$
$$\therefore QP = \frac{11}{6} \times 5$$
$$= 9\frac{1}{6} \text{ cm}$$

(ii) $$\frac{AC}{AQ} = \frac{AB}{AP}$$
$$\frac{AC}{AC + 6.5} = \frac{6}{6+5}$$
$$11AC = 6(AC + 6.5)$$
$$= 6AC + 39$$
$$5AC = 39$$
$$\therefore AC = 7.8 \text{ cm}$$

8. **(i)**

Model		**Actual**
1 cm	represents	7.5 m
24.4 cm	represents	(24.4×7.5) m
		= 183 m

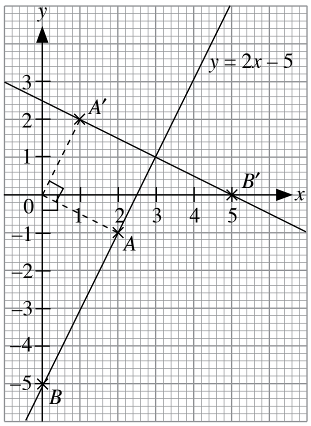
$\therefore$ The actual height of the building is 183 m.

(ii) Actual		**Model**
12 m	is represented by	1 cm
1 m	is represented by	$\frac{1}{12}$ cm
183 m	is represented by	$\left(183 \times \frac{1}{12}\right)$ cm
		= 15.25 cm

∴ The height of the model building is 15.25 cm.

9. $y = 2x - 5$

x	0	2	4
y	–5	–1	3



From graph, after an anticlockwise rotation of 90° about the origin, image of the point $A(2, –1)$ is $A'(1, 2)$, and that of $B(0, –5)$ is $B'(5, 0)$.

Gradient of line $A'B' = \frac{\text{rise}}{\text{run}} = \frac{1}{2}$

y-intercept $= 2\frac{1}{2}$

∴ The equation of the image of the line is $y = \frac{1}{2}x + 2\frac{1}{2}$

i.e. $2y = x + 5$.

10. (i) $A' = T_1(A)$

$= \begin{pmatrix} 3 \\ -7 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ -3 \end{pmatrix}$

∴ The image of the point $A(2, 4)$ under T_1 is $(5, –3)$.

(ii) Let the translation vector T_2 be $\begin{pmatrix} a \\ b \end{pmatrix}$.

$\begin{pmatrix} 5 \\ 9 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 5 \\ 9 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

∴ $T_2 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

(iii) $T_2(B) = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ -8 \end{pmatrix}$

$= \begin{pmatrix} 7 \\ -4 \end{pmatrix}$

∴ The image of the point $B(4, –8)$ under T_2 is $(7, –4)$.

Chapter 10 Pythagoras' Theorem

TEACHING NOTES

Suggested Approach

There are many ways of proving the Pythagoras' Theorem. An unofficial tally shows more than 300 ways of doing this. Teachers may use this opportunity to ask students to do a project of finding the best or the easiest method of doing this and get the students to present them to their class (see Performance Task on page 262).

Students should be able to easily recall the previous lesson on similar triangles and apply their understanding in this chapter.

Section 10.1: Pythagoras' Theorem

Students are expected to know that the longest side of a right-angled triangle is known as the hypotenuse. The condition that the triangle must be a right-angled triangle has to be highlighted.

Teachers may wish to prove the Pythagoras' Theorem by showing the activity on the pages 260 and 261 (see Investigation: Pythagoras' Theorem – The Secret of the Rope-Stretchers). Again, it is important to state the theorem applies only to right-angled triangles. The theorem does not hold for other types of triangles.

Section 10.2: Applications of Pythagoras' Theorem in Real-World Contexts

There are many real-life applications of Pythagoras' Theorem which the teachers can show to students. The worked examples and exercises should be more than enough for students to appreciate how the theorem is frequently present in real-life. Teachers should always remind students to check before applying the theorem, that the triangle is a right-angled triangle and that the longest side refers to the hypotenuse.

Section 10.3: Converse of Pythagoras' Theorem

Worked Example 8 provides an example of the converse of Pythagoras' Theorem. Some students should find the converse of the theorem easily manageable while teachers should take note of students who may have difficulty in following. Students should be guided of the importance of giving reasons to justify their answers.

Challenge Yourself

Question 2 requires the arrangement of 3 right-angled triangles such that their hypotenuses form another triangle. Students should be able to do the rest of the questions if they have understood Pythagoras' Theorem.

WORKED SOLUTIONS

Investigation (Pythagoras' Theorem – The Secret of the Rope-Stretchers)

Part I:

In all 3 triangles, AB is the hypotenuse.

1, 2, 3, 4.

	BC	**AC**	**AB**	**BC^2**	**AC^2**	**AB^2**	**$BC^2 + AC^2$**
(a)	3 cm	4 cm	5 cm	9 cm^2	16 cm^2	25 cm^2	25 cm^2
(b)	6 cm	8 cm	10 cm	36 cm^2	64 cm^2	100 cm^2	100 cm^2
(c)	5 cm	12 cm	13 cm	25 cm^2	144 cm^2	169 cm^2	169 cm^2

Table 8.1

The value of AB^2 in table 8.1 is the same as the value of $BC^2 + AC^2$.

Part II:

5. In $\triangle ABC$, AB is the hypotenuse.

6. *Any 6 sets of values of BC, AC and AB can be used. Teachers may wish to have students attempt to get integer values for all 3 sides of the triangle.*

7. The value of AB^2 in table 8.2 is the same as the value of $BC^2 + AC^2$.

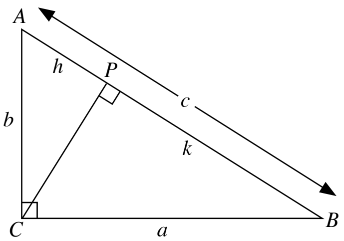
Performance Task (Page 262)

Even though Pythagoras' Theorem was long known years before Pythagoras' time, the theorem was credited to him as he was widely believed to be the first to provide a proof of it, which is shown in Fig. 8.5.

The Babylonians knew about the theorem by the Pythagorean triplets stated found in their remaining text that survived till this day. The Indians were able to list down the Pythagorean triplets, along with a geometrical proof of the Pythagoras' Theorem for a regular right-angled triangle.

The Chinese stated the theorem as the 'Gougu theorem' listed in the Chinese text '*Zhou Bi Suan Jing'* published around the first century B.C. It was also known alternatively as '*Shang Gao Theorem'*, after the Duke of Zhou's astronomer and mathematician, and where the reasoning of Pythagoras' Theorem in '*Zhou Bi Suan Jing'* came from. Some proofs of Pythagoras' Theorem are as follows.

Proof 1: (Using Similar Triangles)



$\angle ACB = \angle BPC = \angle APC = 90°$

Since $\triangle ACB$ is similar to $\triangle APC$,

$$\frac{AB}{AC} = \frac{AC}{AP}$$

i.e. $\frac{c}{b} = \frac{b}{h}$

$b^2 = ch$ — (1)

Since $\triangle ACB$ is similar to $\triangle CPB$,

$$\frac{AB}{CB} = \frac{CB}{PB}$$

i.e. $\frac{c}{a} = \frac{a}{k}$

$a^2 = ck$ — (2)

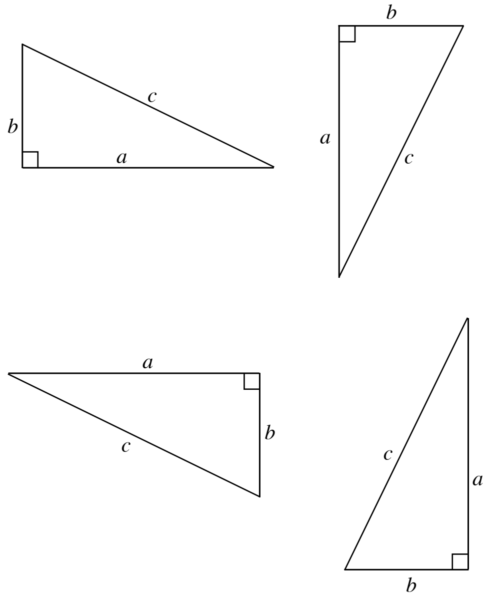
(1) + (2): $b^2 + a^2 = ch + ck$

$= c(h + k)$

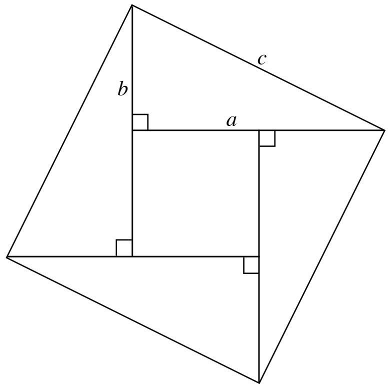
$= c^2$

$\therefore a^2 + b^2 = c^2$

Proof 2: (Using four right-angled triangles)



We can arrange the four triangles to form the following diagram.



The diagram is a large square of length c units, with a smaller square of length $(a - b)$ units.

$\therefore$ Area of large square $= 4 \times$ area of a triangle + area of small square

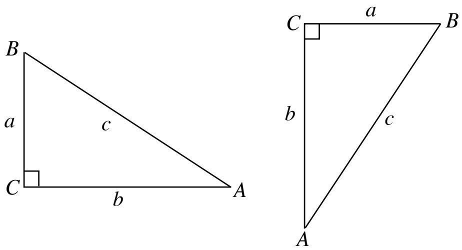
$$c^2 = 4 \times \left(\frac{1}{2} \times a \times b\right) + (a - b)^2$$

$$= 2ab + a^2 - 2ab + b^2$$

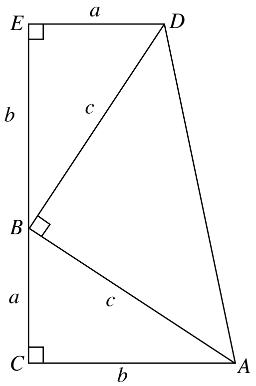
$$= a^2 + b^2$$

$\therefore a^2 + b^2 = c^2$

Proof 3: (Using a trapezium)



By rotating $\triangle ABC$ 90° clockwise, and placing the second triangle on top of the first one, we can get the following trapezium.



First, we show that $\angle ABD = 90°$.

$\angle ABC + \angle BAC = 180° - 90° = 90°$ (sum of $\angle$ in $\triangle ABC$)

$\angle BDE + \angle DBE = 180° - 90° = 90°$ (sum of $\angle$ in $\triangle BDE$)

Since $\angle BAC = \angle DBE$,

$\angle ABC + \angle DBE = 90°$

$\therefore \angle ABD = 180° - 90°$ (adj. $\angle$s on a str. line)

$= 90°$

Area of trapezium = 2 × Area of $\triangle ABC$ + Area of $\triangle ABD$

$$\frac{1}{2} \times (a + b) \times (a + b) = 2 \times \left(\frac{1}{2} \times a \times b\right) + \left(\frac{1}{2} \times c \times c\right)$$

$$\frac{1}{2}(a + b)^2 = ab + \frac{1}{2}c^2$$

$$(a + b)^2 = 2ab + c^2$$

$$c^2 = (a + b)^2 - 2ab$$

$$= a^2 + 2ab + b^2 - 2ab$$

$$= a^2 + b^2$$

$\therefore a^2 + b^2 = c^2$

Practise Now (Page 259)

(a) AB is the hypotenuse.
(b) DE is the hypotenuse.
(c) PQ is the hypotenuse.

Practise Now 1

1. In $\triangle ABC$, $\angle C = 90°$.
Using Pythagoras' Theorem,
$AB^2 = BC^2 + AC^2$
$= 8^2 + 6^2$
$= 64 + 36$
$= 100$
$\therefore AB = \sqrt{100}$ (since $AB > 0$)
$= 10$ cm

2. In $\triangle ABC$, $\angle C = 90°$.
Using Pythagoras' Theorem,
$AB^2 = BC^2 + AC^2$
$= 7^2 + 24^2$
$= 49 + 576$
$= 625$
$\therefore AB = \sqrt{625}$ (since $AB > 0$)
$= 25$ cm

Practise Now 2

1. In $\triangle PQR$, $\angle R = 90°$.
Using Pythagoras' Theorem,
$PQ^2 = QR^2 + PR^2$
$15^2 = 12^2 + PR^2$
$PR^2 = 15^2 - 12^2$
$= 225 - 144$
$= 81$
$\therefore PR = \sqrt{81}$ (since $PR > 0$)
$= 9$ m

2. In $\triangle PQR$, $\angle R = 90°$.
Using Pythagoras' Theorem,
$PQ^2 = QR^2 + PR^2$
$35^2 = QR^2 + 28^2$
$QR^2 = 35^2 - 28^2$
$= 1225 - 784$
$= 441$
$\therefore QR = \sqrt{441}$ (since $QR > 0$)
$= 21$ cm

Practise Now 3

1. (i) In $\triangle ABQ$, $\angle B = 90°$.
Using Pythagoras' Theorem,
$AQ^2 = BQ^2 + AB^2$
$5^2 = BQ^2 + 3^2$
$BQ^2 = 5^2 - 3^2$
$= 25 - 9$
$= 16$
$\therefore BQ = \sqrt{16}$ (since $PR > 0$)
$= 4$ cm

(ii) In $\triangle ABC$, $\angle B = 90°$.
Using Pythagoras' Theorem,
$$AC^2 = CB^2 + AB^2$$
$$= (4 + 4)^2 + 3^2$$
$$= 8^2 + 3^2$$
$$= 64 + 9$$
$$= 73$$
$$\therefore AC = \sqrt{73} \text{ (since } AC > 0)$$
$$= 8.54 \text{ cm (to 3 s.f.)}$$

2. **(i)** In $\triangle GHI$, $\angle I = 90°$.
Using Pythagoras' Theorem,
$$GH^2 = HI^2 + GI^2$$
$$61^2 = HI^2 + 11^2$$
$$HI^2 = 61^2 - 11^2$$
$$= 3721 - 121$$
$$= 3600$$
$$\therefore HI = \sqrt{3600} \text{ (since } HI > 0)$$
$$= 60 \text{ cm}$$

(ii) In $\triangle GRI$, $\angle I = 90°$.
Using Pythagoras' Theorem,
$$GR^2 = RI^2 + GI^2$$
$$= (60 - 21)^2 + 11^2$$
$$= 39^2 + 11^2$$
$$= 1521 + 121$$
$$= 1642$$
$$\therefore GR = \sqrt{1642} \text{ (since } GR > 0)$$
$$= 40.5 \text{ cm (to 3 s.f.)}$$

3. **(i)** In $\triangle HKR$, $\angle R = 90°$.
Using Pythagoras' Theorem,
$$HK^2 = KR^2 + HR^2$$
$$19^2 = 13^2 + HR^2$$
$$HR^2 = 19^2 - 13^2$$
$$= 361 - 169$$
$$= 192$$
$$\therefore HR = \sqrt{192} \text{ (since } HR > 0)$$
$$= 13.9 \text{ cm}$$

(ii) In $\triangle PQR$, $\angle R = 90°$.
Using Pythagoras' Theorem,
$$PQ^2 = QR^2 + PR^2$$
$$33^2 = (QK + 13)^2 + (6 + 13.86)^2$$
$$= (QK + 13)^2 + 19.86^2$$
$$(QK + 13)^2 = 33^2 - 19.86^2$$
$$= 694.58$$
$$\therefore QK + 13 = \pm\sqrt{694.58}$$
$$QK = -13 \pm \sqrt{694.58}$$
$QK = 13.4$ cm (to 3 s.f.) or $QK = -39.4$ cm (to 3 s.f.)
(rejected, since $QK > 0$)

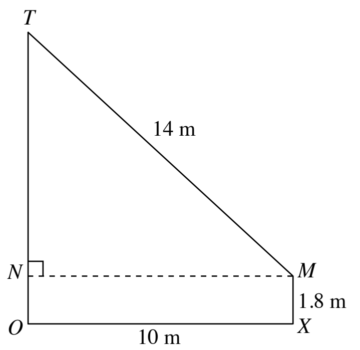
Practise Now 4

1. Let the length of the cable be x m.
Using Pythagoras' Theorem,
$$x^2 = 24^2 + 14^2$$
$$= 576 + 196$$
$$= 772$$
$$\therefore x = \sqrt{772} \text{ (since } x > 0)$$
$$= 27.8 \text{ (to 3 s.f.)}$$
The cable is 27.8 m.

2. Let the vertical height the ladder reached be x m.
Using Pythagoras' Theorem,
$$2.5^2 = x^2 + 1.5^2$$
$$x^2 = 2.5^2 - 1.5^2$$
$$= 6.25 - 2.25$$
$$= 4$$
$$\therefore x = \sqrt{4} \text{ (since } x > 0)$$
$$= 2$$
The ladder reaches 2 m up the wall.

Practise Now 5

1.



Let the height of the tree be OT.
In $\triangle TNM$, $\angle N = 90°$
Using Pythagoras' Theorem,
$$TM^2 = MN^2 + TN^2$$
$$14^2 = 10^2 + TN^2$$
$$TN^2 = 14^2 - 10^2$$
$$= 196 - 100$$
$$= 96$$
$$\therefore TN = \sqrt{96} \text{ (since } TN > 0)$$
$$= 9.798 \text{ m (to 4 s.f.)}$$
$$\therefore OT = 9.798 + 1.8$$
$$= 11.6 \text{ m (to 3 s.f.)}$$
The height of the tree is 11.6 m.

Practise Now 6

1. In $\triangle ABD$, $\angle A = 90°$.

Using Pythagoras' Theorem,

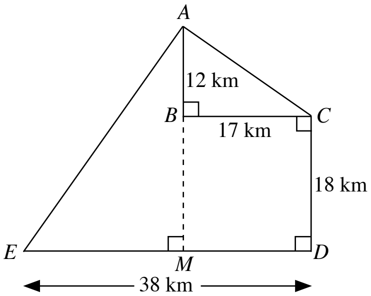
$$BD^2 = DA^2 + BA^2$$
$$(2x + 18)^2 = x^2 + (2x + 12)^2$$
$$4x^2 + 72x + 324 = x^2 + 4x^2 + 48x + 144$$
$$x^2 - 24x - 180 = 0$$
$$(x - 30)(x + 6) = 0$$
$$x = 30 \quad \text{or} \quad x = -6$$

$\therefore x = 30$ (since $x > 0$)

Practise Now 7

(i) $AB = 10 \times 1.2 = 12$ km

$BC = 10 \times 1.7 = 17$ km



In $\triangle ABC$, $\angle B = 90°$.

Using Pythagoras' Theorem,

$AC^2 = CB^2 + AB^2$
$= 17^2 + 12^2$
$= 289 + 144$
$= 433$

$\therefore AC = \sqrt{433}$ (since $AC > 0$)
$= 20.8$ km (to 3 s.f.)

The shortest distance between Port A and Jetty C is 20.8 km.

(ii) Draw a perpendicular line from B to DE cutting DE at M.

In $\triangle AEM$, $\angle M = 90°$.

$AM = 12 + 18$
$= 30$ km

$EM = 38 - 17$
$= 21$ km

Using Pythagoras' Theorem,

$AE^2 = EM^2 + AM^2$
$= 21^2 + 30^2$
$= 441 + 900$
$= 1341$

$\therefore AE = \sqrt{1341}$ (since $AE > 0$)
$= 36.6$ km (to 3 s.f.)

The shortest distance between Port A and Island E is 36.6 km.

Practise Now 8

1. (a) AB is the longest side of $\triangle ABC$.

$AB^2 = 12^2$
$= 144$

$BC^2 + AC^2 = 10^2 + 8^2$
$= 100 + 64$
$= 164$

Since $AB^2 \neq BC^2 + AC^2$, $\triangle ABC$ is not a right-angled triangle.

(b) PQ is the longest side of $\triangle PQR$.

$PQ^2 = 34^2$
$= 1156$

$QR^2 + PR^2 = 16^2 + 30^2$
$= 256 + 900$
$= 1156$

Since $PQ^2 = QR^2 + PR^2$, $\triangle PQR$ is a right-angled triangle where $\angle R = 90°$.

2. (i) XZ is the longest side in $\triangle XYZ$.

$XZ^2 = 51^2$
$= 2601$

$XY^2 + YZ^2 = 45^2 + 24^2$
$= 2025 + 576$
$= 2601$

Since $XZ^2 = XY^2 + YZ^2$, $\triangle XYZ$ is a right-angled triangle where $\angle XYZ = 90°$.

(ii) In XYT, $\angle Y = 90°$

Using Pythagoras' Theorem,

$TX^2 = XY^2 + TY^2$
$= 45^2 + (24 - 14)^2$
$= 45^2 + 10^2$
$= 2025 + 100$
$= 2125$

$\therefore TX = \sqrt{2125}$ (since $TX > 0$)
$= 46.1$ m (to 3 s.f.)

The distance of the tree from X is 46.1 m.

Exercise 10A

1. (a) Using Pythagoras' Theorem,

$a^2 = 20^2 + 21^2$
$= 400 + 441$
$= 841$

$\therefore a = \sqrt{841}$ (since $a > 0$)
$= 29$

(b) Using Pythagoras' Theorem,

$b^2 = 12^2 + 35^2$
$= 144 + 1225$
$= 1369$

$\therefore b = \sqrt{1369}$ (since $b > 0$)
$= 37$

(c) Using Pythagoras' Theorem,

$c^2 = 10^2 + 12^2$
$= 100 + 144$
$= 244$

$\therefore c = \sqrt{244}$ (since $c > 0$)
$= 15.6$ (to 3 s.f.)

(d) Using Pythagoras' Theorem,

$d^2 = 23^2 + 29^2$
$= 529 + 841$
$= 1370$

$\therefore d = \sqrt{1370}$ (since $d > 0$)
$= 37.0$ (to 3 s.f.)

2. (a) Using Pythagoras' Theorem,

$39^2 = a^2 + 15^2$
$a^2 = 39^2 - 15^2$
$= 1521 - 225$
$= 1296$

$\therefore a = \sqrt{1296}$ (since $a > 0$)
$= 36$

(b) Using Pythagoras' Theorem,

$19^2 = b^2 + 14^2$
$b^2 = 19^2 - 14^2$
$= 361 - 196$
$= 165$

$\therefore b = \sqrt{165}$ (since $b > 0$)
$= 12.8$ (to 3 s.f.)

(c) Using Pythagoras' Theorem,

$9.8^2 = c^2 + 6.5^2$
$c^2 = 9.8^2 - 6.5^2$
$= 96.04 - 42.25$
$= 53.79$

$\therefore c = \sqrt{53.79}$ (since $c > 0$)
$= 7.33$ (to 3 s.f.)

(d) Using Pythagoras' Theorem,

$24.7^2 = d^2 + 14.5^2$
$d^2 = 24.7^2 - 14.5^2$
$= 610.09 - 210.25$
$= 399.84$

$\therefore d = \sqrt{399.84}$ (since $d > 0$)
$= 20.0$ (to 3 s.f.)

3. In $\triangle ABC$, $\angle B = 90°$.

Using Pythagoras' Theorem,

$AC^2 = AB^2 + BC^2$
$= 8^2 + 15^2$
$= 64 + 225$
$= 289$

$\therefore AC = \sqrt{289}$ (since $AC > 0$)
$= 17$ cm

4. In $\triangle DEF$, $\angle E = 90°$.

Using Pythagoras' Theorem,

$DF^2 = EF^2 + DE^2$
$= 5.5^2 + 6.7^2$
$= 30.25 + 44.89$
$= 75.14$

$\therefore DF = \sqrt{75.14}$ (since $DF > 0$)
$= 8.67$ m (to 3 s.f.)

5. In $\triangle GHI$, $\angle H = 90°$.

Using Pythagoras' Theorem,

$GI^2 = HI^2 + GH^2$
$65^2 = HI^2 + 33^2$
$HI^2 = 65^2 - 33^2$
$= 4225 - 1089$
$= 3136$

$\therefore HI = \sqrt{3136}$ (since $HI > 0$)
$= 56$ cm

6. In $\triangle MNO$, $\angle N = 90°$.

Using Pythagoras' Theorem,

$MO^2 = MN^2 + NO^2$
$14.2^2 = MN^2 + 11^2$
$MN^2 = 14.2^2 - 11^2$
$= 201.64 - 121$
$= 80.64$

$\therefore MN = \sqrt{80.64}$ (since $MN > 0$)
$= 8.98$ cm (to 3 s.f.)

7. (i) In $\triangle PQS$, $\angle Q = 90°$.

Using Pythagoras' Theorem,

$PS^2 = PQ^2 + QS^2$
$53^2 = 45^2 + QS^2$
$QS^2 = 53^2 - 45^2$
$= 2809 - 2025$
$= 784$

$\therefore QS = \sqrt{784}$ (since $QS > 0$)
$= 28$ cm

(ii) In $\triangle QRS$, $\angle S = 90°$.

Using Pythagoras' Theorem,

$QR^2 = QS^2 + SR^2$
$30^2 = 28^2 + SR^2$
$SR^2 = 30^2 - 28^2$
$= 900 - 784$
$= 116$

$\therefore SR = \sqrt{116}$ (since $QS > 0$)
$= 10.8$ cm (to 3 s.f.)

8. H is the midpoint of UV.

$\therefore HV = \dfrac{15.4}{2} = 7.7$ m

$TV = 9.6$ m (isos. $\triangle TUV$)

In $\triangle THV$, $\angle H = 90°$.

Using Pythagoras' Theorem,

$TV^2 = TH^2 + HV^2$

$9.6^2 = TH^2 + 7.7^2$

$TH^2 = 9.6^2 - 7.7^2$

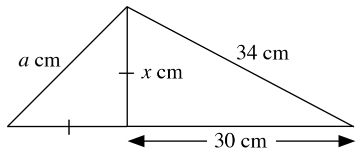
$= 92.16 - 59.29$

$= 32.87$

$\therefore TH = \sqrt{32.87}$ (since $TH > 0$)

$= 5.73$ m (to 3 s.f.)

9. (a)



Let the unknown side be x cm.

Using Pythagoras' Theorem on the right-angled triangle on the right,

$34^2 = x^2 + 30^2$

$x^2 = 34^2 - 30^2$

$= 1156 - 900$

$= 256$

Using Pythagoras' Theorem on the right-angled triangle on the left,

$a^2 = x^2 + x^2$

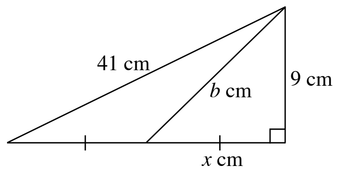
$= 256 + 256$

$= 512$

$\therefore a = \sqrt{512}$ (since $a > 0$)

$= 22.6$ (to 3 s.f.)

(b)



Let the unknown side be x cm.

Using Pythagoras' Theorem on the larger right-angled triangle,

$41^2 = (x + x)^2 + 9^2$

$(2x)^2 = 41^2 - 9^2$

$4x^2 = 1681 - 81$

$4x^2 = 1600$

$x^2 = 400$

Using Pythagoras' Theorem on the smaller right-angled triangle,

$b^2 = x^2 + 9^2$

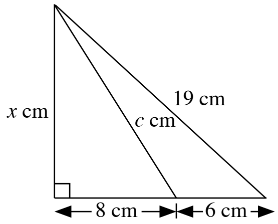
$= 400 + 81$

$= 481$

$\therefore b = \sqrt{481}$ (since $b > 0$)

$= 21.9$ (to 3 s.f.)

(c)



Let the unknown side be x cm.

Using Pythagoras' Theorem on the larger right-angled triangle,

$19^2 = x^2 + (8 + 6)^2$

$19^2 = x + 14^2$

$x^2 = 19^2 - 14^2$

$= 361 - 196$

$= 165$

Using Pythagoras' Theorem on the smaller right-angled triangle,

$c^2 = x^2 + 8^2$

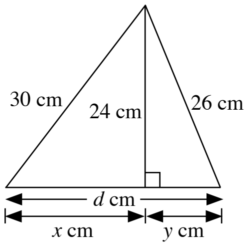
$= 165 + 64$

$= 229$

$\therefore c = \sqrt{229}$ (since $c > 0$)

$= 15.1$

(d)



Let the two unknown sides be x cm and y cm.

Using Pythagoras' Theorem on the right-angled triangle on the left,

$30^2 = x^2 + 24^2$

$x^2 = 30^2 - 24^2$

$= 900 - 576$

$= 324$

Using Pythagoras' Theorem on the right-angled triangle to the right,

$26^2 = y^2 + 24^2$

$y^2 = 26^2 - 24^2$

$= 676 - 576$

$= 100$

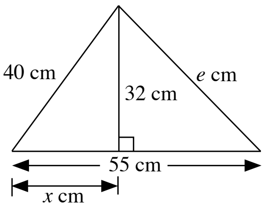
$\therefore d = x + y$

$= \sqrt{324} + \sqrt{100}$ (since $x, y > 0$)

$= 18 + 10$

$= 28$

(e)



Let the unknown side be x cm.
Using Pythagoras' Theorem on the right-angled triangle on the left,

$$40^2 = x^2 + 32^2$$
$$x^2 = 40^2 - 32^2$$
$$= 1600 - 1024$$
$$= 576$$
$$\therefore x = \sqrt{576} \text{ (since } x > 0)$$
$$= 24$$

Using Pythagoras' Theorem on the right-angled triangle on the right,

$$e^2 = (55 - x)^2 + 32^2$$
$$= (55 - 24)^2 + 32^2$$
$$= 31^2 + 32^2$$
$$= 961 + 1024$$
$$= 1985$$
$$\therefore e = \sqrt{1985} \text{ (since } e > 0)$$
$$= 44.6 \text{ (to 3 s.f.)}$$

10. (a) Using Pythagoras' Theorem on the right-angled triangle with one side 27 cm,

$$(2a + a)^2 = 36^2 + 27^2$$
$$(3a)^2 = 1296 + 729$$
$$9a^2 = 2025$$
$$a^2 = 225$$
$$\therefore a = \sqrt{225} \text{ (since } a > 0)$$
$$= 15$$

Using Pythagoras' Theorem on the right-angled triangle with one side a cm,

$$b^2 = a^2 + 60^2$$
$$= 225 + 3600$$
$$= 3825$$
$$\therefore b = \sqrt{3825} \text{ (since } b > 0)$$
$$= 61.8$$

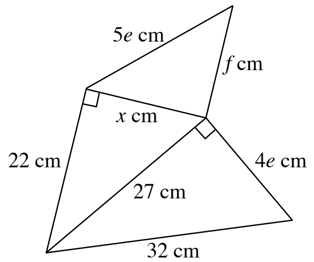
(b) Using Pythagoras' Theorem on the larger right-angled triangle,

$$39^2 = (3c + 4c)^2 + 25^2$$
$$(7c)^2 = 39^2 - 25^2$$
$$49c^2 = 1521 - 625$$
$$49c^2 = 896$$
$$c^2 = \frac{128}{7}$$
$$\therefore c = \sqrt{\frac{128}{7}} \text{ (since } c > 0)$$
$$= 4.28 \text{ (to 3 s.f.)}$$

Using Pythagoras' Theorem on the smaller right-angled triangle,

$$d^2 = (4c)^2 + 25^2$$
$$= 16c^2 + 625$$
$$= 16\left(\frac{128}{7}\right) + 625$$
$$= 917\frac{4}{7}$$
$$\therefore d = \sqrt{917\frac{4}{7}} \text{ (since } d > 0)$$
$$= 30.3$$

(c)



Using Pythagoras' Theorem on the right-angled triangle with side 32 cm,

$$32^2 = 27^2 + (4e)^2$$
$$16e^2 = 32^2 - 27^2$$
$$= 1024 - 729$$
$$= 295$$
$$e^2 = \frac{295}{16}$$
$$\therefore e = \sqrt{\frac{295}{16}} \text{ (since } e > 0)$$
$$= 4.29 \text{ (to 3 s.f.)}$$

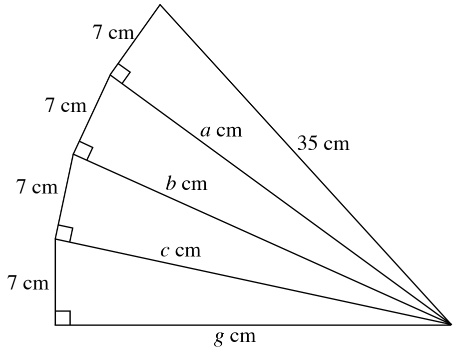
Let the unknown side be x cm.
Using Pythagoras' Theorem on the right-angled triangle with side 22 cm,

$$27^2 = x^2 + 22^2$$
$$x^2 = 27^2 - 22^2$$
$$= 729 - 484$$
$$= 245$$

Using the Pythagoras' Theorem on the right-angled triangle with side $5e$ cm,

$$(5e)^2 = f^2 + x^2$$
$$25e^2 = f^2 + 245$$
$$25\left(\frac{295}{16}\right) = f^2 + 245$$
$$f^2 = 25\left(\frac{295}{16}\right) - 245$$
$$= 215\frac{15}{16}$$
$$\therefore f = \sqrt{215\frac{15}{16}} \text{ (since } f > 0)$$
$$= 14.7 \text{ (to 3 s.f.)}$$

(d)



Let the unknown sides be a cm, b cm and c cm.

Using Pythagoras' Theorem,

$35^2 = a^2 + 7^2$

$a^2 = 35^2 - 7^2$

$b^2 + 7^2 = 35 - 7^2$

$b^2 = 35^2 - 7^2 - 7^2$

$c^2 + 7^2 = 35^2 - 7^2 - 7^2$

$c^2 = 35^2 - 7^2 - 7^2 - 7^2$

$g^2 + 7^2 = 35^2 - 7^2 - 7^2 - 7^2$

$g^2 = 35^2 - 7^2 - 7^2 - 7^2 - 7^2$

$= 1225 - 49 - 49 - 49 - 49$

$= 1029$

$\therefore g = \sqrt{1029}$ (since $g > 0$)

$= 32.1$ (to 3 s.f.)

11. (i) In $\triangle WXY$, $\angle Y = 90°$.

Using Pythagoras' Theorem,

$WX^2 = XY^2 + WY^2$

$(18 + 14)^2 = XY^2 + 24^2$

$XY^2 = 32^2 - 24^2$

$= 1024 - 576$

$= 448$

$XY = \sqrt{448}$ (since $XY > 0$)

$= 21.17$ m (to 4 s.f.)

$\therefore YQ = XY - QX$

$= 21.17 - 9.8$

$= 11.4$ m (to 3 s.f.)

(ii) In $\triangle XPY$, $\angle P = 180° - 90°$ (adj. $\angle$s on a str. line)

$= 90°$

Using Pythagoras' Theorem,

$XY^2 = YP^2 + XP^2$

$448 = YP^2 + 14^2$

$YP^2 = 448 - 14^2$

$= 448 - 196$

$= 252$

$YP = \sqrt{252}$ (since $YP > 0$)

$= 15.87$ m (to 4 s.f.)

$\therefore$ Area of $\triangle XPY = \frac{1}{2} \times 14 \times 15.87$

$= 111$ m^2 (to 3 s.f.)

12. In $\triangle HBK$, $\angle B = 90°$.

Using Pythagoras' Theorem,

$HK^2 = BK^2 + BH^2$

$22^2 = BK^2 + 15^2$

$BK^2 = 22^2 - 15^2$

$= 484 - 225$

$= 259$

$\therefore BK = \sqrt{259}$ (since $BK > 0$)

$= 16.09$ cm (to 4 s.f.)

In $\triangle ABC$, $\angle B = 90°$.

Using Pythagoras' Theorem,

$AC^2 = AB^2 + BC^2$

$43^2 = (AH + 15)^2 + (16.09 + 19)^2$

$(AH + 15)^2 = 43^2 - 35.09^2$

$AH + 15 = \sqrt{43^2 - 35.09^2}$

$\therefore AH = -15 + \sqrt{43^2 - 35.09^2}$

$= 9.85$ cm (to 3 s.f.)

or

$AH + 15 = -\sqrt{43^2 - 35.09^2}$

$AH = -15 - \sqrt{43^2 - 35.09^2}$

$= -39.9$ cm (to 3 s.f.)

(rejected, since $AH > 0$)

13. In $\triangle EPF$, $\angle P = 90°$.

Using Pythagoras' Theorem,

$EF^2 = PF^2 + PE^2$

$23^2 = 13^2 + PE^2$

$PE^2 = 23^2 - 13^2$

$= 529 - 169$

$= 360$

$PE = \sqrt{360}$ (since $PE > 0$)

$= 18.97$ m (to 4 s.f.)

In $\triangle DPE$, $\angle DPE = 180° - 90°$ (adj. $\angle$s on a str. line)

$= 90°$

Using Pythagoras' Theorem,

$DE^2 = PD^2 + PE^2$

$31^2 = PD^2 + 360$

$PD = 31^2 - 360$

$= 961 - 360$

$= 601$

$PD = \sqrt{601}$ (since $PD > 0$)

$= 24.52$ m (to 4 s.f.)

In $\triangle DGF$, $\angle G = 90°$

Using Pythagoras' Theorem,

$DF^2 = FG^2 + DG^2$

$(24.52 + 13)^2 = FG^2 + 32^2$

$FG^2 = (24.52 + 13)^2 - 32^2$

$FG = \sqrt{(24.52 + 13)^2 - 32^2}$ (since $FG > 0$)

$= 19.59$ m (to 4 s.f.)

$\therefore$ Area of the figure

= Area of $\triangle EPF$ + Area of $\triangle DPE$ + Area of $\triangle DGF$

$= \frac{1}{2} \times 13 \times 18.97 \times \frac{1}{2} \times 24.52 \times 18.97 + \frac{1}{2} \times 32 \times 19.59$

$= 669 \text{ m}^2$ (to 3 s.f.)

Exercise 10B

1. Let the length of each cable be x m.

Using Pythagoras' Theorem,

$x^2 = 47^2 + 18^2$
$= 2209 + 324$
$= 2533$

$\therefore x = \sqrt{2533}$ (since $x > 0$)
$= 50.3$ (to 3 s.f.)

The length of each cable is 50.3 m,

2. Let the length of the barricade be x m.

Using Pythagoras' Theorem,

$x^2 = 50^2 + 50^2$
$= 2500 + 2500$
$= 5000$

$\therefore x = \sqrt{5000}$ (since $x > 0$)
$= 70.7$ (to 3 s.f.)

The length of the barricade is 70.7 m.

3. Let the distance Ethan has to swim be x m.

Using Pythagoras' Theorem,

$x^2 = 50^2 + 30^2$
$= 2500 + 900$
$= 3400$

$\therefore x = \sqrt{3400}$ (since $x > 0$)
$= 58.3$ (to 3 s.f.)

The distance Ethan has to swim is 58.3 m.

4. Let the vertical height the ladder reached be x m.

Using Pythagoras' Theorem,

$5^2 = x^2 + 1.8^2$
$x^2 = 5^2 - 1.8^2$
$= 25 - 3.24$
$= 21.76$

$\therefore x = \sqrt{21.76}$ (since $x > 0$)
$= 4.66$ (to 3 s.f.)

The ladder reaches 4.66 m up the wall.

5. Let the width of the screen be x inches.

Using Pythagoras' Theorem,

$30^2 = x^2 + 18^2$
$x^2 = 30^2 - 18^2$
$= 900 - 324$
$= 576$

$\therefore x = \sqrt{576}$ (since $x > 0$)
$= 24$

The width of the screen is 24 inches.

6. Let the length of the cable be x m.

Using Pythagoras' Theorem,

$x^2 = 16^2 + (37 - 30^2)$
$= 16^2 + 7^2$
$= 256 + 49$
$= 305$

$\therefore x = \sqrt{305}$ (since $x > 0$)
$= 17.5$ (to 3 s.f.)

The length of the cable is 17.5 m.

7. In $\triangle AED$, $\angle E = 90°$.

Using Pythagoras' Theorem,

$AD^2 = DE^2 + AE^2$
$= 8^2 + 8^2$
$= 64 + 64$
$= 128$

$AD = \sqrt{128}$ (since $AD > 0$)
$= 11.31$ (to 4 s.f.)

In $\triangle BCD$, $\angle C = 90°$.

Using Pythagoras' Theorem,

$DB^2 = BC^2 + DC^2$
$= 14^2 + 14^2$
$= 196 + 196$
$= 392$

$DB = \sqrt{392}$ (since $DB > 0$)
$= 19.80$ (to 4 s.f.)

$\therefore$ Total length = 11.31 + 19.80
= 31.1 cm (to 3 s.f.)

The total length is 31.1 cm.

8. The diagonals of a rhombus bisect each other and are at right angles to each other.

Let the length of each side of the coaster be x cm.

Using Pythagoras' Theorem,

$x^2 = \left(\frac{10}{2}\right)^2 + \left(\frac{24}{2}\right)^2$
$= 5^2 + 12^2$
$= 25 + 144$
$= 169$

$\therefore x = \sqrt{169}$ (since $x > 0$)
$= 13$

The length of each side of the coaster is 13 cm.

9. **(i)** In $\triangle BKQ$, $\angle B = 90°$.

Using Pythagoras' Theorem,

$KQ^2 = BQ^2 + BK^2$
$21^2 = BQ^2 + 17.2^2$
$BQ^2 = 21^2 - 17.2^2$
$= 441 - 295.84$
$= 145.16$

$\therefore BQ = \sqrt{145.16}$ (since $BQ > 0$)
$= 12.0$ m (to 3 s.f.)

The height above the ground at which the spotlight Q is mounted, BQ, is 12.0 m.

(ii) In $\triangle BHP$, $\angle B = 90°$.

Using Pythagoras' Theorem,

$HP^2 = BP^2 + BH^2$

$39^2 = (12.05 + 12.7)^2 + BH^2$

$BH^2 = 39^2 - 24.75^2$

$BH = \sqrt{39^2 - 24.75^2}$ (since $BH > 0$)

$= 30.14$ m (to 4 s.f.)

$\therefore HK = BH - BK$

$= 30.14 - 17.2$

$= 12.9$ m (to 3 s.f.)

The distance between the projections of the light beams, HK, is 12.9 m.

10. (i) In $\triangle PQR$, $\angle Q = 90°$.

Using Pythagoras' Theorem,

$PR^2 = RQ^2 + PQ^2$

$= 1.1^2 + 4.2^2$

$= 1.21 + 17.64$

$= 18.85$

$\therefore PR = \sqrt{18.85}$ (since $PR > 0$)

$= 4.34$ m (to 3 s.f.)

The length of the pole is 4.34 m.

(ii) In $\triangle XQY$, $\angle Q = 90°$.

Using Pythagoras' Theorem,

$XY^2 = QY^2 + QX^2$

$18.85 = (YR + 1.1)^2 + (4.2 - 0.9)^2$

$(YR + 1.1)^2 = 18.85 - 3.3^2$

$YR + 1.1 = \pm\sqrt{18.85 - 3.3^2}$

$\therefore YR = -1.1 + \sqrt{18.85 - 3.3^2}$

$YR = 1.72$ m (to 3 s.f.)

or

$YR = -1.1 - \sqrt{18.85 - 3.3^2}$

$YR = -3.92$ m (to 3 s.f.)

(rejected, since $YR > 0$)

The distance, YR, is 1.72 m.

11. In $\triangle FGH$, $\angle G = 90°$.

Using Pythagoras' Theorem,

$FH^2 = GH^2 + GF^2$

$(4x + 1)^2 = (x + 1)^2 + (3x + 6)^2$

$16x^2 + 8x + 1 = x^2 + 2x + 1 + 9x^2 + 36x + 36$

$6x^2 - 30x - 36 = 0$

$x^2 - 5x - 6 = 0$

$(x - 6)(x + 1) = 0$

$x = 6$ or $x = -1$

When $x = 6$,	When $x = -1$,
$FG = 3(6) + 6$	$FG = 3(-1) + 6$
$= 24$ m	$= 3$ m
$GH = 6 + 1$	$GH = -1 + 1$
$= 7$ m	$= 0$ m
	$x = -1$ is rejected since $GH > 0$.

$\therefore$ Area of campsite $= 24 \times 7$

$= 168 \text{ m}^2$

The area of the campsite is 168 m^2.

12. The side $(x + 2)$ cm is the longest side.

Using Pythagoras' Theorem,

$(x + 2)^2 = x^2 + (x + 1)^2$

$x^2 + 4x + 4 = x^2 + x^2 + 2x + 1$

$x^2 - 2x - 3 = 0$

$(x - 3)(x + 1) = 0$

$\therefore x = 3$ or $x = -1$ (rejected, since $x > 0$)

The value of x is 3.

13. (i) $HL = 9 - 2 = 7$ cm

$OL = 6$ cm

In $\triangle HLO$, $\angle L = 90°$.

Using Pythagoras' Theorem,

$OH^2 = HL^2 + OL^2$

$= 7^2 + 6^2$

$= 49 + 36$

$= 85$

$\therefore OH = \sqrt{85}$ (since $OH > 0$)

$= 9.22$ cm (to 3 s.f.)

The length of the zip is 9.22 cm.

(ii) In $\triangle HMN$, $\angle M = 90°$.

Using Pythagoras' Theorem,

$HN^2 = NM^2 + HM^2$

$= 6^2 + 2^2$

$= 36 + 4$

$= 40$

Let the length of NK be x cm,

the length of OK be y cm.

In $\triangle HKN$, $\angle K = 90°$.

Using Pythagoras' Theorem,

$HN^2 = NK^2 + HK^2$

$x^2 + HK^2 = 40$

$HK^2 = 40 - x^2$

$(\sqrt{85} - OK)^2 = 40 - x^2$

$85 - 2\sqrt{85}\,y + y^2 = 40 - x^2$

$y^2 = 2\sqrt{85}\,y - 45 - x^2$

In $\triangle OKN$, $\angle K = 180° - 90°$ (adj. $\angle$s on a str. line)

$= 90°$

Using Pythagoras' Theorem,

$ON^2 = NK^2 + OK^2$

$9^2 = x^2 + y^2$

$81 = x^2 + 2\sqrt{85}\,y - 45 - x^2$

$2\sqrt{85}\,y = 126$

$y = \frac{63}{\sqrt{85}}$

$\therefore y^2 = 2\sqrt{85}\,y - 45 - x^2$

$$\left(\frac{63}{\sqrt{85}}\right)^2 = 2\sqrt{85}\left(\frac{63}{\sqrt{85}}\right) - 45 - x^2$$

$$x^2 = 126 - 45 - \frac{3969}{85}$$

$$= 34\frac{26}{35}$$

$$\therefore x = \sqrt{34\frac{26}{85}} \text{ (since } x > 0)$$

$$= 5.86 \text{ (to 3 s.f.)}$$

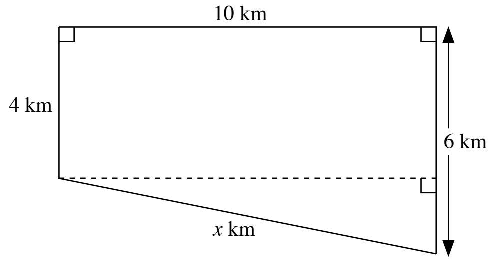
The length of the second zip is 5.86 cm.

14. Distance travelled due North $= 40 \times \frac{6}{60}$

$= 4$ km

Distance travelled due South $= 30 \times \frac{12}{60}$

$= 6$ km



Let the shortest distance be x km.

Using Pythagoras' Theorem,

$$x^2 = 10^2 + (6 - 4)^2$$

$$= 100 + 4$$

$$= 104$$

$$\therefore x = \sqrt{104} \text{ (since } x > 0)$$

$$= 10.2 \text{ (to 3 s.f.)}$$

The shortest distance between the courier and his starting point is 10.2 km.

15. (a) (i) Length of each side of square tabletop

$$= \frac{132}{4}$$

$$= 33 \text{ cm}$$

(ii) Let the radius of the round tabletop be r cm.

$$2\pi r = 132$$

$$2 \times \frac{22}{7} \times r = 132$$

$$\therefore r = 21$$

The radius is 21 cm.

(b) Area of square tabletop $= 33^2$

$= 1089 \text{ cm}^2$

Area of round tabletop $= \pi r^2$

$= \frac{22}{7} \times 21^2$

$= 1386 \text{ cm}^2$

(c) (i) Length of each side of table

$$= \frac{132}{3}$$

$$= 44 \text{ cm}$$

(ii) The height of the equilateral triangle bisects the side opposite it.

Let the height of the equilateral triangle be h cm.

Using Pythagoras' Theorem,

$$44^2 = h^2 + \left(\frac{44}{2}\right)^2$$

$$h^2 = 44^2 - 22^2$$

$$= 1936 - 484$$

$$= 1452$$

$$h = \sqrt{1452} \text{ (since } h > 0)$$

$$= 38.11 \text{ (to 4 s.f.)}$$

Area of tabletop $= \frac{1}{2} \times 44 \times 38.11$

$= 838 \text{ cm}^2$ (to 3 s.f.)

(d) The tabletop in the shape of a circle should be chosen since it has the greatest area.

Exercise 10C

1. (a) AC is the longest side of $\triangle ABC$.

$$AC^2 = 65^2$$

$$= 4225$$

$$AB^2 + BC^2 = 16^2 + 63^2$$

$$= 256 + 3969$$

$$= 4225$$

Since $AC^2 = AB^2 + BC^2$, $\triangle ABC$ is a right-angled triangle where $\angle B = 90°$.

(b) EF is the longest side of $\triangle DEF$.

$$EF^2 = 27^2$$

$$= 729$$

$$DF^2 + DE^2 = 21^2 + 24^2$$

$$= 441 + 576$$

$$= 1017$$

Since $EF^2 \neq DF^2 + DE^2$, $\triangle DEF$ is not a right-angled triangle.

(c) GH is the longest side in $\triangle GHI$.

$$GH^2 = 7.5^2$$

$$= 56.25$$

$$HI^2 + GI^2 = 7.1^2 + 2.4^2$$

$$= 50.41 + 5.76$$

$$= 56.17$$

Since $GH^2 \neq HI^2 + GI^2$, $\triangle GHI$ is not a right-angled triangle.

(d) MN is the longest side in $\triangle MNO$.

$MN^2 = \left(\frac{5}{13}\right)^2$

$= \frac{25}{169}$

$NO^2 + MO^2 = \left(\frac{3}{13}\right)^2 + \left(\frac{4}{13}\right)^2$

$= \frac{9}{169} + \frac{16}{169}$

$= \frac{25}{169}$

Since $MN^2 = NO^2 + MO^2$, $\triangle MNO$ is a right-angled triangle where $\angle O = 90°$.

2. PR is the longest side is $\triangle PQR$.

$PR^2 = 30^2$

$= 900$

$PQ^2 + QR^2 = 19^2 + 24^2$

$= 361 + 576$

$= 937$

Since $PR^2 \neq PQ^2 + QR^2$, $\triangle PQR$ is not a right-angled triangle.

3. $ST = \frac{7}{12}$ cm

$TU = \frac{5}{6}$ cm $= \frac{10}{12}$ cm

$SU = \frac{1}{3}$ cm $= \frac{4}{12}$ cm

TU is the longest side in $\triangle STU$.

$TU^2 = \left(\frac{10}{12}\right)^2$

$= \frac{100}{144}$

$SU^2 + ST^2 = \left(\frac{4}{12}\right)^2 + \left(\frac{7}{12}\right)^2$

$= \frac{16}{144} + \frac{49}{144}$

$= \frac{65}{144}$

Since $TU^2 \neq SU^2 + ST^2$, $\triangle STU$ is not a right-angled triangle.

4. In $\triangle PQS$, $\angle P = 90°$.

Using Pythagoras' Theorem,

$SQ^2 = PQ^2 + PS^2$

$= 40^2 + 30^2$

$= 1600 + 900$

$= 2500$

$SQ = \sqrt{2500}$ (since $SQ > 0$)

$= 50$ m

$\frac{SX}{SQ} = \frac{16}{16 + 9}$

$SX = \frac{16}{25} \times 50$

$= 32$ m

$QX = 50 - 32$

$= 18$ m

To show Jun Wei stops at X is to show RX is perpendicular to QS.

We need to show $\triangle SXR$ and $\triangle QXR$ are right-angled triangles.

RS is the longest side in $\triangle SXR$.

$RS^2 = 40^2$

$= 1600$

$SX^2 + RX^2 = 32^2 + 24^2$

$= 1024 + 576$

$= 1600$

Since $RS^2 = SX^2 + RX^2$, $\triangle SXR$ is a right-angled triangle where $\angle X = 90°$.

QR is the longest side in $\triangle QXR$.

$QR^2 = 30^2$

$= 900$

$RX^2 + QX^2 = 24^2 + 18^2$

$= 576 + 324$

$= 900$

Since $QR^2 = RX^2 + QX^2$, $\triangle QXR$ is a right-angled triangle where $\angle X = 90°$.

$\therefore$ Jun Wei stops at X.

5. Since m and n are positive integers,

$m^2 + n^2 > m^2 - n^2$

Also,

$(m - n)^2 > 0$

$m^2 - 2mn + n^2 > 0$

$m^2 + n^2 > 2mn$

c is the longest side in the triangle.

$c^2 = (m^2 + n^2)$

$= m^4 + 2m^2n^2 + n^4$

$a^2 + b^2 = (m^2 - n^2)^2 + (2mn)^2$

$= m^4 - 2m^2n^2 + n^4 + 4m^2n^2$

$= m^4 + 2m^2n^2 + n^4$

Since $c^2 = a^2 + b^2$, then the triangle is a right-angled triangle.

Review Exercise 10

1. (a) Using Pythagoras' Theorem,

$a^2 = 6.3^2 + 9.6^2$

$= 39.69 + 92.16$

$= 131.85$

$\therefore a = \sqrt{131.85}$ (since $a > 0$)

$= 11.5$ (to 3 s.f.)

(b) Using Pythagoras' Theorem,

$13.5^2 = b^2 + 8.7^2$

$b^2 = 13.5^2 - 8.7^2$

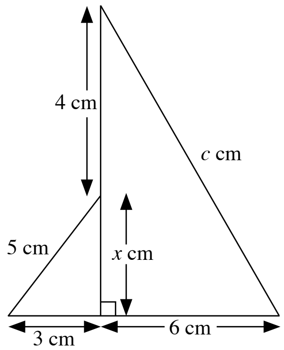
$= 182.25 - 75.69$

$= 106.56$

$\therefore b = \sqrt{106.56}$ (since $b > 0$)

$= 10.3$ (to 3 s.f.)

(c)



Let the unknown side be x cm.

Using Pythagoras' Theorem on the smaller right-angled triangle,

$5^2 = x^2 + 3^2$

$x^2 = 5^2 - 3^2$

$= 25 - 9$

$= 16$

$x = \sqrt{16}$ (since $x > 0$)

$= 4$

Using Pythagoras' Theorem on the larger right-angled triangle,

$c^2 = 6^2 + (x + 4)^2$

$= 6^2 + 8^2$

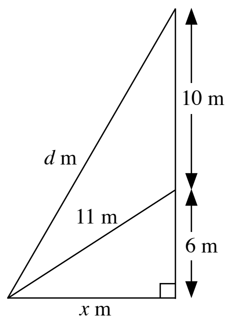
$= 36 + 64$

$= 100$

$\therefore c = \sqrt{100}$ (since $c > 0$)

$= 10$

(d)



Let the unknown side be x m.

Using Pythagoras' Theorem on the smaller right-angled triangle,

$11^2 = x^2 + 6^2$

$x = 11^2 - 6^2$

$= 121 - 36$

$= 85$

Using Pythagoras' Theorem on the larger right-angled triangle,

$d^2 = x^2 + (10 + 6)^2$

$= 85 + 16^2$

$= 85 + 256$

$= 341$

$\therefore d = \sqrt{341}$ (since $c > 0$)

$= 18.5$ (to 3 s.f.)

2. **(i)** Let the side of the square be x cm.

Using Pythagoras' Theorem,

$42.5^2 = x^2 + x^2$

$2x^2 = 1806.25$

$x = 903.125$

$x = \sqrt{903.125}$ (since $x > 0$)

$= 30.05$ (to 4 s.f.)

$\therefore$ Perimeter of the square $= 4 \times 30.05$

$= 120$ cm (to 3 s.f.)

(ii) Area of the square $= 30.05^2$

$= 903$ cm^2 (to 3 s.f.)

3. Let the height of the briefcase be x cm.

Using Pythagoras' Theorem,

$37^2 = x^2 + 30^2$

$x^2 = 37^2 - 30^2$

$= 1369 - 900$

$= 469$

$\therefore x = \sqrt{469}$ (since $x > 0$)

$= 21.7$ (to 3 s.f.)

The height of the briefcase is 21.7 cm.

4. Let the perpendicular distance from F to GH be x cm.

The perpendicular distance from F to GH bisects GH.

Using Pythagoras' Theorem,

$2^2 = x^2 + 1^2$

$x^2 = 2^2 - 1^2$

$= 4 - 1$

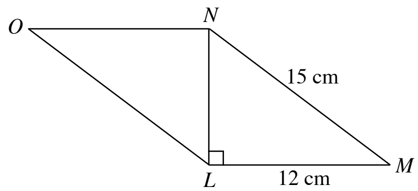
$= 3$

$\therefore x = \sqrt{3}$ (since $x > 0$)

$= 1.73$

The perpendicular distance from F to GH is 1.73 cm.

5.



Let the length of LN be x cm.

In $\triangle LMN$, $\angle L = 90°$.

Using Pythagoras' Theorem,

$MN^2 = LN^2 + LM^2$

$15^2 = LN^2 + 12^2$

$LN^2 = 15^2 - 12^2$

$= 225 - 144$

$= 81$

$LN = \sqrt{81}$ (since $LN > 0$)

$= 9$

$\therefore$ Area of stained glass $= 12 \times 9$

$= 108$ cm^2

6. **(i)** Let the length of the other diagonal be x cm.

The diagonals of a rhombus bisect and are at right angles to each other.

Using Pythagoras' Theorem,

$$52^2 = \left(\frac{x}{2}\right)^2 + \left(\frac{48}{2}\right)^2$$

$$\frac{x^2}{4} = 52^2 - 24^2$$
$$= 2704 - 576$$
$$= 2128$$
$$x^2 = 8512$$
$$\therefore x = \sqrt{8512} \text{ (since } x > 0)$$
$$= 92.26$$
$$= 92.3 \text{ (to 3 s.f.)}$$

The length of the other diagonal is 92.3 cm.

(ii) Area of the floor tile $= 4 \times \left(\frac{1}{2} \times \frac{92.26}{2} \times \frac{48}{2}\right)$

$$= \frac{1}{2} \times 92.26 \times 48$$
$$= 2210 \text{ cm}^2 \text{ (to 3 s.f.)}$$

The area of the floor tile is 2210 cm^2.

7. **(i)** In $\triangle ABD$, $\angle A = 90°$.

Using Pythagoras' Theorem,

$$BD^2 = AD^2 + AB^2$$
$$= 48^2 + 36^2$$
$$= 2304 + 1296$$
$$= 3600$$
$$\therefore BD = \sqrt{3600} \text{ (since } BD > 0)$$
$$= 60 \text{ cm}$$

(ii) BC is the longest side in $\triangle BCD$.

$$BC^2 = 87^2$$
$$= 7569$$
$$BD^2 + CD^2 = 60^2 + 63^2$$
$$= 3600 + 3969$$
$$= 7569$$

Since $BC^2 = BD^2 + CD^2$, $\triangle BCD$ is a right-angled triangle where $\angle D = 90°$.

8. **(i)** $AP = 28 - 6$

$= 22$ m

$CR = 15 - 6$

$= 9$ m

Area of shaded region $DPQR$

= Area of $ABCD$ – area of $\triangle ADP$ – area of $\triangle CDR$ – area of $PBRQ$

$$= (28 \times 15) - \left(\frac{1}{2} \times 22 \times 15\right) - \left(\frac{1}{2} \times 28 \times 9\right) - 6^2$$
$$= 420 - 165 - 126 - 36$$
$$= 93 \text{ m}^2$$

(ii) In $\triangle ADP$, $\angle A = 90°$.

Using Pythagoras' Theorem,

$$DP^2 = AP^2 + AD^2$$
$$= 22^2 + 15^2$$
$$= 484 + 225$$
$$= 709 \text{ m}$$
$$DP = \sqrt{709} \text{ (since } DP > 0)$$
$$= 26.6 \text{ m (to 3 s.f.)}$$

(iii) Let the length of AX be x m.

$$\frac{1}{2} \times 22 \times 15 = \frac{1}{2} \times \sqrt{709} \times x$$
$$x = \frac{22 \times 15}{\sqrt{709}}$$
$$= 12.4 \text{ (to 3 s.f.)}$$

The length of AX is 12.4 m.

9. **(i)** In $\triangle FTK$, $\angle T = 90°$.

Using Pythagoras' Theorem,

$$FK^2 = KT^2 + FT^2$$
$$18^2 = 12.5^2 + FT^2$$
$$FT^2 = 18^2 - 12.5^2$$
$$= 324 - 156.25$$
$$= 167.75$$
$$\therefore FT = \sqrt{167.75} \text{ (since } FT > 0)$$
$$= 13.0 \text{ m (to 3 s.f.)}$$

The height of the pole is 13.0 m.

(ii) $\frac{HT}{KT} = \frac{2}{3+2}$

$$HT = 12.5 \times \frac{2}{5}$$
$$= 5 \text{ m}$$

In $\triangle FTH$, $\angle T = 90°$.

Using Pythagoras' Theorem,

$$FH^2 = HT^2 + FT^2$$
$$= 5^2 + 167.75$$
$$= 25 + 167.75$$
$$= 192.75$$
$$\therefore FH = \sqrt{192.75} \text{ (since } FH > 0)$$
$$= 13.9 \text{ m (to 3 s.f.)}$$

The distance FH is 13.9 m.

10. Let the length of the diagonal be x m.

Using Pythagoras' Theorem,

$$x^2 = 80^2 + 60^2$$
$$= 6400 + 3600$$
$$= 10\,000$$
$$x = \sqrt{10\,000} \text{ (since } x > 0)$$
$$= 100$$

$\therefore$ Time taken to complete run $= \frac{100}{7.5}$

$$= 13\frac{1}{3} \text{ s (to 3 s.f.)}$$

Farhan takes $13\frac{1}{3}$ s to complete his run.

Challenge Yourself

1. **(a)** $6^2 + 8^2 = 36 + 64$
$= 100$
$= 10^2$

6, 8 and 10 form a Pythagorean Triple.

(b) **(i)** $c^2 = 12^2 + 16^2$
$= 144 + 256$
$= 400$

$\therefore c = \sqrt{400}$ (since $c > 0$)
$= 20$

The Pythagorean Triple is 12, 16 and 20.

(ii) $7^2 + 24^2 = 49 + 576 = 625 = 25^2$

A Pythagorean Triple is 7, 24 and 25.

Alternatively,

$3^2 + 4^2 = 9 + 16 = 25 = 5^2$

Multiply throughout by 25,

$(3 \times 5)^2 + (4 \times 5)^2 = (5 \times 5)^2$

$15^2 + 20^2 = 25^2$

A Pythagorean Triple is 15, 20 and 25.

(c) **(i)** $(3n)^2 + (4n)^2 = 9n^2 + 16n^2$
$= 25n^2$

(ii) $25n^2 = (5n)^2$

Let $n = 7$.

$(3 \times 7)^2 + (4 \times 7)^2 = (5 \times 7)^2$

$21^2 \times 28^2 = 35^2$

The Pythagorean Triple is 21, 28 and 35.

(d) **(i)** When $n = 24$,

$1 + 2n = 1 + 2(24)$
$= 49$
$= 7^2$

$n + 1 = 24 + 1$
$= 25$

The Pythagorean Triple is 7, 24 and 25.

(ii) $1 + 2n = 42$

$2n = 41$

$n = 20\frac{1}{2}$

n is not an integer, so a Pythagorean Triple cannot be obtained.

(iii) When $k = 9$,

$1 + 2n = 9^2$

$2n = 81$

$n = 40$

$n + 1 = 40 + 1$
$= 41$

The Pythagorean Triple is 9, 40 and 41.

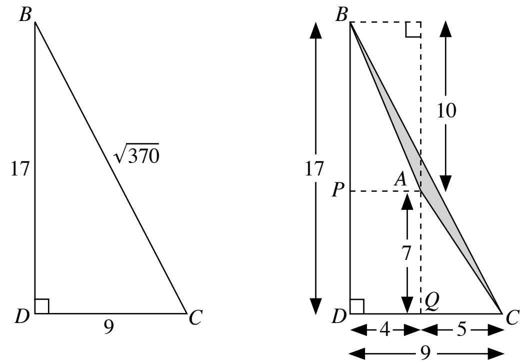
2. $\triangle ABC$ is such that $BC^2 = 370$, $AC^2 = 74$ and $AB^2 = 116$.

The hint is $370^2 = 9^2 + 17^2$, $74 = 5^2 + 7^2$, $116 = 4^2 + 10^2$.

The key is to observe that $17 = 7 + 10$,
$9 = 5 + 4$

So starting with $BC^2 = 9^2 + 17^2$, we have the diagram below.

Then we try to construct the point A as follows.



$\therefore$ Area of $\triangle ABC$

= area of $\triangle BCD$ – area of $\triangle ABP$ – area of $\triangle ACQ$ – area of rectangle $APDQ$

$= \frac{1}{2} \times 9 \times 17 - \frac{1}{2} \times 4 \times 10 - \frac{1}{2} \times 5 \times 7 - 7 \times 4$

$= 11$ units^2

3. Let the diameter of A_1, A_2 and A_3 be d_1, d_2 and d_3.

Using Pythagoras' Theorem,

$\therefore d_1^2 = d_2^2 + d_3^2$

$A_1 = \frac{1}{2} \times \pi \times \left(\frac{d_1}{2}\right)^2$
$= \frac{\pi}{8}(d_1^2)$

$A_2 + A_3 = \frac{1}{2} \times \pi \times \left(\frac{d_2}{2}\right)^2 + \frac{1}{2} \times \pi \times \left(\frac{d_3}{2}\right)^2$
$= \frac{\pi}{8}(d_2^2 + d_3^2)$
$= \frac{\pi}{8}(d_1^2)$

Since $A_1 = A_2 + A_3$, the relatioship still holds true.

4. **(i)** Let the length of each side of the equilateral triangle be x cm, the height of the equilateral triangle be h cm.

Area of equilateral triangle
= Area of square
$= 3^2$
$= 9 \text{ cm}^2$

The height of an equilateral triangle bisects the side.

Using Pythagoras' Theorem,

$$x^2 = h^2 + \left(\frac{x}{2}\right)^2$$

$$h^2 = x^2 - \left(\frac{x}{2}\right)^2$$

$$= x - \frac{x^2}{4}$$

$$= \frac{3}{4}x^2$$

$$\therefore h = \sqrt{\frac{3}{4}x^2} \quad (\text{since } h > 0)$$

$$= \frac{\sqrt{3}}{2}x$$

$$\therefore \frac{1}{2} \times x \times \frac{\sqrt{3}}{2}x = 9$$

$$\frac{\sqrt{3}}{4}x^2 = 9$$

$$x^2 = \frac{36}{\sqrt{3}}$$

$$\therefore x = \sqrt{\frac{36}{\sqrt{3}}} \quad (\text{since } x > 0)$$

$$= 4.56 \text{ (to 3 s.f.)}$$

The length of each side of the equilateral triangle is 4.56 cm.

(ii) No. From above, $h = \frac{\sqrt{3}}{2}x$. If x is an integer, h is never an integer and therefore the area of the triangle will not be an integer. Thus, the side of the square is never an integer.

This applies for the converse.

Chapter 11 Trigonometric Ratios

TEACHING NOTES

Suggested Approach

Teachers may want to introduce this topic by stating some of the uses of trigonometry such as surveying, engineering, physics and other physical sciences etc. Teachers can also introduce this chapter from a historical perspective. For instance, teachers can show old trigonometric tables to students, and explaining how the people in the past studied trigonometry before calculators became common. Depending on the profiles on the students, teachers may want to introduce the basic trigonometric ratios one at a time, or present them together.

Section 11.1: Trigonometric Ratios

Teachers should guide the students through the activity on page 283 (see Investigation: Trigonometric Ratios). Just like Pythagoras' Theorem, it is important to emphasise that trigonometric ratios are applicable only to right-angled triangles. Students should not attempt to use trigonometric ratios in other types of triangles.

To help students to memorise the trigonometric ratios easily, teachers may wish to use the mnemonic 'TOA-CAH-SOH' (means 'Big-foot lady' in a Chinese dialect). Students may need practice to identify the opposite, adjacent and hypotenuse sides with reference to a given angle as they may find the ratios confusing at the initial stage.

In using a calculator, it is important to remind pupils to check and see that the MODE is set as DEG.

The examination requirements state students are to give answers correct to 3 significant figures and angles in degree to correct to 1 decimal place. Therefore, students should develop the habit of working with 4 or 5 significant figures and angles in degree to 2 decimal places and give the final answer correct to the required accuracy.

Section 11.2: Applications of Trigonometric Ratios to Find Unknown Sides of Right-Angled Triangles

Students are required to solve simple right-angled triangles in this section. They are expected to understand, write and express their working in explicit form before using a calculator to evaluate the expression. For example, when $\sin 72° = \frac{x}{12}$, they must first write

$x = 12 \sin 72°$
$\quad = 11.4$ (to 3 s.f.)

and not state the answer 11.4, corrected to 3 significant figures, outright.

Section 11.3: Applications of Trigonometric Ratios to Find Unknown Angles in Right-Angled Triangles

Previously, students are taught to find the sides given the angles of a right-angled triangle. Here, they will do the reverse, i.e. finding the angles of a right-angled triangle, given the sides.

Teachers should remind students to choose the correct trigonometric ratio that can be used to find the angle.

In finding an unknown side or angle, the angle properties and Pythagoras' Theorem students have picked up at this stage can be used as well. Students should practise using different approaches to a particular problem so as to appreciate the concepts they have learnt thus far.

Section 11.4 Applications of Trigonometric Ratios in Real-World Contexts

In this section, students will learn how trigonometry is used in real-life situations. Teachers are encouraged to work through as many worked examples as possible. Students should also work through some questions of similar type.

The usage of a clinometer is one of the learning experiences that students should be placed in groups and get involved in (see Investigation: Using a Clinometer to Find the Height of the Object). Teachers may want to prepare several clinometers for demonstration purposes, or ask students to make their own. Some suitable objects to measure include the heights of a tree, a school block or a flag pole. Students should be reminded to include the height they hold the clinometer above the ground in performing calculations.

Challenge Yourself

For Question 1, students should recall that the diagonal BD of a kite bisects the angles $\angle B$ and $\angle D$. In Question 2, students can substitute suitable values to see how $\sin x$ and $\cos x$ varies from 0° to 90°.

WORKED SOLUTIONS

Investigation (Trigonometric Ratios)

1. The triangle should be the size of half a paper.
2. One possible set of values is $AC = 8$ cm, $AB = 9.2$ cm, $BC = 4.6$ cm.
3. $\frac{BC}{AB} = \frac{4.6}{8} = 0.575$

   $\frac{AC}{AB} = \frac{8}{9.2} = 0.870$ (to 3 s.f.)

   $\frac{BC}{AC} = \frac{4.6}{9.2} = 0.5$
4. **(a)** The two triangles are similar. The corresponding angles in both triangles are equal.
   (b) The values of the ratios are the same.
   (c) The two right-angled triangles must be similar triangles.
5. **(a)** AB is the hypotenuse.
   (b) BC is the opposite side.
   (c) AC is the adjacent side.
6. $\frac{BC}{AB} = \frac{\text{opp}}{\text{hyp}}$, $\frac{AC}{AB} = \frac{\text{adj}}{\text{hyp}}$, $\frac{BC}{AC} = \frac{\text{opp}}{\text{adj}}$

Thinking Time (Page 284)

$\sin 50° \neq \sin 30°$, $\cos 50° \neq \cos 30°$, $\tan 50° \neq \tan 30°$

Trigonometric ratios depend on the value of the angle, as they affect the lengths of the sides of the triangle, and hence the ratios.

Teachers may want students to repeat the Investigation on page 283, replacing $\angle A = 30°$ *with* $\angle X = 50°$ to verify that the ratios are not equal.

Investigation (Using a Clinometer to Find the Height of an Object)

1. $\angle ABC = 90°$

   $\angle ABD = 90° - \angle x$

   In $\triangle ABD$, since $\angle ADB = 90°$,

   $90° - \angle x + \angle y + 90° = 180°$

   $\angle x = \angle y$
3. **(i)** Let the distance between the student and the object be k m.
   (ii) Let the angle of elevation be $a°$.
   (iii) Let the vertical height of the clinometer above the ground be h m, the vertical height between the top of the object and the clinometer be p m.

   $\therefore \tan a° = \frac{p}{k}$

   $p = k \tan a°$

   Height of the object $= h + p$

   $= h + k \tan a°$

Practise Now (Page 285)

1. **(i)** AB is the hypotenuse.
   (ii) BC is the side opposite $\angle A$.
   (iii) AC is the side adjacent to $\angle A$.
2. **(i)** $\sin P = \frac{\text{opp}}{\text{hyp}} = \frac{QR}{PQ} = \frac{3}{5}$

   (ii) $\cos P = \frac{\text{adj}}{\text{hyp}} = \frac{PR}{PQ} = \frac{4}{5}$

   (iii) $\tan P = \frac{\text{opp}}{\text{adj}} = \frac{QR}{PR} = \frac{3}{4}$

   (iv) $\sin Q = \frac{\text{opp}}{\text{hyp}} = \frac{PR}{PQ} = \frac{4}{5}$

   (v) $\cos Q = \frac{\text{adj}}{\text{hyp}} = \frac{QR}{PQ} = \frac{3}{5}$

   (vi) $\tan Q = \frac{\text{opp}}{\text{adj}} = \frac{PR}{QR} = \frac{4}{3}$
3. **(i)** $\sin X = \frac{\text{opp}}{\text{hyp}} = \frac{YZ}{XY} = \frac{a}{c}$

   (ii) $\cos X = \frac{\text{adj}}{\text{hyp}} = \frac{XZ}{XY} = \frac{b}{c}$

(iii) $\tan X = \dfrac{\text{opp}}{\text{adj}}$

$= \dfrac{YZ}{XZ}$

$= \dfrac{a}{b}$

(iv) $\sin Y = \dfrac{\text{opp}}{\text{hyp}}$

$= \dfrac{XZ}{XY}$

$= \dfrac{b}{c}$

(v) $\cos Y = \dfrac{\text{adj}}{\text{hyp}}$

$= \dfrac{YZ}{XY}$

$= \dfrac{a}{c}$

(vi) $\tan Y = \dfrac{\text{opp}}{\text{adj}}$

$= \dfrac{XZ}{YZ}$

$= \dfrac{b}{a}$

Practise Now 1

(a) Sequence of calculator keys:

[cos] [2] [4] [=]

$\therefore \cos 24° = 0.914$ (to 3 s.f.)

(b) Sequence of calculator keys:

[tan] [7] [4] [.] [6] [=]

$\therefore \tan 74.6° = 3.63$ (to 3 s.f.)

(c) Sequence of calculator keys:

[sin] [7] [2] [.] [1] [5] [=]

$\therefore \sin 72.15° = 0.952$ (to 3 s.f.)

(d) Sequence of calculator keys:

[3] [sin] [4] [8] [+] [2] [cos] [3] [9] [=]

$\therefore 3 \sin 48° + 2 \cos 39° = 3.78$ (to 3 s.f.)

(e) Sequence of calculator keys:

[5] [÷] [tan] [1] [8] [.] [3] [=]

$\therefore \dfrac{5}{\tan 18.3°} = 15.1$ (to 3 s.f.)

(f) Sequence of calculator keys:

[(] [tan] [4] [8] [.] [3] [–] [sin] [2] [8] [.] [7] [)]
[÷] [(] [cos] [1] [5] [+] [cos] [3] [5] [)] [=]

$\therefore \dfrac{\tan 48.3° - \sin 28.7°}{\cos 15° + \cos 35°} = 0.360$ (to 3 s.f.)

Practise Now 2

(a) $\sin \angle BAC = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{BC}{AB}$

$\sin 30° = \dfrac{x}{11}$

$\therefore x = 11 \sin 30°$

$= 5.5$

(b) $\sin \angle QPR = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{QR}{PQ}$

$\sin 67° = \dfrac{8.7}{y}$

$\therefore y = \dfrac{8.7}{\sin 67°}$

$= 9.45$ (to 3 s.f.)

Practise Now 3

1. $\cos \angle YXZ = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{XZ}{XY}$

$\cos 58° = \dfrac{4.9}{XY}$

$\therefore XY = \dfrac{4.9}{\cos 58°}$

$= 9.25$ m (to 3 s.f.)

2. $\cos \angle ABC = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{BC}{AB}$

$\cos 47° = \dfrac{BC}{18}$

$\therefore BC = 18 \cos 47°$

$= 12.3$ m (to 3 s.f.)

Practise Now 4

1. $\tan \angle QPR = \dfrac{\text{opp}}{\text{adj}} = \dfrac{PR}{PQ}$

$\tan 53° = \dfrac{QR}{20}$

$\therefore QR = 20 \tan 53°$

$= 26.5$ cm (to 3 s.f.)

2. $\tan \angle YXZ = \dfrac{\text{opp}}{\text{adj}} = \dfrac{YZ}{XZ}$

$\tan 35° = \dfrac{12}{XZ}$

$\therefore XZ = \dfrac{12}{\tan 35°}$

$= 17.1$ cm (to 3 s.f.)

Practise Now 5

1. (i) $\cos \angle ABC = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{BC}{AB}$

$\cos 27° = \dfrac{BC}{23}$

$\therefore BC = 23 \cos 27°$

$= 20.5$ m (to 3 s.f.)

(ii) Method 1:

$\sin \angle ABC = \frac{\text{opp}}{\text{hyp}} = \frac{AC}{AB}$

$\sin 27° = \frac{AC}{23}$

$\therefore AC = 23 \sin 27°$

$= 10.4$ m (to 3 s.f.)

Method 2:

In $\triangle ABC$, $\angle C = 90°$.

Using Pythagoras' Theorem,

$AB^2 = AC^2 + BC^2$

$23^2 = AC^2 + 20.49^2$

$AC^2 = 23^2 - 20.49^2$

$= 529 - 419.8401$

$= 109.1599$

$\therefore AC = \sqrt{109.1599}$ (since $AC > 0$)

$= 10.4$ m (to 3 s.f.)

2. **(i)** $\cos \angle QPR = \frac{\text{adj}}{\text{hyp}} = \frac{PQ}{PR}$

$\cos 46° = \frac{PQ}{9.2}$

$\therefore PQ = 9.2 \cos 46°$

$= 6.39$ m (to 3 s.f.)

(ii) $\sin \angle QPR = \frac{\text{opp}}{\text{hyp}} = \frac{QR}{PR}$

$\sin 46° = \frac{QR}{9.2}$

$\therefore QR = 9.2 \sin 46°$

$= 6.62$ m (to 3 s.f.)

(iii) $\sin \angle PSR = \frac{\text{opp}}{\text{hyp}} = \frac{PR}{PS}$

$\sin 48° = \frac{9.2}{PS}$

$\therefore PS = \frac{9.2}{\sin 48°}$

$= 12.4$ m (to 3 s.f.)

(iv) $\tan \angle PSR = \frac{\text{opp}}{\text{adj}} = \frac{PR}{RS}$

$\tan 48° = \frac{9.2}{RS}$

$\therefore RS = \frac{9.2}{\tan 48°}$

$= 8.28$ m (to 3 s.f.)

(v) Perimeter of the quadrilateral $PQRS$

$= PQ + QR + RS + PS$

$= 6.391 + 6.618 + 12.38 + 8.284$

$= 33.7$ m (to 3 s.f.)

(vi) Area of the quadrilateral $PQRS$

$=$ Area of $\triangle PQR$ + Area of $\triangle PRS$

$= \frac{1}{2} \times PQ \times QR + \frac{1}{2} \times PR \times RS$

$= \frac{1}{2} \times 6.391 \times 6.618 + 1.2 \times 9.2 \times 8.284$

$= 59.3 \text{ m}^2$

Practise Now (Page 297)

(a) Sequence of calculator keys:

2nd F	sin	0	.	7	8	=

$\therefore \angle A = \sin^{-1}(0.78)$

$= 51.3°$ (to 1 d.p.)

(b) Sequence of calculator keys:

2nd F	cos	0	.	3	5	=

$\therefore \angle B = \cos^{-1}(0.35)$

$= 69.5°$ (to 1 d.p.)

(c) Sequence of calculator keys:

2nd F	tan	1	.	2	3	=

$\therefore \angle C = \tan^{-1}(1.23)$

$= 50.9°$ (to 1 d.p.)

Practise Now 6

(a) $\sin x° = \frac{11}{19}$

$\therefore x° = \sin^{-1}\left(\frac{11}{19}\right)$

$= 35.4°$ (to 1 d.p.)

$\therefore x = 35.4$

(b) $\cos y° = \frac{9.1}{21}$

$\therefore y° = \cos^{-1}\left(\frac{9.1}{21}\right)$

$= 64.3°$ (to 1 d.p.)

$\therefore y = 64.3$

(c) $\tan z° = \frac{29.2}{18.8}$

$\therefore z° = \tan^{-1}\left(\frac{29.2}{18.8}\right)$

$= 57.2°$ (to 1 d.p.)

$\therefore z = 57.2$

Practise Now 7

1. **(i)** In $\triangle KBC$,

$\tan 42° = \frac{BC}{7.6}$

$\therefore BC = 7.6 \tan 42°$

$= 6.843$ m (to 4 s.f.)

In $\triangle ABC$,

$\sin \angle BAC = \frac{6.843}{17.3}$

$\therefore \angle BAC = \sin^{-1}\left(\frac{6.843}{17.3}\right)$

$= 23.3°$ (to 1 d.p.)

(ii) In $\triangle ABC$, $\angle C = 90°$.

Using Pythagoras' Theorem,

$$AB^2 = AC^2 + BC^2$$
$$17.3^2 = AC^2 + 6.843^2$$
$$AC^2 = 17.3^2 - 6.843^2$$
$$= 299.29 - 46.826\,649$$
$$= 252.463\,351$$
$$\therefore AC = \sqrt{252.463\,351} \quad (\text{since } AC > 0)$$
$$= 15.89 \text{ m (to 4 s.f.)}$$
$$\therefore KA = AC - KC$$
$$= 15.89 - 7.6$$
$$= 8.29 \text{ m (to 3 s.f.)}$$

2. **(i)** In $\triangle PQS$,

$$\tan \angle PQS = \frac{3}{4}$$
$$\therefore \angle PQS = \tan^{-1}\left(\frac{3}{4}\right)$$
$$= 36.9° \text{ (to 1 d.p.)}$$

(ii) In $\triangle PQS$, $\angle P = 90°$.

Using Pythagoras' Theorem,

$$QS^2 = PQ^2 + PS^2$$
$$= 4^2 + 3^2$$
$$= 16 + 9$$
$$= 25$$
$$\therefore QS = \sqrt{25} \quad (\text{since } QS > 0)$$
$$= 5$$
$$\angle QRS = 180° - 90° - 36.87° \text{ (int. } \angle\text{s. } PQ \,//\, SR)$$
$$= 53.13° \text{ (to 2 d.p.)}$$

In $\triangle QRS$,

$$\tan 53.13° = \frac{5}{QR}$$
$$\therefore QR = \frac{5}{\tan 53.13°}$$
$$= 3.75 \text{ cm (to 3 s.f.)}$$

Practise Now 8

1. $\tan 54° = \dfrac{PQ}{28}$

$$\therefore PQ = 28 \tan 54°$$
$$= 38.5 \text{ m (to 3 s.f.)}$$

The height of the flagpole is 38.5 m.

2. $\tan 37° = \dfrac{40}{FA}$

$$\therefore FA = \frac{40}{\tan 37°}$$
$$= 53.1 \text{ m (to 3 s.f.)}$$

The distance is 53.1 m.

3. $\tan 40° = \dfrac{18.4 - 1.6}{AB}$

$$\therefore AB = \frac{16.8}{\tan 40°}$$
$$= 20.0 \text{ m (to 3 s.f.)}$$

Practise Now 9

$$\tan \angle TAB = \frac{13}{24}$$
$$\therefore \angle TAB = \tan^{-1}\left(\frac{13}{24}\right)$$
$$= 28.4° \text{ (to 1 d.p.)}$$

Practise Now 10

In $\triangle CHW$, $\angle H = 90°$.

$$\tan 32.4° = \frac{26.5}{CH}$$
$$\therefore CH = \frac{26.5}{\tan 32.4°}$$
$$= 41.76 \text{ m (to 4 s.f.)}$$

$BW = CH = 41.76$ m

In $\triangle ABW$,

$$\tan 49.6° = \frac{AB}{41.76}$$
$$\therefore AB = 41.76 \tan 49.6°$$
$$= 49.07 \text{ m (to 4 s.f.)}$$

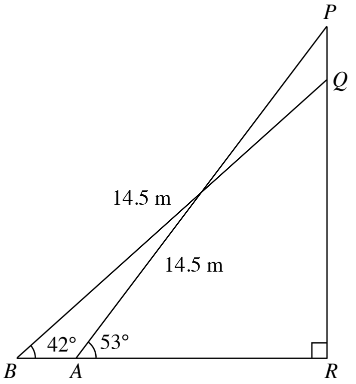
$BC = WH = 26.5$ m

$$\therefore AB = AB + BC$$
$$= 49.07 + 26.5$$
$$= 75.6 \text{ m}$$

The height of the mast is 75.6 m.

Practise Now 11

In the figure, AP and BQ represent the two positions of the straight pole and PQ represents the signboard.



In $\triangle APR$,

$$\sin 53° = \frac{PR}{14.5}$$
$$\therefore PR = 14.5 \sin 53°$$
$$= 11.58 \text{ m (to 4 s.f.)}$$

In $\triangle BQR$,

$$\sin 42° = \frac{QR}{14.5}$$
$$\therefore QR = 14.5 \sin 42°$$
$$= 9.702 \text{ m (to 4 s.f.)}$$

$\therefore PQ = PR - QR$
$= 11.58 - 9.702$
$= 1.88$ m (to 3 s.f.)

The height of the signboard is 1.88 m.

Exercise 11A

1. (a) (i) PQ is the hypotenuse.
(ii) PR is the side opposite $\angle a$.
(iii) QR is the side adjacent to $\angle a$.
(b) (i) XY is the hypotenuse.
(ii) XZ is the side opposite $\angle a$.
(iii) YZ is the side adjacent to $\angle a$.

2. (a) (i) $\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{BC}{AB} = \frac{5}{13}$

(ii) $\cos A = \frac{\text{adj}}{\text{hyp}} = \frac{AC}{AB} = \frac{12}{13}$

(iii) $\tan A = \frac{\text{opp}}{\text{adj}} = \frac{BC}{AC} = \frac{5}{12}$

(iv) $\sin B = \frac{\text{opp}}{\text{hyp}} = \frac{AC}{AB} = \frac{12}{13}$

(v) $\cos B = \frac{\text{adj}}{\text{hyp}} = \frac{BC}{AB} = \frac{5}{13}$

(vi) $\tan B = \frac{\text{opp}}{\text{adj}} = \frac{AC}{BC} = \frac{12}{5}$

(b) (i) $\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{BC}{AB} = \frac{24}{25}$

(ii) $\cos A = \frac{\text{adj}}{\text{hyp}} = \frac{AC}{AB} = \frac{7}{25}$

(iii) $\tan A = \frac{\text{opp}}{\text{adj}} = \frac{BC}{AC} = \frac{24}{7}$

(iv) $\sin B = \frac{\text{opp}}{\text{hyp}} = \frac{AC}{AB} = \frac{7}{25}$

(v) $\cos B = \frac{\text{adj}}{\text{hyp}} = \frac{BC}{AB} = \frac{24}{25}$

(vi) $\tan B = \frac{\text{opp}}{\text{adj}} = \frac{AC}{BC} = \frac{7}{24}$

3. (a) (i) $\sin P = \frac{\text{opp}}{\text{hyp}} = \frac{QR}{PQ} = \frac{y}{z}$

(ii) $\cos P = \frac{\text{adj}}{\text{hyp}} = \frac{PR}{PQ} = \frac{x}{z}$

(iii) $\tan P = \frac{\text{opp}}{\text{adj}} = \frac{QR}{PR} = \frac{y}{x}$

(iv) $\sin Q = \frac{\text{opp}}{\text{hyp}} = \frac{PR}{PQ} = \frac{x}{z}$

(v) $\cos Q = \frac{\text{adj}}{\text{hyp}}$
$= \frac{QR}{PQ}$
$= \frac{y}{z}$

(vi) $\tan Q = \frac{\text{opp}}{\text{adj}}$
$= \frac{PR}{QR}$
$= \frac{x}{y}$

(b) **(i)** $\sin P = \frac{\text{opp}}{\text{hyp}}$
$= \frac{QR}{PQ}$
$= \frac{z}{x}$

(ii) $\cos P = \frac{\text{adj}}{\text{hyp}}$
$= \frac{PR}{PQ}$
$= \frac{y}{x}$

(iii) $\tan P = \frac{\text{opp}}{\text{adj}}$
$= \frac{QR}{PR}$
$= \frac{z}{y}$

(iv) $\sin Q = \frac{\text{opp}}{\text{hyp}}$
$= \frac{PR}{PQ}$
$= \frac{y}{x}$

(v) $\cos Q = \frac{\text{adj}}{\text{hyp}}$
$= \frac{QR}{PQ}$
$= \frac{z}{x}$

(vi) $\tan Q = \frac{\text{opp}}{\text{adj}}$
$= \frac{PR}{QR}$
$= \frac{y}{z}$

4. **(a)** Sequence of calculator keys:

[tan] [4] [7] [=]

∴ tan 47° = 1.07 (to 3 s.f.)

(b) Sequence of calculator keys:

[sin] [7] [5] [.] [3] [=]

∴ sin 75.3° = 0.967 (to 3 s.f.)

(c) Sequence of calculator keys:

[cos] [3] [0] [.] [1] [9] [=]

∴ cos 30.19° = 0.864 (to 3 s.f.)

(d) Sequence of calculator keys:

[sin] [3] [5] [+] [cos] [4] [9] [=]

∴ sin 35° + cos 49° = 1.23 (to 3 s.f.)

(e) Sequence of calculator keys:

[2] [cos] [4] [2] [.] [3] [+] [3] [sin] [1] [6] [.] [8] [=]

∴ 2 cos 42.3° + 3 sin 16.8° = 2.35 (to 3 s.f.)

(f) Sequence of calculator keys:

[sin] [7] [1] [.] [6] [×] [tan] [1] [6] [.] [7] [=]

∴ sin 71.6° × tan 16.7° = 0.285 (to 3 s.f.)

5. **(a)** Sequence of calculator keys:

[5] [tan] [6] [1] [.] [4] [÷] [2] [cos] [1] [0] [.] [3] [=]

$\therefore \frac{5\tan 61.4°}{2\cos 10.3°} = 4.66$ (to 3 s.f.)

(b) Sequence of calculator keys:

[4] [(] [sin] [2] [2] [.] [5] [)] [x^2] [÷]
[cos] [6] [7] [.] [5] [=]

$\therefore \frac{4(\sin 22.5°)^2}{\cos 67.5°} = 1.53$ (to 3 s.f.)

(c) Sequence of calculator keys:

[(] [tan] [1] [5] [+] [cos] [3] [3] [)] [÷]
[sin] [7] [8] [.] [4] [=]

$\therefore \frac{\tan 15° + \cos 33°}{\sin 78.4°} = 1.13$ (to 3 s.f.)

(d) Sequence of calculator keys:

[tan] [4] [7] [.] [9] [÷] [(] [cos] [8] [4]
[–] [sin] [6] [3] [)] [=]

$\therefore \frac{\tan 47.9°}{\cos 84° - \sin 63°} = -1.41$ (to 3 s.f.)

(e) Sequence of calculator keys:

[(] [cos] [6] [7] [+] [sin] [8] [9] [)] [÷] [(] [tan]
[6] [3] [.] [4] [×] [cos] [1] [5] [.] [5] [)] [=]

$\therefore \frac{\cos 67° + \sin 89°}{\tan 63.4° \times \cos 15.5°} = 0.723$ (to 3 s.f.)

(f) Sequence of calculator keys:

[(] [sin] [2] [4] [.] [6] [÷] [cos] [6] [2] [.] [1]
[)] [÷] [(] [tan] [2] [1] [+] [cos] [1] [4] [)] [=]

$\therefore \frac{\sin 24.6° \div \cos 62.1°}{\tan 21° + \cos 14°} = 0.657$ (to 3 s.f.)

(g) Sequence of calculator keys:

(sin 5 7 – cos 7 3) ÷ (tan 1 5 . 3 × sin 8 3 . 4) =

$\therefore \dfrac{\sin 57° - \cos 73°}{\tan 15.3° \times \sin 83.4°} = 2.01$ (to 3 s.f.)

(h) Sequence of calculator keys:

(cos 2 4 . 7 × sin 3 5 . 1) ÷ tan 5 7 – cos 1 5) =

$\therefore \dfrac{\cos 24.7° \times \sin 35.1°}{\tan 57° - \cos 15°} = 0.910$ (to 3 s.f.)

Exercise 11B

1. **(a)** $\sin 67° = \dfrac{a}{15}$

$\therefore a = 15 \sin 67°$

$= 13.8$ (to 3 s.f.)

(b) $\sin 15° = \dfrac{9.7}{b}$

$\therefore b = \dfrac{9.7}{\sin 15°}$

$= 37.5$ (to 3 s.f.)

2. **(a)** $\cos 36° = \dfrac{a}{13.5}$

$\therefore a = 13.5 \cos 36°$

$= 10.9$ (to 3 s.f.)

(b) $\cos 61° = \dfrac{17}{b}$

$\therefore b = \dfrac{17}{\cos 61°}$

$= 35.1$ (to 3 s.f.)

3. **(a)** $\tan 28° = \dfrac{a}{14}$

$\therefore a = 14 \tan 28°$

$= 7.44$ (to 3 s.f.)

(b) $\tan 62.5° = \dfrac{13}{b}$

$\therefore b = \dfrac{13}{\tan 62.5°}$

$= 6.77$ (to 3 s.f.)

4. **(a)** $\sin 34° = \dfrac{a}{12}$

$\therefore a = 12 \sin 34°$

$= 6.71$ (to 3 s.f.)

$\therefore b = 12 \cos 34°$

$= 9.95$ (to 3 s.f.)

(b) $\cos 43° = \dfrac{c}{16}$

$\therefore c = 16 \cos 43°$

$= 11.7$ (to 3 s.f.)

$\sin 43° = \dfrac{d}{16}$

$\therefore d = 16 \sin 43°$

$= 10.9$ (to 3 s.f.)

(c) $\tan 44.2° = \dfrac{e}{7}$

$\therefore e = 7 \tan 44.2°$

$= 6.81$ (to 3 s.f.)

$\cos 44.2° = \dfrac{7}{f}$

$\therefore f = \dfrac{7}{\cos 44.2°}$

$= 9.76$ (to 3 s.f.)

(d) $\tan 21.5° = \dfrac{8.9}{g}$

$\therefore g = \dfrac{8.9}{\tan 21.5°}$

$= 22.6$ (to 3 s.f.)

$\sin 21.5° = \dfrac{8.9}{h}$

$\therefore h = \dfrac{8.9}{\sin 21.5°}$

$= 24.3$ (to 3 s.f.)

5. **(i)** In $\triangle ABH$,

$\sin 56° = \dfrac{AH}{8.9}$

$\therefore AH = 8.9 \sin 56°$

$= 7.38$ m (to 3 s.f.)

(ii) **Method 1:**

In $\triangle ABH$,

$\cos 56° = \dfrac{BH}{8.9}$

$\therefore BH = 8.9 \cos 56°$

$= 4.977$ m (to 4 s.f.)

In $\triangle ABC$,

$\cos 56° = \dfrac{8.9}{BC}$

$\therefore BC = \dfrac{8.9}{\cos 56°}$

$= 15.92$ m (to 4 s.f.)

$\therefore HC = BC - BH$

$= 15.92 - 4.977$

$= 10.9$ m (to 3 s.f.)

Method 2:

In $\triangle ABC$,

$\angle ACB = 180° - 90° - 56°$ ($\angle$ sum of $\triangle ABC$)

$= 34°$

In $\triangle AHC$, $\angle ACH = 34°$.

From **(i)**,

$\tan 34° = \dfrac{7.378}{HC}$

$\therefore HC = \dfrac{7.378}{\tan 34°}$

$= 10.9$ m (to 3 s.f.)

6. **(i)** In $\triangle QST$, $\angle T = 180° - 90°$ (adj. $\angle$s a str. line)
$= 90°$

$\sin 60° = \frac{TQ}{25}$

$\therefore TQ = 25 \sin 60°$
$= 21.7$ cm (to 3 s.f.)

(ii) **Method 1:**

In $\triangle QST$,

$\cos 60° = \frac{TS}{25}$

$TS = 25 \cos 60°$
$= 12.5$ cm

In $\triangle PQT$, $\angle Q = 180° - 90°$ (adj. $\angle$s on a str. line)
$= 90°$

$\cos 60° = \frac{25}{PS}$

$\therefore PS = \frac{25}{\cos 60°}$
$= 50$ cm

$\therefore PT = PS - TS$
$= 50 - 12.5$
$= 37.5$ cm

Method 2:

In $\triangle PQS$,

$\angle PQS = 180° - 90°$ (adj. $\angle$s on a str. line)
$= 90°$

$\angle QPS = 180° - 90° - 60°$ ($\angle$ sum of $\triangle PQS$)
$= 30°$

In $\triangle PQT$, $\angle QPT = 30°$.

From **(i)**,

$\tan 30° = \frac{21.65}{PT}$

$\therefore PT = \frac{21.65}{\tan 30°}$
$= 37.5$ cm (to 3 s.f.)

(iii) In $\triangle PQR$,

$\tan 60° = \frac{PQ}{25}$

$\therefore PQ = 25 \tan 60°$
$= 43.30$ cm (to 4 s.f.)

In $\triangle QRS$,

$\tan 45° = \frac{QR}{25}$

$\therefore QR = 25 \tan 45°$
$= 25$ cm

$\therefore PR = PQ + QR$
$= 43.30 + 25$
$= 68.3$ cm (to 3 s.f.)

7. **(i)** In $\triangle VWX$,

$\cos 63° = \frac{WX}{154}$

$\therefore WX = 154 \cos 63°$
$= 69.91$ m

$\sin 63° = \frac{VX}{154}$

$\therefore VX = 154 \sin 63°$
$= 137.22$ cm

In $\triangle VXY$, $\angle Y = 180° - 90°$ (adj. $\angle$s on a str. line)
$= 90°$

Using Pythagoras' Theorem,

$VX^2 = XY^2 + VY^2$

$137.22^2 = XY^2 + 88^2$

$XY^2 = 137.22^2 - 88^2$

$\therefore XY = \sqrt{137.22^2 - 88^2}$ (since $XY > 0$)
$= 105.29$

In $\triangle VYZ$,

$\tan 46° = \frac{88}{YZ}$

$\therefore YZ = \frac{88}{\tan 46°}$
$= 84.98$

$\sin 46° = \frac{88}{VZ}$

$\therefore VZ = \frac{88}{\sin 46°}$
$= 122.33$

$\therefore$ Perimeter of the figure
$= WX + XY + YZ + VZ + WV$
$= 69.91 + 105.29 + 84.98 + 122.33 + 154$
$= 537$ cm (to 3 s.f.)

(ii) Area of the figure
= Area of $\triangle WXV$ + Area of $\triangle VXZ$
$= \frac{1}{2} \times WX \times VX + \frac{1}{2} \times XZ \times VY$
$= \frac{1}{2} \times 69.91 \times 137.22 + \frac{1}{2} \times (105.29 + 84.98) \times 88$
$= 13\,200$ m^2 (to 3 s.f.)

8. Since y is inversely proportiona to $(\tan x)^2$,

then $y = \frac{k}{(\tan x)^2}$, where k is a constant.

When $x = 30°$, $y = 2$,

$2 = \frac{k}{(\tan 30°)^2}$

$k = 2(\tan 30°)^2$
$= \frac{2}{3}$

When $x = 2(30°) = 60°$,

$\therefore y = \frac{2}{3} \times \frac{1}{(\tan 60°)^2}$
$= \frac{2}{3} \times \frac{1}{3}$
$= \frac{2}{9}$

Exercise 11C

1. (a) Sequence of calculator keys:

| 2nd F | sin | 0 | . | 5 | 2 | 7 | = |

$\therefore \angle A = \sin^{-1}(0.527)$
$= 31.8°$ (to 1 d.p.)

(b) Sequence of calculator keys:

| 2nd F | cos | 0 | . | 7 | 2 | 5 | = |

$\therefore \angle B = \cos^{-1}(0.725)$
$= 43.5°$ (to 1 d.p.)

(c) Sequence of calculator keys:

| 2nd F | tan | 2 | . | 5 | 6 | = |

$\therefore \angle C = \tan^{-1}(2.56)$
$= 68.7°$ (to 1 d.p.)

2. (a) $\sin a° = \frac{12}{26}$

$\therefore a° = \sin^{-1}\left(\frac{12}{26}\right)$
$= 27.5°$ (to 1 d.p.)
$\therefore a = 27.5$

(b) $\cos b° = \frac{10}{17}$

$\therefore b° = \cos^{-1}\left(\frac{10}{17}\right)$
$= 54.0°$ (to 1 d.p.)
$\therefore b = 54.0$

(c) $\tan c° = \frac{27}{11}$

$\therefore c° = \tan^{-1}\left(\frac{27}{11}\right)$
$= 67.8°$ (to 1 d..)
$\therefore c = 67.8$

(d) $\cos d° = \frac{17.6}{20}$

$\therefore d° = \cos^{-1}\left(\frac{17.6}{20}\right)$
$= 28.4°$ (to 1 d.p.)
$\therefore b = 28.4$

(e) $\sin e° = \frac{15}{22.7}$

$\therefore e° = \sin^{-1}\left(\frac{15}{22.7}\right)$
$= 41.4°$ (to 1 d.p.)
$\therefore e = 41.4$

(f) $\tan f° = \frac{12.5}{14}$

$\therefore f° = \tan^{-1}\left(\frac{12.5}{14}\right)$
$= 41.8°$ (to 1 d.p.)
$\therefore f = 41.8$

(g) $\tan g° = \frac{14.7}{12.9}$

$\therefore g° = \tan^{-1}\left(\frac{14.7}{12.9}\right)$
$= 48.7°$ (to 1 d.p.)
$\therefore g = 48.7$

(h) $\cos h° = \frac{15.8}{21.2}$

$\therefore h° = \cos^{-1}\left(\frac{15.8}{21.2}\right)$
$= 41.8°$ (to 1 d.p.)
$\therefore h = 41.8$

(i) $\sin i° = \frac{32.75}{41.62}$

$\therefore i° = \sin^{-1}\left(\frac{32.75}{41.62}\right)$
$= 51.9°$ (to 1 d.p.)
$\therefore i = 51.9$

3. (i) Let the point where the perpendicular line from A to CD meets CD be X.
In $\triangle AXD$,
$\angle AXD = 90°$ and $XD = (7 - 4) = 3$ m

$\cos \angle ADC = \frac{3}{7}$

$\therefore \angle ADC = \cos^{-1}\left(\frac{3}{7}\right)$
$= 64.6°$ (to 1 d.p.)

(ii) In $\triangle AXD$, $\angle X = 90°$.
Using Pythagoras' Theorem,
$AD^2 = AX^2 + XD^2$
$7^2 = AX^2 + 3^2$
$AX^2 = 7^2 - 3^2$
$= 49 - 9$
$= 40$
$\therefore AX = \sqrt{40}$ (since $AX > 0$)
$= 6.32$ m (to 3 s.f.)
$\therefore BC = AX$
$= 6.32$ m

4. (i) In $\triangle HMN$,

$\cos 38° = \frac{MN}{9.2}$

$\therefore MN = 9.2 \cos 38°$
$= 7.250$ cm (to 4 s.f.)

In $\triangle LMN$,

$\sin \angle MLN = \frac{7.250}{15.5}$

$\therefore \angle MLN = \sin^{-1}\left(\frac{7.250}{15.5}\right)$
$= 27.9°$ (to 1 d.p.)

(ii) In $\triangle HMN$,

$\sin 38° = \dfrac{HN}{9.2}$

$\therefore HN = 9.2 \sin 38°$

$= 5.664$ cm (to 4 s.f.)

In $\triangle LMN$, $\angle N = 90°$.

Using Pythagoras' Theorem,

$LM^2 = MN^2 + LN^2$

$15.5^2 = 7.250^2 + LN^2$

$LN^2 = 15.5^2 - 7.250^2$

$= 240.25 - 52.5625$

$= 187.6875$

$\therefore LN = \sqrt{187.6875}$ (since $LN > 0$)

$= 13.70$ cm (to 4 s.f.)

$\therefore HL = LN - HN$

$= 13.70 - 5.664$

$= 8.04$ cm (to 3 s.f.)

5. **(i)** In $\triangle PQR$,

$\sin \angle PQR = \dfrac{7.6}{17.4}$

$\therefore \angle PQR = \sin^{-1}\left(\dfrac{7.6}{17.4}\right)$

$= 25.90°$ (to 2 d.p.)

In $\triangle PQK$,

$\angle QPK = 180° - 137° - 25.90°$ ($\angle$ sum of $\triangle PQK$)

$= 17.1°$ (to 1 d.p.)

(ii) In $\triangle PKR$,

$\angle PRK = 180° - 137°$ (adj. $\angle$s on a str. line)

$= 43°$

$\tan 43° = \dfrac{7.6}{KR}$

$\therefore KR = \dfrac{7.6}{\tan 43°}$

$= 8.150$ m (to 4 s.f.)

In $\triangle PQR$, $\angle R = 90°$.

Using Pythagoras' Theorem,

$PQ^2 = QR^2 + PR^2$

$17.4^2 = QR^2 + 7.6^2$

$QR^2 = 17.4^2 - 7.6^2$

$= 302.76 - 57.76$

$= 245$

$\therefore QR = \sqrt{245}$ (since $QR > 0$)

$= 15.65$ m (to 4 s.f.)

$\therefore QK = QR - KR$

$= 15.65 - 8.150$

$= 7.50$ m (to 3 s.f.)

6. $\dfrac{TH}{HU} \times 100\% = 120\%$

$\dfrac{TH}{HU} = \dfrac{6}{5}$

$\therefore TH = \dfrac{6}{5+6} \times TU$

$= \dfrac{6}{11} \times 11$

$= 6$ cm

$HU = 11 - 6$

$= 5$ cm

In $\triangle STH$,

Area of $\triangle STH = 21$

$\dfrac{1}{2} \times 6 \times SH = 21$

$SH = 7$

$\tan \angle STH = \dfrac{7}{6}$

$\therefore \angle STH = \tan^{-1}\left(\dfrac{7}{6}\right)$

$= 49.40°$ (to 2 d.p.)

In $\triangle SUH$,

$\tan \angle SUH = \dfrac{7}{5}$

$\therefore \angle SUH = \tan^{-1}\left(\dfrac{7}{5}\right)$

$= 54.46°$ (to 2 d.p.)

In $\triangle STU$,

$\angle TSU = 180° - 49.40° - 54.46°$ ($\angle$ sum of $\triangle STU$)

$= 76.1°$ (to 1 d.p.)

7. **(i)** $\dfrac{WK}{ZY} = \dfrac{6}{13}$

$\therefore WK = \dfrac{6}{13} \times 7.8$

$= 3.6$ m

In $\triangle KWZ$,

$\tan 62° = \dfrac{3.6}{KZ}$

$\therefore KZ = \dfrac{3.6}{\tan 62°}$

$= 1.914$ m (to 4 s.f.)

Let the point where the perpendicular line from X to ZY meets ZY be L.

$LY = 7.8 - 1.914 - 4.7$

$= 1.186$ m

In $\triangle LXY$,

$\tan \angle XYL = \dfrac{3.6}{1.186}$

$\tan \angle XYZ = \dfrac{3.6}{1.186}$

$\therefore \angle XYZ = \tan^{-1}\left(\dfrac{3.6}{1.186}\right)$

$= 71.8°$ (to 1 d.p.)

(ii) In $\triangle KWZ$,

$\sin 62° = \frac{3.6}{WZ}$

$\therefore WZ = \frac{3.6}{\sin 62°}$

$= 4.077$ m (to 4 s.f.)

In $\triangle LXY$,

$\sin 71.77° = \frac{3.6}{XY}$

$\therefore XY = \frac{3.6}{\sin 71.77}$

$= 3.790$ m (to 4 s.f.)

$\therefore$ Perimeter of the trapezium $WXYZ$

$= WX + XY + YZ + WZ$

$= 4.7 + 3.790 + 7.8 + 4.077$

$= 20.4$ m (to 3 s.f.)

8. Let AH be h units.

In $\triangle ABH$,

$\tan 35° = \frac{h}{BH}$

$\therefore BH = \frac{h}{\tan 35°}$

$HC = \frac{2h}{\tan 35°}$

In $\triangle ACH$,

$\tan \angle ACH = \frac{h}{\frac{2h}{\tan 35°}}$

$\tan \angle ACB = \frac{\tan 35°}{2}$

$\therefore \angle ACB = \tan^{-1}\left(\frac{\tan 35°}{2}\right)$

$= 19.3°$ (to 1 d.p.)

Exercise 11D

1. $\tan 32° = \frac{TB}{34}$

$\therefore TB = 34 \tan 32°$

$= 21.2$ m (to 3 s.f.)

The height of the Christmas tree is 21.2 m.

2. $\tan 27° = \frac{7.7}{AQ}$

$\therefore AQ = \frac{7.7}{\tan 27°}$

$= 15.1$ m (to 3 s.f.)

The distance AQ is 15.1 m.

3. $\cos 53° = \frac{AB}{120}$

$\therefore AB = 120 \cos 53°$

$= 72.2$ m (to 3 s.f.)

The distance AB is 72.2 m.

4. $\tan \angle PRQ = \frac{82}{62}$

$\therefore \angle PRQ = \tan^{-1}\left(\frac{82}{62}\right)$

$= 52.9°$ (to 1 d.p.)

5. **(i)** Let the height of the nail above the ground be h m.

$\sin 60° = \frac{h}{5}$

$\therefore h = 5 \sin 60°$

$= 4.33$ (to 3 s.f.)

The nail is 4.33 m above the ground.

(ii) Let the distance of the foot of the ladder from the base of the wall be d m.

$\cos 60° = \frac{d}{5}$

$\therefore d = 5 \cos 60°$

$= 2.5$

The distance of the foot of the ladder from the base of the wall is 2.5 m.

6. Let the angle the rope makes with the water be $a°$.

$\sin a° = \frac{3.5}{12}$

$\therefore a° = \sin^{-1}\left(\frac{3.5}{12}\right)$

$= 17.0°$ (to 1 d.p.)

The rope makes an angle of 170° with the water.

7. Let the point where the horizontal line from Lixin's eyes meets the status be X.

$\tan 42° = \frac{MX}{7.05}$

$\therefore MX = 7.05 \tan 42°$

$= 6.348$ m (to 4 s.f.)

$\therefore$ Height of the statue $= 6.348 + 1.55$

$= 7.90$ m (to 3 s.f.)

The height of the statue is 7.90 m.

8. In $\triangle HRW$,

$\tan 24.3° = \frac{8}{HR}$

$\therefore HR = \frac{8}{\tan 24.3°}$

$= 17.72$ m (to 4 s.f.)

$WQ = HR = 17.72$ m

In $\triangle PQW$,

$\tan 35.4° = \frac{PQ}{17.72}$

$\therefore PQ = 17.72 \tan 35.4°$

$= 12.59$ m (to 4 s.f.)

$\therefore$ Height of the flagpole PR

$= PQ + QR$

$= 12.59 + 8$

$= 20.6$ m (to 3 s.f.)

9. Let the angle the plank makes with the wall be $a°$.

$$\cos a° = \frac{1.8}{4 - 1.2}$$

$$\therefore a° = \cos^{-1}\left(\frac{1.8}{2.8}\right)$$

$$= 50.0° \text{ (to 1 d.p.)}$$

The plank makes an angle of 50.0° with the wall.

10. Let the height in which the pendulum rises above Y be h cm.

$$\cos\left(\frac{30°}{2}\right) = \frac{45 - h}{45}$$

$$\cos 15° = \frac{45 - h}{45}$$

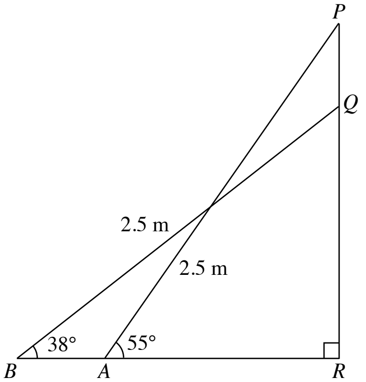
$$45 \cos 15° = 45 - h$$

$$\therefore h = 45 - 45 \cos 15°$$

$$= 1.53 \text{ m (to 3 s.f.)}$$

The height is 1.53 cm.

11. In the figure, AP and BQ represent the two positions of the ladder and PQ represents the window.



In $\triangle APR$,

$$\sin 55° = \frac{PR}{2.5}$$

$$\therefore PR = 2.5 \sin 55°$$

$$= 2.048 \text{ m (to 4 s.f.)}$$

In $\triangle BQR$,

$$\sin 38° = \frac{QR}{2.5}$$

$$\therefore QR = 2.5 \sin 38°$$

$$= 1.539 \text{ (to 4 s.f.)}$$

$$\therefore PQ = PQ - QR$$

$$= 2.048 - 1.539$$

$$= 0.509 \text{ m (to 3 s.f.)}$$

$$= 50.9 \text{ cm}$$

The height of the window is 50.9 cm.

12. (i) In $\triangle ACH$,

$$\sin 35° = \frac{CH}{36}$$

$$\therefore CH = 36 \sin 35°$$

$$\therefore CD = 36 \sin 35° + 18$$

$$= 38.6 \text{ m (to 3 s.f.)}$$

(ii) In $\triangle ACH$,

$$\cos 35° = \frac{AH}{36}$$

$$\therefore AH = 36 \cos 35°$$

$$\therefore AF = AH - FH$$

$$= AH - GD$$

$$= 36 \cos 35° - 20$$

In $\triangle AEF$, $\angle A = 90°$.

Using Pythagoras' Theorem,

$$AE^2 = AF^2 + EF^2$$

$$36^2 = (36 \cos 35° - 20)^2 + EF^2$$

$$EF^2 = 36^2 - (36 \cos 35° - 20)^2$$

$$\therefore EF = \sqrt{36^2 - (36 \cos 35° - 20)^2} \quad \text{(since } EF > 0\text{)}$$

$$= 34.7 \text{ m (to 3 s.f.)}$$

(iii) In $\triangle AEF$,

$$\cos \angle EAF = \frac{36 \cos 35° - 20}{36}$$

$$\therefore \angle EAF = \cos^{-1}\left(\frac{36 \cos 35° - 20}{36}\right)$$

$$= 74.72° \text{ (to 2 d.p.)}$$

$\therefore$ Angle in which the jib has rotated

$$= \angle EAC$$

$$= 74.72° - 35°$$

$$= 39.7° \text{ (to 1 d.p.)}$$

13. Let the height of the tree be h m,
the distance QB be d m.

In $\triangle TBQ$,

$$\tan 32° = \frac{h}{d}$$

$$\therefore d = \frac{h}{\tan 32°} \quad \text{— (1)}$$

In $\triangle TBP$,

$$\tan 23° = \frac{h}{d + 10}$$

$$d + 10 = \frac{h}{\tan 23°}$$

$$\therefore d = \frac{h}{\tan 23°} - 10 \quad \text{— (2)}$$

(1) = (2):

$$\frac{h}{\tan 32°} = \frac{h}{\tan 23°} - 10$$

$$h \tan 23° = h \tan 32° - 10 \tan 32° \tan 23°$$

$$h \tan 32° - h \tan 23° = 10 \tan 32° \tan 23°$$

$$h(\tan 32° - \tan 23°) = 10 \tan 32° \tan 23°$$

$$\therefore h = \frac{10 \tan 32° \tan 23°}{\tan 32° - \tan 23°}$$

$$= 13.2 \text{ (to 3 s.f.)}$$

The height of the tree is 13.2 m.

Review Exercise 11

1. (a) $\sin 43° = \dfrac{12}{a}$

$\therefore a = \dfrac{12}{\sin 43°}$

$= 17.6$ (to 3 s.f.)

(b) $\tan 52° = \dfrac{b}{9.8}$

$\therefore b = 9.8 \tan 52°$

$= 12.5$ (to 3 s.f.)

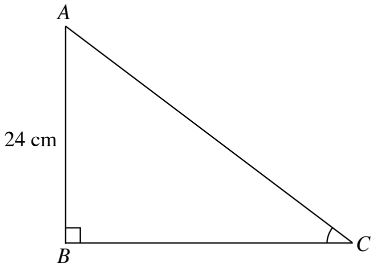
(c) $\cos c° = \dfrac{7.6}{14.3}$

$\therefore c° = \cos^{-1}\left(\dfrac{7.6}{14.3}\right)$

$= 57.9°$ (to 1 d.p.)

$\therefore c = 57.9$

2.



(i) $\sin \angle ACB = \dfrac{3}{5}$

$\dfrac{24}{AC} = \dfrac{3}{5}$

$\therefore AC = 24 \times \dfrac{5}{3}$

$= 40$ m

(ii) In $\triangle ABC$, $\angle B = 90°$.

Using Pythagoras' Theorem,

$AC^2 = BC^2 + AC^2$

$40^2 = BC^2 + 24^2$

$BC^2 = 40^2 - 24^2$

$= 1600 - 576$

$= 1024$

$\therefore BC = \sqrt{1024}$ (since $BC > 0$)

$= 32$ m

(iii) $\cos \angle ACB + \tan \angle BAC = \dfrac{32}{40} + \dfrac{32}{24}$

$= 2\dfrac{2}{15}$

3. (i) $\cos 60° = \dfrac{2}{OQ}$

$\therefore OQ = \dfrac{2}{\cos 60°}$

$= 4$ units

(ii) In $\triangle OPQ$, $\angle P = 90°$.

Using Pythagoras' Theorem,

$OQ^2 = OP^2 + PQ^2$

$4^2 = 2^2 + h^2$

$h^2 = 4^2 - 2^2$

$= 16 - 4$

$= 12$

$\therefore h = \sqrt{12}$ (since $h > 0$)

$= 3.46$ (to 3 s.f.)

4. In $\triangle KXZ$, $\sin X = \dfrac{ZK}{XZ}$.

In $\triangle HXZ$, $\sin Z = \dfrac{XH}{XZ}$.

$\therefore \dfrac{\sin X}{\sin Z} = \dfrac{ZK}{XZ} \div \dfrac{XH}{XZ}$

$= \dfrac{ZK}{XZ} \times \dfrac{XZ}{XH}$

$= \dfrac{ZK}{XH}$

$= \dfrac{4}{3}$ (given)

5. Let the angle between each diagonal and the breadth of the rectangle be $a°$.

$\tan a° = \dfrac{19.2}{12.4}$

$\therefore a° = \tan^{-1}\left(\dfrac{19.2}{12.4}\right)$

$= 57.1°$ (to 1 d.p.)

The angle is 57.1°.

6. (i) In $\triangle ABC$,

$\sin \angle BAC = \dfrac{6.9}{13.2}$

$\therefore \angle BAC = \sin^{-1}\left(\dfrac{6.9}{13.2}\right)$

$= 31.52°$ (to 2 d.p.)

$\angle ABH + \angle BAH = \angle BHC$ (ext. $\angle$ = sum of int. opp. $\angle$s)

$\angle ABH + 31.52° = 46°$

$\therefore \angle ABH = 46° - 31.52°$

$= 14.5°$ (to 1 d.p.)

(ii) In $\triangle BCH$,

$\tan 46° = \dfrac{6.9}{HC}$

$\therefore HC = \dfrac{6.9}{\tan 46°}$

$= 6.663$ m (to 4 s.f.)

In $\triangle ABC$, $\angle C = 90°$.

Using Pythagoras' Theorem,

$AB^2 = BC^2 + AC^2$

$13.2^2 = 6.9^2 + AC^2$

$AC^2 = 13.2^2 - 6.9^2$

$= 174.24 - 47.61$

$= 126.63$

$\therefore AC = \sqrt{126.63}$ (since $AC > 0$)

$= 11.25$ m (to 4 s.f.)

$\therefore AH = AC - HC$

$= 11.25 - 6.663$

$= 4.59$ m (to 4 s.f.)

7. (i) In $\triangle QPK$,

$\sin 30° = \dfrac{QK}{50}$

$\therefore QK = 50 \sin 30°$

$= 25$ cm

(ii) In $\triangle QPK$,

$\cos 30° = \dfrac{PK}{50}$

$\therefore PK = 50 \cos 30°$

$= 43.30$ cm (to 4 s.f.)

In $\triangle RPK$,

$\tan 46° = \dfrac{KR}{43.30}$

$\therefore KR = 43.30 \tan 46°$

$= 44.84$ cm (to 4 s.f.)

$\therefore QR = QK + KR$

$= 25 + 44.84$

$= 69.8$ cm (to 3 s.f.)

8. (i) In $\triangle VXZ$,

$\tan \angle VZX = \dfrac{35}{54}$

$\therefore \angle VZX = \tan^{-1}\left(\dfrac{35}{54}\right)$

$= 32.9°$ (to 1 d.p.)

(ii) In $\triangle XYZ$,

$\sin 54° = \dfrac{XY}{54}$

$\therefore XY = 54 \sin 54°$

$= 43.7$ m (to 3 s.f.)

(iii) In $\triangle XYZ$,

$\angle YXZ = 180° - 90° - 54°$ ($\angle$ sum of $\triangle XYZ$)

$= 36°$

In $\triangle VWX$,

$\angle VXW = 180° - 90° - 36°$ (adj. $\angle$s on a str. line)

$= 54°$

$\sin 54° = \dfrac{VW}{35}$

$\therefore VW = 35 \sin 54°$

$= 28.3$ m (to 3 s.f.)

9. In $\triangle ABL$,

$\tan 66.75° = \dfrac{AB}{15}$

$\therefore AB = 15 \tan 66.75°$

$= 34.91$ m (to 4 s.f.)

In $\triangle ABU$,

$\tan 27.4° = \dfrac{AU}{34.91}$

$\therefore AU = 34.91 \tan 27.4°$

$= 18.10$ m (to 4 s.f.)

$\therefore$ Height of the window $UL = AU - AL$

$= 18.10 - 15$

$= 3.10$ m (to 3 s.f.)

10. In $\triangle FHT$,

$\tan 54° = \dfrac{72}{HF}$

$\therefore HF = \dfrac{72}{\tan 54°}$

$= 52.31$ m (to 4 s.f.)

In $\triangle FKT$,

$\tan 47° = \dfrac{FK}{72}$

$\therefore FK = 72 \tan 47°$

$= 77.21$ m (to 4 s.f.)

$\therefore HK = HF + FK$

$= 52.31 + 77.21$

$= 130$ m (to 3 s.f.)

11. Let the height of the mast be h m.

$\tan 32° = \dfrac{h}{57}$

$\therefore h = 57 \tan 32°$

$= 35.6$ (to 3 s.f.)

The height of the mast is 35.6 m.

12. (i) Let the height of the window sill above the ground be h m.

$\sin 61° = \dfrac{h}{6.5}$

$\therefore h = 6.5 \sin 61°$

$= 5.69$ (to 3 s.f.)

The height of the window sill above the ground is 5.69 m.

(ii) Let the distance of the foot of the ladder from the base of the wall be d m.

$\cos 61° = \frac{d}{6.5}$

$\therefore d = 6.5 \cos 61°$

$= 3.15$ (to 3 s.f.)

The distance of the foot of the ladder from the base of the wall is 3.15 m.

Challenge Yourself

1. BD bisects $\angle ADC$, i.e. $\angle ADB = \angle CDB = 20°$.

Let AB be x units and AD be y units.

$\angle A = \angle C = 90°$

$\angle D = \angle D = 20°$

$\angle B = \angle B = 180° - 90° - 20° = 70°$

$BC = AB = x$

$CD = AD = y$

BD is common side in $\triangle ADB$ and $\triangle CDB$.

$\therefore \triangle ADB \equiv \triangle CDB$

$\frac{1}{2} \times x \times y = \frac{900}{2}$

$xy = 900$ — (1)

$\tan 20° = \frac{x}{y}$

$x = y \tan 20°$ — (2)

Substitute (2) into (1):

$(y \tan 20°)y = 900$

$y^2 = \frac{900}{\tan 20°}$

$\therefore y = \sqrt{\frac{900}{\tan 20°}}$ (since $y > 0$)

$= 49.73$ (to 4 s.f.)

Substitute $y = 49.73$ into (1):

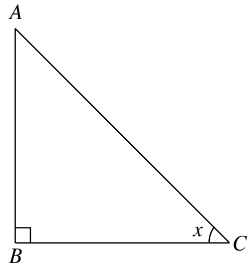
$x = \frac{900}{49.73}$

$= 18.10$ (to 4 s.f.)

$\therefore$ Perimeter $= 2(49.73 + 18.10)$

$= 136$ units (to 3 s.f.)

2. Consider $\triangle ABC$ where $\angle B = 90°$.



$\sin x = \frac{AB}{AC}$, $\cos x = \frac{BC}{AC}$

When $\sin x < \cos x$,

$\frac{AB}{AC} < \frac{BC}{AC}$

$AB < BC$

Let $AB = BC$.

$\tan x = \frac{AB}{BC}$

$= 1$

$\therefore x = \tan^{-1}(1)$

$= 45°$

$\therefore AB < BC$ when $x < 45°$

$\therefore 0° < x < 45°$

Chapter 12 Volume and Surface Area of Pyramids, Cones and Spheres

TEACHING NOTES

Suggested Approach

In Secondary One, students have learnt to find the volume and surface area of cubes, cuboids, prisms and cylinders. Here, they will learn to determine the volume and surface area of other regular figures, the pyramid, cone and sphere. By the end of this chapter, students are to be familiar with the various formulas in calculating the volume and surface area, as well as the various real-life examples of such figures. When the value of π is not stated, students are to use the value in the calculator. In some problems, students are expected to recall and apply Pythagoras' Theorem.

Section 12.1: Volume and Surface Area of Pyramids

As an introduction, teachers can show students some real-life examples of pyramids and question students on the properties of pyramids (see Class Discussion: What are Pyramids?)

Teachers should go through the part of a pyramid. Following that, students should observe and recognise the various types of pyramids. Teachers should indicate that the pyramids studied in this chapter are right pyramids, where the apex is vertically above the centre of the base and the base is a regular polygon.

The activity in determining the volume of a pyramid is to enable students to appreciate the relation between the volume of a pyramid and its corresponding prism (see Investigation: Volume of Pyramids).

Section 12.2: Volume and Surface Area of Cones

Similar to pyramids, teachers can start off with an activity to introduce cones (see Class Discussion: What are Cones?).

To improve and enhance understanding, students should learn and explain the features of a cone and state the differences between a cone, a cylinder and a pyramid (see Journal Writing on page 329, and Investigation: Comparison between a Cone and a Pyramid).

Proceeding on, students should realise that the volume and total surface area of a cone is analogous to the volume and total surface area of a pyramid. The curved surface area of a cone is one unique calculation that has to be noted.

Section 12.3: Volume and Surface Area of Spheres

Besides the volume and surface area of a sphere, students have to be aware of the volume and total surface area of a hemisphere, or half a sphere as well. Teachers should demonstrate how the volume and surface area of a sphere can be obtained (see Investigation: Volume of Spheres and Investigation: Surface Area of Spheres), and show the simple steps in deriving the volume and total surface area of a hemisphere (see Thinking Time on page 342). This will minimise the formulas students need to recall.

Section 12.4 Volume and Surface Area of Composite Solids

In this section, students are required to make calculations involving the various composite solids made up of regular figures. Besides the ones covered in this chapter, regular figures from Secondary One, such as cubes, cuboids, prisms and cylinders may be included. Weaker students may need a revision of their formulas for volume and total surface area.

In calculating the total surface area, students must be careful not to include any sides that are overlapping. It is good practice to state and calculate the volume and total surface area part by part.

Challenge Yourself

A regular tetrahedron is a solid made up of four equilateral triangular faces. Therefore, it is a pyramid regardless of which side it lies on. This information is required for Question 1.

For Question 2, to derive and prove the statement, students should observe that the length from the centre of the top of the hemisphere to the side of the depth of the water in the sphere is also its radius. After applying Pythagoras' Theorem, the statement should follow after a little logical reasoning.

WORKED SOLUTIONS

Class Discussion (What are Pyramids?)

1. The pyramids are made up of one base and four triangular faces joined to the sides of the base. The four triangles are joined by a single point at the other end.
2. The slanted faces of the pyramids are congruent, isosceles triangles.
3. The bases of these pyramids are squares.
4. The vertex of a pyramid is the point where the vertices of the triangle are joined to the vertices of the base. The apex of a pyramid is the point vertically above the base, where the triangles are joined to each other.
5. The cross sections of a pyramid are squares and are not uniform throughout the pyramid.
6. The food pyramids, human pyramid and rice dumplings are pyramids and they have the same features as the pyramids in Fig. 10.1.
7. Three more real-life examples of pyramids are the roof of a house, tents, packets of milk etc.

Thinking Time (Page 319)

The slant edge is the hypotenuse of a right-angled triangle, together with the height of the pyramid and half of the diagonal of the base.

The slant height is the hypotenuse of another right-angled triangle, together with the height of the pyramid and half the side of the base.

The slant faces of regular pyramids are congruent, isosceles triangles.

Journal Writing (Page 319)

Prisms have two polygonal bases that are congruent and parallel to each other while pyramids have only one polygonal base with an apex vertically above it.

The sides of a prism are made up of rectangles while the sides of a pyramid are triangles that are joined at the apex.

The cross-section of a prism is uniform while the cross-section of a pyramid is non-uniform.

Investigation (Volume of Pyramids)

It will take 3 times to fill the prism completely.
Volume of pyramid = $\underline{3}$ × volume of corresponding prism

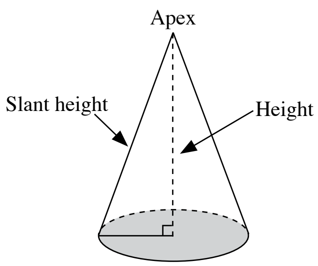
Class Discussion (What are Cones?)

1. The cones have a circular base with a curved surface and an apex opposite the base.
2. The base of a cone and a cylinder is a circle. The sides of a cone and a cylinder are curved surfaces.

   A cone has one circular base while a cylinder has two circular bases. A cone has an apex opposite its base while a cylinder does not have an apex. The cross-section of a cone is non-uniform while the cross-section of a cylinder is uniform.
3. Both the cone and pyramid have one base only. Both the cone and pyramid have an apex. The cross-section of both the cone and pyramid are non-uniform.

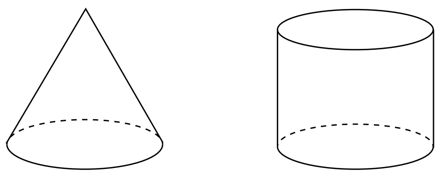
   The base of a cone is a circle while the base of a pyramid is a polygon. The side of a cone is a curved surface while the sides of a pyramid are made up of triangles. The cross-section of a cone is a circle while the cross-section of a pyramid is a polygon.
4. Three more real-life examples of cones are traffic cones, tents and mountains etc.

Journal Writing (Page 329)



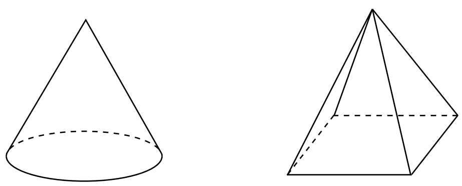
A cone is a solid in which the base is bounded by a simple closed curve and the curved surface tapers into a point called the apex, which is opposite the base. If the apex is vertically above the centre of the circular base, we call the cone a right circular cone.

The perpendicular height (or height) of a cone is the perpendicular distance from the apex to the base of the cone. The slant height of a right circular cone is the distance from the apex to the circumference of the base.



Cone Cylinder

A cone has one circular base while a cylinder has two circular bases.
A cone has an apex opposite its base while a cylinder does not have an apex. The cross-section of a cone is non-uniform while the cross-section of a cylinder is uniform.



Cone Pyramid

The base of a cone is a circle while the base of a pyramid is a polygon. The side of a cone is a curved surface while the sides of a pyramid are made up of triangles. The cross-section of a cone is a circle while the cross-section of a pyramid is a polygon.

Investigation (Comparison between a Cone and a Pyramid)

1. The polygon will become a circle.
2. The pyramid will become a cone.

Thinking Time (Page 331)

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a cylinder = $\pi r^2 h$

Since the cone and cylinder have the same base and same height,

$\therefore$ Volume of cone = $\frac{1}{3}$ × volume of a cylinder

Investigation (Curved Surface Area of Cones)

If the number of sectors is increased indefinitely, then the shape in Fig. 10.15(b) will become a <u>rectangle</u> $PQRS$.

Since $PQ + RS$ = circumference of the base circle in Fig. 10.15(a), then the length of the rectangle is $PQ = \underline{\pi r}$.

Since PS = slant height of the cone in Fig. 10.15(a), then the breadth of the rectangle is $PS = \underline{l}$.

$\therefore$ Curved surface area of cone = area of rectangle

$$= \underline{PQ} \times \underline{PS}$$
$$= \underline{\pi rl}$$

Thinking Time (Page 334)

Total surface of a solid cone

= Curved surface area of cone + base area of cone

$= \pi rl + \pi r^2$

Thinking Time (Page 338)

A hemisphere is half a sphere.

Some real-life examples of hemispheres are bowls, stadium domes, the base of a tilting doll etc.

Class Discussion (Is the King's Crown Made of Pure Gold?)

$$\text{Density of the crown} = \frac{11.6 \text{ kg}}{714 \text{ cm}^3}$$
$$= \frac{(11.6 \times 1000)\text{ g}}{714 \text{ cm}^3}$$
$$= \frac{11\,600 \text{ g}}{714 \text{ cm}^3}$$
$$= 16.2 \text{ g/cm}^3 \text{ (to 3 s.f.)}$$

Since 16.2 g/cm^3 ≠ 19.3 g/cm^3, the crown was not made of pure gold.

Investigation (Volume of Spheres)

Volume of cylinder = $\pi r^2 h$

$= \pi \times r^2 \times 2r$

$= \underline{2\pi r^3}$

Volume of sphere = $\frac{2}{3}$ × volume of cylinder

$= \frac{2}{3} \times \underline{2\pi r^3}$

$= \underline{\frac{4}{3}\pi r^3}$

Investigation (Surface Area of Spheres)

Part I:

Length of second piece of twine = $2\pi rh$

$= 2\pi \times r \times r$

$= \underline{2\pi r^2}$

Curved surface area of sphere = 2 × length of first piece of twine

= 2 × length of second piece of twine

$= 2 \times \underline{2\pi r^2}$

$= \underline{4\pi r^2}$

Part II:

4. 4 circles are covered completely with the orange skin.
5. Surface area of the orange = $4\pi r^2$

Thinking Time (Page 342)

Total surface of a solid hemisphere

= Curved surface area of hemisphere + Base area of hemisphere

$= \frac{1}{2} \times 4\pi r^2 + \pi r^2$

$= 2\pi r^2 + \pi r^2$

$= 3\pi r^2$

Practise Now 1

1. Volume of triangular pyramid

   $= \frac{1}{3}$ × base area × height

   $= \frac{1}{3} \times 36 \times 7$

   $= 84 \text{ cm}^3$

2. Volume of the pyramid

   $= \frac{1}{3}$ × base area × height

   $= \frac{1}{3} \times 229 \times 229 \times 146$

   $= 2\,550\,000 \text{ m}^3$ (to 3 s.f.)

Practise Now 2

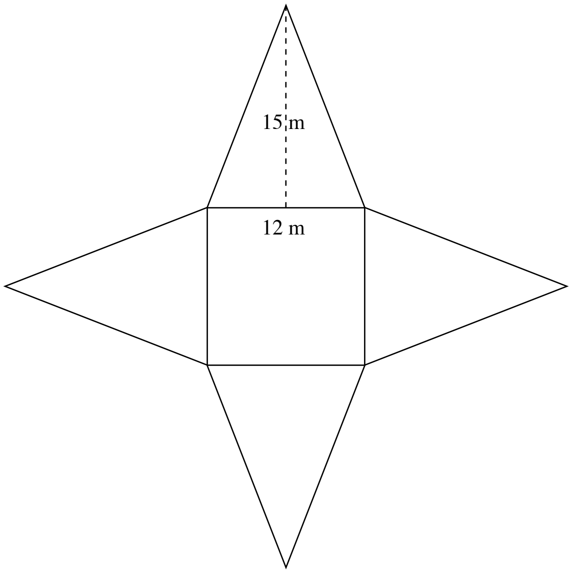
Volume of pyramid = $\frac{1}{3}$ × base area × height

$75 = \frac{1}{3} \times (5 \times 5) \times \text{height}$

$75 = \frac{25}{3} \times \text{height}$

$\therefore$ Height = 9 m

Practise Now 3



Area of each triangular face $= \frac{1}{2} \times 12 \times 15$

$= 90 \text{ m}^2$

Area of square base $= 12 \times 12$

$= 144 \text{ m}^2$

$\therefore$ Total surface area of pyramid

$= 4 \times$ area of each triangular face + area of square base

$= 4 \times 90 + 144$

$= 504 \text{ m}^2$

Practise Now 4

(i) Total surface area of pyramid

$= 4 \times$ area of each triangular face + area of square base

Area of each triangular face

$= \frac{\text{Total surface area of pyramid} - \text{area of square base}}{4}$

$= \frac{161 - (7 \times 7)}{4}$

$= \frac{112}{4}$

$= 28 \text{ cm}$

Area of $\triangle VQR = \frac{1}{2} \times 7 \times VB = 28$

$\frac{7}{2} \times VB = 28$

$VB = 8 \text{ cm}$

(ii) Let the point where the vertical from V meets the square base be P.

$PB = \frac{1}{2} \times PQ$

$= \frac{1}{2} \times 7$

$= 3.5 \text{ cm}$

In $\triangle VPB$, $\angle P = 90°$.

Using Pythagoras' Theorem,

$VB^2 = VP^2 + PB^2$

$8^2 = VP^2 + 3.5^2$

$VP^2 = 8^2 - 3.5^2$

$= 64 - 12.25$

$= 51.75$

$\therefore VP = \sqrt{51.75}$ (since $VP > 0$)

$\therefore$ Volume of pyramid $= \frac{1}{3} \times$ base area $\times$ height

$= \frac{1}{3} \times 7 \times 7 \times \sqrt{51.75}$

$= 117 \text{ cm}^3$ (to 3 s.f.)

Practise Now 5

1. Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 8^2 \times 17$

$= 362\frac{2}{3}\pi$

$= 1140 \text{ cm}^3$ (to 3 s.f.)

2. Volume of cone $= \frac{1}{3}\pi r^2 h$

$84\pi = \frac{1}{3} \times \pi \times 6^2 \times h$

$84\pi = 12\pi h$

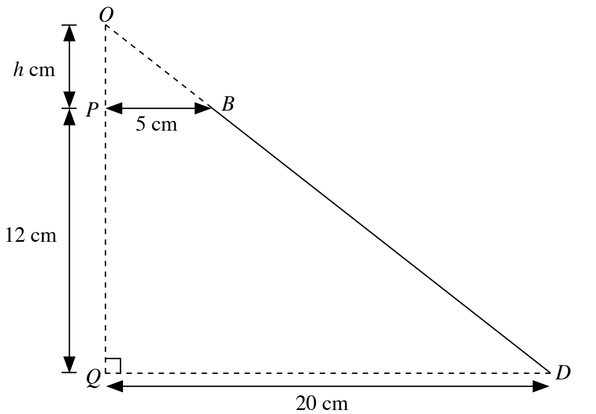
$\therefore h = 7 \text{ m}$

The height of the cone is 7 m.

Practise Now 6

Let the height of the smaller cone be h cm.

Then the height of the bigger cone is $(h + 12)$ cm.



Since $\triangle OPB$ is similar to $\triangle OQD$,

$$\frac{OP}{OQ} = \frac{PB}{QD}$$

$$\frac{h}{h+12} = \frac{5}{20}$$

$$\frac{h}{h+12} = \frac{1}{4}$$

$$4h = h + 12$$

$$3h = 12$$

$$h = 4$$

$\therefore$ Height of bigger cone $= 12 + 4$
$= 16$ cm

$\therefore$ Volume of frustum = volume of bigger cone – volume of smaller cone

$$= \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi(R^2 H - r^2 h)$$

$$= \frac{1}{3}\pi(20^2 \times 16 - 5^2 \times 4)$$

$$= \frac{1}{3}\pi(6300)$$

$$= 2100\pi$$

$= 6600$ cm^3 (to 3 s.f.)

Practise Now 7

1. Total surface area of cone $= \pi rl + \pi r^2$

$= \pi \times 9 \times 5 + \pi + 9^2$

$= 45\pi + 81\pi$

$= 126\pi$

$= 396$ cm^2 (to 3 s.f.)

2. Total surface area of cone $= \pi rl + \pi r^2$

$350 = \pi \times 8 \times l + \pi \times 8^2$

$= 8\pi l + 64\pi$

$8\pi l = 350 - 64\pi$

$$\therefore l = \frac{350 - 64\pi}{8\pi}$$

$$= \frac{350 - 64 \times 3.142}{8 \times 3.142}$$

$= 5.92$ m (to 3 s.f.)

Practise Now 8

1. Let the slant height of the cone be l m.

Using Pythagoras' Theorem, $l = \sqrt{8^2 + 15^2}$

$= 17$

$\therefore$ Curved surface area of cone $= \pi rl$

$= \pi \times 8 \times 17$

$= 136\pi$

$= 427$ m^2 (to 3 s.f.)

2. Let the height of the cone be h cm.

Using Pythagoras' Theorem, $h = \sqrt{12^2 + 7^2}$

$= 9.747$ (to 4 s.f.)

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 7^2 \times 9.747$

$= 500$ cm^3 (to 3 s.f.)

Practise Now 9

1. Radius of ball bearing $= 0.4 \div 2$

$= 0.2$ cm

Volume of ball bearing $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 0.2^3$

$= \frac{4\pi}{375}$ cm^3

Mass of 5000 ball bearings

= volume of 5000 ball bearings × density

$= 5000 \times \frac{4\pi}{375} \times 11.3$

$= 1890$ g (to 3 s.f.)

2. Volume of basketball $= 5600$

$$\frac{4}{3}\pi r^3 = 5600$$

$$r^3 = \frac{4200}{\pi}$$

$$\therefore r = \sqrt[3]{\frac{4200}{\pi}}$$

$= 11.0$ cm (to 3 s.f.)

The radius of the basketball is 11.0 cm.

Practise Now 10

Radius of sphere $= 25 \div 2$

$= 12.5$ cm

Surface area of sphere $= 4\pi r^2$

$= 4 \times \pi \times 12.5^2$

$= 6.25\pi$

$= 1960$ cm^2 (to 3 s.f.)

Practise Now 11

Curved surface area of hemisphere $= 200$ cm^2

$$\frac{1}{2} \times 4\pi r^2 = 200$$

$$2\pi r^2 = 200$$

$$r^2 = \frac{100}{\pi}$$

$$\therefore r = \sqrt{\frac{100}{\pi}} \quad (\text{since } r > 0)$$

$= 5.64$ cm (to 3 s.f.)

Practise Now 12

Height of cone $= \frac{3}{4} \times$ height of cylinder

$= \frac{3}{4} \times 3r$

$= \frac{9}{4}r$

Volume of cone $= \frac{1}{3}\pi r^2\left(\frac{9}{4}r\right)$

$= \frac{3}{4}\pi r^3$

Since volume of cone = 10 l = 10 000 cm^3,

then $\frac{3}{4}\pi r^3 = 10\ 000$

$r^3 = \frac{10\ 000 \times 4}{3\pi}$

$= \frac{40\ 000}{3\pi}$

Volume of cylinder $= \pi r^2(3r)$

$= 3\pi r^3$

$= 3\pi \times \frac{40\ 000}{3\pi}$

= 40 000 cm^3

= 40 l

∴ Amount of water needed to fill container completely = 40 + 10

= 50 l

Practise Now 13

(a) (i) Radius of hemisphere = 30 ÷ 2

= 15 cm

Height of cone = 50 – 15

= 35 cm

Volume of solid = volume of cone + volume of hemisphere

$= \frac{1}{3} \times \pi \times 15^2 \times 35 + \frac{1}{2} \times \frac{4}{3} \times \pi \times 15^3$

$= 2625\pi + 2250\pi$

$= 4875\pi$

= 15 300 cm^3

(ii) Using Pythagoras' Theorem,

Slant height of cone $= \sqrt{15^2 + 35^2}$

= 38.08 cm (to 4 s.f.)

Total surface area of solid

= curved surface area of cone

+ curved surface area of hemisphere

$= \pi \times 15 \times 38.08 + 2 \times \pi \times 15^2$

$= 571.2\pi + 450\pi$

$= 1021.2\pi$

= 3210 cm^2 (to 3 s.f.)

(b) (i) Volume of cylinder $= 4875\pi$

$\pi(12.5^2)h = 4875\pi$

$\therefore h = \frac{4875\pi}{156.25\pi}$

= 31.2 cm

The height of the cylinder is 31.2 cm.

(ii) Surface area of the cylinder

$= 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 12.5^2 + 2 \times \pi \times 12.5 \times 31.2$

$= 312.5\pi + 780\pi$

$= 1092.5\pi$ cm^2

Exercise 12A

1. Volume of triangular pyramid $= \frac{1}{3} \times$ base area × height

$= \frac{1}{3} \times 15 \times 4$

= 20 cm^3

2. Volume of pyramid $= \frac{1}{3} \times$ base area × height

$= \frac{1}{3} \times 23 \times 6$

= 46 cm^3

3. Base area of pyramid $= \frac{1}{2} \times 7 \times 4$

= 14 cm^2

Volume of pyramid $= \frac{1}{3} \times$ base area × height

$= \frac{1}{3} \times 14 \times 5$

$= 23\frac{1}{3}$ m^3

4. Base area of pyramid = 10 × 6

= 60 cm^2

Volume of pyramid $= \frac{1}{3} \times$ base area × height

$100 = \frac{1}{3} \times 60 \times$ height

100 = 20 × height

∴ Height = 5 cm

5. Base area of pyramid $= \frac{1}{2} \times 5 \times 8$

= 20 cm^2

Volume of pyramid $= \frac{1}{3} \times$ base area × height

$50 = \frac{1}{3} \times 20 \times$ height

$50 = \frac{20}{3} \times$ height

∴ Height = 7.5 cm

6. Volume of pyramid $= \frac{1}{3} \times$ base area × height

$100 = \frac{1}{3} \times$ base area × 12

100 = 4 × base area

∴ Base area = 25 m^2

Let the length of the square base be x.

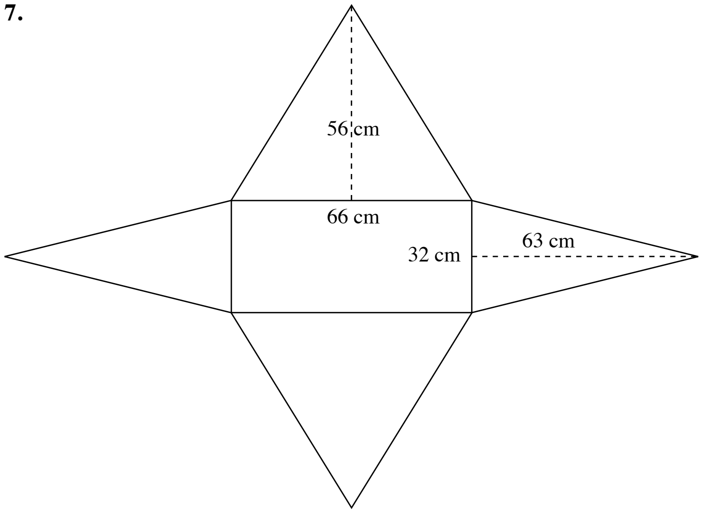
$x^2 = 25$

$\therefore x = \sqrt{25}$ (since $x > 0$)

= 5 m

The length of its square base is 5 m.

7.



Area of triangular face $OWX = \frac{1}{2} \times 66 \times 56$

$= 1848 \text{ cm}^2$

Area of triangular face $OXY = \frac{1}{2} \times 32 \times 63$

$= 1008 \text{ cm}^2$

Area of rectangular base $= 66 \times 32$

$= 2112 \text{ cm}^2$

$\therefore$ Total surface area of pyramid

$= 2 \times$ area of $OWX + 2 \times$ area of $OXY +$ area of square base

$= 2 \times 1848 + 2 \times 1008 + 2112$

$= 7824 \text{ cm}^2$

8. Volume of paper weight $= \frac{1}{3} \times$ base area $\times$ height

$= \frac{1}{3} \times 6 \times 6 \times 7$

$= 84 \text{ cm}^3$

Mass of 4 paper weights = volume of 4 paper weights $\times$ density

$= 4 \times 84 \times 3.1$

$= 1041.6$ g

9. Volume of pyramid = mass of pyramid $\div$ density

$= 500 \div 6$

$= 83\frac{1}{3} \text{ cm}^3$

Volume of pyramid $= \frac{1}{3} \times$ base area $\times$ height

$83\frac{1}{3} = \frac{1}{3} \times 30 \times$ height

$83\frac{1}{3} = 10 \times$ height

$\therefore$ Height $= 8\frac{1}{3}$ cm

10. (i) Let the point where the vertical from V meets the rectangular base be P.

$PQ = 15 \div 2$

$= 7.5$ cm

Using Pythagoras's Theorem, $PV = \sqrt{16^2 - 7.5^2}$

$= 14.13$ cm (to 4 s.f.)

$= 14.1$ cm (to 3 s.f.)

The height of the pyramids is 10.1 cm.

(ii) Volume of pyramid $= \frac{1}{3} \times$ base area $\times$ height

$= \frac{1}{3} \times 15 \times 9 \times 14.13$

$= 636 \text{ cm}^3$ (to 3 s.f.)

11. (i) Volume of pyramid $= \frac{1}{3} \times$ base area $\times$ height

$180 = \frac{1}{3} \times 10 \times 8 \times$ height

$180 = \frac{80}{3} \times$ height

$\therefore$ Height $= 6.75$ cm

(ii) Let the slant height from V to PQ be l_1 cm,
the slant height from V to QR be l_2 cm.

Using Pythagoras' Theorem,

$l_1 = \sqrt{6.75^2 + 4^2}$

$= 7.846$ (to 4 s.f.)

$l_2 = \sqrt{6.75^2 + 5^2}$

$= 8.400$ (to 4 s.f.)

$\therefore$ Total surface area of pyramid

= Area of all triangular faces + area of square base

$= 2 \times \left(\frac{1}{2} \times 10 \times 7.846 + \frac{1}{2} \times 8 \times 8.400\right) + 10 \times 8$

$= 2(39.23 + 33.6) + 80$

$= 145.66 + 80$

$= 226 \text{ cm}^2$ (to 3 s.f.)

12. (i) Volume of pyramid $= \frac{1}{3} \times$ base area $\times$ height

$700 = \frac{1}{3} \times 16 \times 14 \times$ height

$700 = \frac{224}{3} \times$ height

$\therefore$ Height $= 9.375$ cm

(ii) Let the slant height from the top of the pyramid to the side with 16 m be l_1 cm,
the slant height from the top of the pyramid to the side with 14 m be l_2 cm.

Using Pythagoras' Theorem,

$l_1 = \sqrt{9.375^2 + 7^2}$

$= 11.70$ (to 4 s.f.)

$l_2 = \sqrt{9.375^2 + 8^2}$

$= 12.32$ (to 4 s.f.)

$\therefore$ Total surface area of pyramid

= Area of all triangular faces + area of square base

$= 2 \times \left(\frac{1}{2} \times 16 \times 11.70 \times \frac{1}{2} \times 14 \times 12.32\right) + 16 \times 14$

$= 2(93.6 + 86.24) + 224$

$= 359.68 + 224$

$= 584 \text{ cm}^2$ (to 3 s.f.)

13. Volume of pyramid $= \frac{1}{3} \times$ base area $\times$ height

$= \frac{1}{3} \times 15 \times 10 \times 20$

$= 1000 \text{ cm}^3$

Volume of cubical tank $= l^3$

$= 30^3$

$= 27\ 000 \text{ cm}^3$

Volume of water left in tank after pyramid is removed

$= 27\ 000 - 1000$

$= 26\ 000 \text{ cm}^3$

Let the depth of the remaining water in the tank be d cm.

$30 \times 30 \times d = 26\ 000$

$900\ d = 26\ 00$

$\therefore d = 28\frac{8}{9}$

The depth of the remaining water is $28\frac{8}{9}$ cm.

14. Let WX be a, XY be b and the height of the pyramid be h.

i.e. $a > b$

Using Pythagoras' Theorem,

$VA = \sqrt{h^2 + \left(\frac{b}{2}\right)^2}$

$= \sqrt{h^2 + \frac{b^2}{4}}$

$VB = \sqrt{h^2 + \left(\frac{a}{2}\right)^2}$

$= \sqrt{h^2 + \frac{a^2}{4}}$

Since $a > b$, $VB > VA$,

$\therefore$ the slant height VA is shorter than VB.

15. (i) Let the slant height be l cm.

Using Pythagoras' Theorem, $l = \sqrt{8^2 - \left(\frac{8}{2}\right)^2}$

$= \sqrt{64 - 16}$

$= \sqrt{48}$

$= 6.928$ cm (to 4 s.f.)

$= 6.93$ cm (to 3 s.f.)

(ii) Base area of tetrahedron $= \frac{1}{2} \times 8 \times 6928$

$= 27.712 \text{ cm}^2$

Let the height of the tetrahedron be h cm.

Using Pythagoras's Theorem, $h = \sqrt{8^2 - \left(\frac{2}{3} \times 6.928\right)^2}$

$= 6.532$ cm (to 4 s.f.)

$\therefore$ Volume of tetrahedron $= \frac{1}{3} \times \text{base area} \times \text{height}$

$= \frac{1}{3} \times 27.712 \times 6.532$

$= 60.3 \text{ cm}^3$

Exercise 12B

1. (a) Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 6^2 \times 14$

$= 168\pi$

$= 528 \text{ cm}^3$ (to 3 s.f.)

(b) Volume of cone $= \frac{1}{3} \times \text{base area} \times \text{height}$

$= \frac{1}{3} \times 154 \times 5$

$= 256\frac{2}{3} \text{ cm}^3$

(c) Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times \left(\frac{7}{2}\right)^2 \times 14$

$= 57\frac{1}{6}\pi$

$= 180 \text{ cm}^3$ (to 3 s.f.)

(d) Circumference $= 132$

$2\pi r = 132$

$\therefore r = \frac{132}{2\pi}$

$= \frac{66}{\pi}$

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times \left(\frac{66}{\pi}\right)^2 \times 28$

$= \frac{40\ 656}{\pi}$

$= 12\ 900 \text{ mm}^3$

2. Volume of cone $= \frac{1}{3}\pi r^2 h$

$320\pi = \frac{1}{3} \times \pi \times 8^2 \times h$

$320\pi = 21\frac{1}{3}\pi h$

$h = 15$ cm

3. Volume of cone $= \frac{1}{3} \times \text{base area} \times \text{height}$

$160 = \frac{1}{3} \times 20 \times \text{height}$

$160 = \frac{20}{3} \times \text{height}$

$\therefore$ Height $= 24$ m

4. Volume of cone $= \frac{1}{3}\pi r^2 h$

$132 = \frac{1}{3} \times \frac{22}{7} \times r^2 \times 14$

$132 = \frac{44}{3} \times r^2$

$r^2 = 9$

$\therefore r = \sqrt{9}$ (since $r > 0$)

$= 3$ cm

5. (a) Total surface area of cone $= \pi rl + \pi r^2$

$= \pi \times 4 \times 7 + \pi \times 4^2$

$= 28\pi + 16\pi$

$= 44\pi$

$= 138 \text{ cm}^2$ (to 3 s.f.)

(b) Radius of cone $= 28 \div 2$

$= 14$ mm

Total surface area of cone $= \pi rl + \pi r^2$

$= \pi \times 14 \times 30 \times \pi \times 14^2$

$= 420\pi + 196\pi$

$= 616\pi$

$= 1940$ mm^2 (to 3 s.f.)

(c) Circumference $= 132$

$2\pi r = 132$

$\therefore r = \dfrac{132}{2\pi}$

$= \dfrac{66}{\pi}$

Total surface area of cone $= \pi rl + \pi r^2$

$= \pi \times \dfrac{66}{\pi} \times 25 + \pi \times \left(\dfrac{66}{\pi}\right)^2$

$= 1650 + \dfrac{4356}{\pi}$

$= 3040$ cm^2 (to 3 s.f.)

6. Curved surface area of cone $= 84\pi$ mm^2

$\pi(6)l = 84\pi$

$6\pi l = 84\pi$

$\therefore l = \dfrac{84\pi}{6\pi}$

$= 14$ mm

The slant height of the cone is 14 mm.

7. Total surface area of cone $= 1000$ cm^2

$\pi(15)l + \pi(15^2) = 1000$

$15\pi l + 225\pi = 1000$

$15\pi l = 1000 - 225\pi$

$\therefore l = \dfrac{1000 - 225\pi}{15\pi}$

$= \dfrac{1000 - 225 \times 3.142}{15 \times 3.142}$

$= 6.22$ cm (to 3 s.f.)

The slant height of the cone is 6.22 cm.

8. Curved surface area of cone $= 251$ m^2

$\pi r(5) = 251$

$5\pi r = 251$

$\therefore r = \dfrac{251}{5\pi}$

$= 16.0$ m (to 3 s.f.)

9. Radius of conical funnel $= 23.2 \div 2$

$= 11.6$ cm

Volume of conical funnel $= \dfrac{1}{3}\pi r^2 h$

$= \dfrac{1}{3} \times \pi \times 11.6^2 \times 42$

$= 1883.84\pi$ cm^3

Radius of cylindrical $= 16.2 \div 2$

$= 8.1$ cm

Volume of cylindrical tin $= 1883.84\pi$ cm^3

$\pi(8.1^2)h = 1883.84\pi$

$65.61\pi h = 1883.84\pi$

$\therefore h = \dfrac{1883.84\pi}{65.61\pi}$

$= 28.7$ cm (to 3 s.f.)

10. Volume of conical block of silver $= \dfrac{1}{3}\pi r^2 h$

$= \dfrac{1}{3} \times \pi \times 12^2 \times 16$

$= 768\pi$ cm^3

Radius of a coin $= 1\dfrac{1}{2} \div 2$

$= \dfrac{3}{4}$ cm

Volume of a coin $= \pi r^2 h$

$= \pi \times \left(\dfrac{3}{4}\right)^2 \times \dfrac{1}{6}$

$= \dfrac{3}{32}\pi$ cm^3

$\therefore$ Number of coins that can be made

$= \dfrac{\text{Volume of conical block of silver}}{\text{Volume of a coin}}$

$= \dfrac{768\pi}{\frac{3}{32}\pi}$

$= 8192$

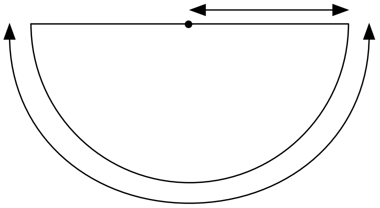
11. Circumference of base of cone $= 2\pi r$

$= 2 \times \pi \times 10$

$= 20\pi$ cm

Observe that $\dfrac{\text{slant height}}{\text{circumference of cone}} = \dfrac{20}{20\pi} = \dfrac{1}{\pi} = \dfrac{1}{2} \times \dfrac{1}{2\pi}$.

$\therefore$ The net of the cone is a semicircle, where the slant height is the radius of the semicircle.



12. **(i)** Let the diameter of the semircircle be d_s cm,
the diameter of the base of the cone be d_c cm.

Circumference of semicircle $= \dfrac{1}{2}\pi d_s$

$= \dfrac{1}{2} \times \pi \times 10$

$= 5\pi$ cm

Circumference of base of cone $= 5\pi$ cm

$\pi d_c = 5\pi$

$\therefore d_c = 5$

The diameter of the base of the cone is 5 cm.

(ii) Radius of the cone $= 5 \div 2$

$= 2.5$ cm

Slant height of the cone $= 10 \div 2$

$= 5$ cm

Curved surface area of the cone $= \pi rl$

$= \pi \times 2.5 \times 5$

$= 12.5\pi$

$= 39.3$ cm^2 (to 3 s.f.)

13. Using Pythagoras' Theorem,

$l = \sqrt{5^2 + 12^2}$

$= 13$ cm

Curved surface area of the cone $= \pi rl$

$= \pi \times 5 \times 13$

$= 65\pi$

$= 204$ cm^2 (to 3 s.f.)

14. Using Pythagoras' Theorem,

$h = \sqrt{20^2 - 8^2}$

$= 18.33$ cm (to 4 s.f.)

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 8^2 \times 18.33$

$= 391.04\pi$

$= 1230$ cm^3 (to 3 s.f.)

15. (i) Using Pythagoras' Theorem,

$r^2 = 21^2 - 17^2$

$= 152$

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 152 \times 17$

$= 861\frac{1}{3}\pi$

$= 2710$ mm^3 (to 3 s.f.)

(ii) Total surface area of cone $= \pi rl + \pi r^2$

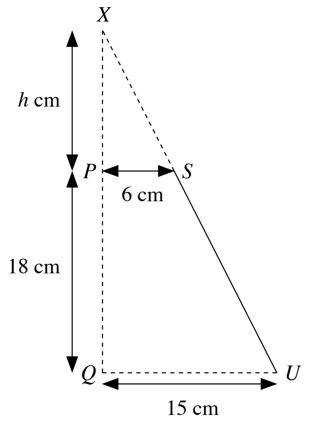
$= \pi \times \sqrt{152} \times 21 + \pi \times 152$

$= 21\sqrt{152}\,\pi + 152\pi$

$= 1290$ mm^2 (to 3 s.f.)

16. Let the height of the smaller cone be h cm.

Then the height of the bigger cone is $(h + 18)$ cm.



Since $\triangle XPS$ is similar to $\triangle XQU$,

$\frac{XP}{XQ} = \frac{PS}{QU}$

$\frac{h}{h+18} = \frac{6}{15}$

$15h = 6h + 108$

$9h = 108$

$h = 12$

$\therefore$ Height of bigger cone $= 18 + 12$

$= 30$ cm

$\therefore$ Volume of frustum

$=$ volume of bigger cone $-$ volume of smaller cone

$= \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(R^2 H - r^2 h)$

$= \frac{1}{3}\pi(15^2 \times 30 - 6^2 \times 12)$

$= \frac{1}{3}\pi(6318)$

$= 2016\pi$

$= 6620$ cm^3 (to 3 s.f.)

17. (i) Radius of cone $= 14 \div 2$

$= 7$ cm

Total surface area of solid $= 2\pi rl$

$= 2 \times \pi \times 7 \times 15$

$= 210\pi$

$= 660$ cm^2 (to 3 s.f.)

(ii) Using Pythagoras' Theorem,

$h = \sqrt{15^2 - 7^2}$

$= 13.27$ cm (to 4 s.f.)

Volume of solid $= 2 \times \frac{1}{3}\pi r^2 h$

$= \frac{2}{3} \times \pi \times 7^2 \times 13.27$

$= 1360$ cm^3 (to 3 s.f.)

18. Total surface area of cone $= 1240$ m^2

$\pi(13.5)l + \pi(13.5)^2 = 1240$

$13.5\pi l + 182.25\pi = 1240$

$13.5\pi l = 1240 - 182.25\pi$

$\therefore l = \frac{1240 - 182.25\pi}{13.5\pi}$

$= 15.74$ m (to 4 s.f.)

Using Pythagoras' Theorem,

$h = \sqrt{15.74^2 - 13.5^2}$

$= 8.093$ m (to 4 s.f.)

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 13.5^2 \times 8.093$

$= 1540$ m^3 (to 3 s.f.)

Exercise 12C

1. (a) Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 8^3$

$= 682\frac{2}{3}\pi$

$= 2140$ cm^3 (to 3 s.f.)

(b) Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 14^3$

$= 3658\frac{2}{3}\pi$

$= 11\,500$ mm^3 (to 3 s.f.)

(c) Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 4^3$

$= 85\frac{1}{3}\pi$

$= 268$ m^3 (to 3 s.f.)

2. (a) Volume of sphere $= 1416$ cm^3

$\frac{4}{3}\pi r^3 = 1416$

$r^3 = \frac{1062}{\pi}$

$\therefore r = \sqrt[3]{\frac{1062}{\pi}}$

$= 6.97$ cm (to 3 s.f.)

(b) Volume of sphere $= 12\,345$ mm^3

$\frac{4}{3}\pi r^3 = 12\,345$

$r^3 = \frac{37\,035}{4\pi}$

$\therefore r = \sqrt[3]{\frac{37\,035}{4\pi}}$

$= 14.3$ mm (to 3 s.f.)

(c) Volume of sphere $= 780$ m^3

$\frac{4}{3}\pi r^3 = 780$

$r^3 = \frac{585}{\pi}$

$\therefore r = \sqrt[3]{\frac{585}{\pi}}$

$= 5.71$ m (to 3 s.f.)

(d) Volume of sphere $= 972\pi$ cm^3

$\frac{4}{3}\pi r^3 = 972\pi$

$r^3 = 729$

$\therefore r = \sqrt[3]{729}$

$= 9$ cm

(e) Volume of sphere $= 498\pi$ mm^3

$\frac{4}{3}\pi r^3 = 498\pi$

$r^3 = 373\frac{1}{2}$

$\therefore r = \sqrt[3]{373\frac{1}{2}}$

$= 7.20$ mm (to 3 s.f.)

(f) Volume of sphere $= 15\frac{3}{16}\pi$ m^3

$\frac{4}{3}\pi r^3 = 15\frac{3}{16}\pi$

$r^3 = \frac{729}{64}$

$\therefore r = \sqrt[3]{\frac{729}{64}}$

$= 2.25$ m (to 3 s.f.)

3. (a) Surface area of sphere $= 4\pi r^2$

$= 4 \times \pi \times 12^2$

$= 576\pi$

$= 1810$ cm^2 (to 3 s.f.)

(b) Surface area of sphere $= 4\pi r^2$

$= 4 \times \pi \times 9^2$

$= 324\pi$

$= 1020$ mm^2 (to 3 s.f.)

(c) Surface area of sphere $= 4\pi r^2$

$= 4 \times \pi \times 3^2$

$= 36\pi$

$= 113$ m^2 (to 3 s.f.)

4. Total surface area of hemisphere $= \pi r^2 + \frac{1}{2} \times 4\pi r^2$

$= 3\pi r^2$

$= 3 \times \pi \times 7^2$

$= 147\pi$

$= 147 \times 3.142$

$= 462$ cm^2 (to 3 s.f.)

5. (a) Surface area of sphere $= 210$ cm^2

$4\pi r^2 = 210$

$r^2 = \frac{210}{4\pi}$

$\therefore r = \sqrt{\frac{210}{4\pi}}$ (since $r > 0$)

$= 4.09$ cm (to 3 s.f.)

(b) Surface area of sphere $= 7230$ mm^2

$4\pi r^2 = 7230$

$r^2 = \frac{7230}{4\pi}$

$\therefore r = \sqrt{\frac{7230}{4\pi}}$ (since $r > 0$)

$= 24.0$ mm (to 3 s.f.)

(c) Surface area of sphere $= 3163$ m^2

$4\pi r^2 = 3163$

$r^2 = \frac{3163}{4\pi}$

$\therefore r = \sqrt{\frac{3163}{4\pi}}$ (since $r > 0$)

$= 15.9$ m (to 3 s.f.)

(d) Surface area of sphere $= 64\pi$ cm^2

$4\pi r^2 = 64\pi$

$r^2 = 16$

$\therefore r = \sqrt{16}$ (since $r > 0$)

$= 4$ cm

(e) Surface area of sphere $= 911\pi \text{ mm}^2$

$$4\pi r^2 = 911\pi$$

$$r^2 = \frac{911}{4}$$

$$\therefore r = \sqrt{\frac{911}{4}} \text{ (since } r > 0)$$

$$= 15.1 \text{ mm (to 3 s.f.)}$$

(f) Surface area of sphere $= 49\pi \text{ m}^2$

$$4\pi r^2 = 49\pi$$

$$r^2 = \frac{49}{4}$$

$$\therefore r = \sqrt{\frac{49}{4}} \text{ (since } r > 0)$$

$$= 3.5 \text{ m}$$

6. Curved surface area of hemisphere $= 364.5\pi \text{ cm}^2$

$$\frac{1}{2} \times 4\pi r^2 = 364.5\pi$$

$$2\pi r^2 = 364.5\pi$$

$$r^2 = \frac{364.5}{2}$$

$$\therefore r = \sqrt{\frac{364.5}{2}} \text{ (since } r > 0)$$

$$= 13.5 \text{ cm}$$

7. Radius of a ball bearing $= 0.7 \div 2$

$$= 0.35 \text{ cm}$$

Volume of a ball bearing $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \pi \times 0.35^3$$

$$= 0.1796 \text{ cm}^3 \text{ (to 4 s.f.)}$$

Mass of a ball bearing $= 0.1796 \times 7.85$

$$= 1.40986 \text{ g}$$

Number of ball bearings $= \frac{1000}{1.40986}$

$$= 709 \text{ (to the nearest whole number)}$$

8. Volume of hollow aluminium sphere

$$= \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \pi \times 30^3 - \frac{4}{3} \times \pi \times 20^3$$

$$= 36\,000\pi - 10\,666\frac{2}{3}\pi$$

$$= 25\,333\frac{1}{3}\pi \text{ cm}^3$$

Mass of hollow aluminium sphere $= 25\,333\frac{1}{3}\pi \times 2.7$

$$= 215\,000 \text{ g (to 3 s.f.)}$$

$$= 215 \text{ kg}$$

9. Radius of hemisphere $= 2 \div 2$

$$= 1 \text{ cm}$$

Volume of a hemisphere $= \frac{1}{2} \times \frac{4}{3}\pi r^3$

$$= \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \pi \times 1^3$$

$$= \frac{2}{3}\pi \text{ cm}^3$$

Volume of sphere $= \frac{2}{3}\pi \times 54$

$$\frac{4}{3}\pi R^3 = 36\pi$$

$$R^3 = 27$$

$$\therefore R = \sqrt[3]{27}$$

$$= 3 \text{ cm}$$

10. Radius of sphere $= 26.4 \div 2$

$$= 13.2 \text{ cm}$$

Volume of acid in the sphere $= \frac{1}{2} \times \frac{4}{3}\pi r^3$

$$= \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \pi \times 13.2^3$$

$$= 1533.312\pi \text{ cm}^3$$

Radius of beaker $= 16 \div 2$

$$= 8 \text{ cm}$$

Volume of acid in the beaker $= 1533.312\pi$

$$\pi R^2 d = 1533.312\pi$$

$$\pi \times 8^2 \times d = 1533.312\pi$$

$$64\pi d = 1533.312\pi$$

$$\therefore d = \frac{1533.312}{64}$$

$$= 24.0 \text{ cm (to 3 s.f.)}$$

The depth of the acid in the beaker is 24.0 cm.

11. Radius of cylindrical tin $= 18 \div 2$

$$= 9 \text{ cm}$$

Volume of water in the cylindrical tin $= \pi r^2 h$

$$= \pi \times 9^2 \times 13.2$$

$$= 1069.2\pi \text{ cm}^3$$

Radius of spherical ball bearing $= 9.3 \div 2$

$$= 4.65 \text{ cm}$$

Volume of spherical ball bearing $= \frac{4}{3}\pi R^3$

$$= \frac{4}{3} \times \pi \times 4.65^3$$

$$= 134.0595\pi \text{ cm}^3$$

Volume of water and spherical ball bearing

$$= 1069.2\pi + 134.0595\pi$$

Volume in the cylindrical tin = 1203.2595π

$$\pi \times 9^2 \times H = 1203.2595\pi$$
$$81\pi H = 1203.2595\pi$$
$$\therefore H = \frac{1203.2595}{81}$$
$$= 14.86 \text{ cm (to 2 d.p.)}$$

The new height of water in the tin is 14.86 cm.

12. Volume of sphere = 850 m^3

$$\frac{4}{3}\pi r^3 = 850$$
$$r^3 = \frac{1275}{2\pi}$$
$$\therefore r = \sqrt[3]{\frac{1275}{2\pi}}$$
$$= 5.876 \text{ m (to 4 s.f.)}$$

Surface area of sphere = $4\pi r^2$

$$= 4 \times \pi \times 5.876^3$$
$$= 434 \text{ m}^2 \text{ (to 3 s.f.)}$$

13. Surface area of basketball = 1810 cm^2

$$4\pi r^2 = 1810$$
$$r^2 = \frac{1810}{4\pi}$$
$$\therefore r = \sqrt{\frac{1810}{4\pi}} \quad (\text{since } r > 0)$$
$$= 12.00 \text{ cm (to 4 s.f.)}$$

Volume of basketball = $\frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \pi \times 12.00^3$$
$$= 2304\pi$$
$$= 7240 \text{ cm}^3 \text{ (to 3 s.f.)}$$

14. Curved surface area of hemisphere = $\frac{1}{2} \times 4\pi r^2$

$$= 2\pi r^2 \text{ units}^2$$

Flat surface area of hemisphere = πr^2 units^2

Ratio of red paint to yellow paint = $2\pi r^2 : \pi r^2$

$$= 2 : 1$$

15. **(i)** Radius of sphere = radius of cylindrical can

$$= 3.4 \text{ cm}$$

Diameter of sphere = 3.4×2

$$= 6.8 \text{ cm}$$

Depth of water in the can when the sphere was placed inside

= 6.8 cm

Surface area of can in contact with water

$= \pi r^2 + 2\pi rh$

$= \pi \times 3.4^2 + 2 \times \pi \times 3.4 \times 6.8$

$= 11.56\pi + 46.24\pi$

$= 57.8\pi$

$= 182 \text{ cm}^2$ (to 3 s.f.)

(ii) Volume of water in the can when the sphere was placed inside

$= \pi r^2 h - \frac{4}{3}\pi r^3$

$= \pi \times 3.4^2 \times 6.8 - \frac{4}{3} \times \pi \times 3.4^3$

$= 3.4^2\pi\left(6.8 - 4\frac{8}{15}\right)$

$= \left(3.4^2 \times 2\frac{4}{15} \times \pi\right) \text{ cm}^3$

Let the depth of water in the can before the sphere was placed inside be d cm.

Volume of water = $\left(3.4^2 \times 2\frac{4}{15} \times \pi\right) \text{ cm}^3$

$$\pi r^2 d = 3.4^2 \times 2\frac{4}{15} \times \pi$$
$$\pi(3.4^2)d = 3.4^2 \times 2\frac{4}{15} \times \pi$$
$$\therefore d = \frac{3.4^2 \times 2\frac{4}{15} \times \pi}{3.4^2\pi}$$
$$= 2\frac{4}{15}$$

The depth of water in the can was $2\frac{4}{15}$ cm.

Exercise 12D

1. Radius of cylinder = $12 \div 2$

$$= 6 \text{ m}$$

Total surface area of rocket

= Flat surface of cylinder + curved surface area of cylinder + curved surface area of cone

$= \pi \times 6^2 + 2 \times \pi \times 6 \times 42 + \pi \times 6 \times 15$

$= 36\pi + 504\pi + 90\pi$

$= 630\pi$

$= 1980 \text{ m}^2$ (to 3 s.f.)

2. Volume of remaining solid

= Volume of cylinder – volume of cone

$= \pi \times 6^2 \times 15 - \frac{1}{3} \times \pi \times 3^2 \times 15$

$= 540\pi - 45\pi$

$= 495\pi$

$= 1560 \text{ cm}^3$ (to 3 s.f.)

3. **(i)** Volume of the solid

= Volume of hemisphere + volume of cylinder

$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 7^3 + \pi \times 7^2 \times 10$

$= 228\frac{2}{3}\pi + 490\pi$

$= 718\frac{2}{3}\pi$

$= 2260 \text{ cm}^3$ (to 3 s.f.)

(ii) Total surface area of the solid

$= \text{Flat surface of cylinder} + \text{curved surface area of cylinder} + \text{curved surface area of hemisphere}$

$= \pi \times 7^2 + 2 \times \pi \times 7 \times 10 + \frac{1}{2} \times 4 \times \pi \times 7^2$

$= 49\pi + 140\pi + 98\pi$

$= 287\pi$

$= 902 \text{ cm}^2$ (to 3 s.f.)

4. **(i)** Volume of the solid

$= \text{Volume of hemisphere} + \text{volume of cone}$

$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 21^3 + \frac{1}{3} \times \pi \times 21^2 \times 28$

$= 6174\pi + 4116\pi$

$= 10\,290\pi$

$= 32\,300 \text{ cm}^3$ (to 3 s.f.)

(ii) Total surface area of the solid

$= \text{Curved surface area of hemisphere} + \text{curved surface area of cone}$

$= \frac{1}{2} \times 4 \times \pi \times 21^2 + \pi \times 21 \times 35$

$= 882\pi + 73.5\pi$

$= 1617\pi$

$= 5080 \text{ cm}^3$ (to 3 s.f.)

5. $7\ l = 7000 \text{ cm}^3$

Height of the cone $= \frac{3}{5} \times 4r$

$= \frac{12r}{5}$ cm

Volume of cone $= 7000 \text{ cm}^3$

$\frac{1}{3} \times \pi \times r^2 \times \frac{12r}{5} = 7000$

$\frac{4}{5}\pi r^3 = 7000$

$r^3 = \frac{8750}{\pi}$

Volume of cylindrical container $= \pi r^2 h$

$= \pi \times r^2 \times 4r$

$= 4\pi r^3$

$= 4\pi\left(\frac{8750}{\pi}\right)$

$= 35\,000 \text{ cm}^3$

Amount of water needed $= 7000 + 35\,000$

$= 42\,000 \text{ cm}^3$

$= 42\ l$

6. **(i)** Radius of cylinder $= 8 \div 2$

$= 4$ m

Total surface area of solid cylinder with conical ends

$= 2 \times \text{curved surface area of cone} + \text{curved surface area of cylinder}$

$= 2 \times \pi \times 4 \times 6 + 2 \times \pi \times 4 \times 8$

$= 48\pi + 64\pi$

$= 112\pi$

$= 352 \text{ m}^2$ (to 3 s.f.)

(ii) Using Pythagoras' Theorem,

$h = \sqrt{6^2 - 4^2}$

$= 4.472$ m (to 4 s.f.)

Volume of the solid cylinder with conical ends

$= 2 \times \text{volume of cone} + \text{volume of cylinder}$

$= 2 \times \frac{1}{3} \times \pi \times 4^2 \times 4.472 + \pi \times 4^2 \times 8$

$= 552 \text{ m}^3$ (to 3 s.f.)

7. Radius of the cylinder $= 4.7 \div 2$

$= 2.35$ m

Height of cylinder $= 16.5 - 2.35$

$= 14.15$ m

Capacity of tank

$= \text{volume of hemisphere} + \text{volume of cylinder}$

$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 2.35^3 + \pi \times 2.35^2 \times 14.15$

$= 273 \text{ m}^3$ (to 3 s.f.)

8. Volume of cone $=$ volume of ball

$\frac{1}{3} \times \pi \times 4^2 \times h = \frac{4}{3} \times \pi \times 3^2$

$\frac{16}{3}\pi h = 36\pi$

$\therefore h = 36 \times \frac{3}{16}$

$= 6.75$ cm

9. **(i)** Volume of cone $= 1\frac{1}{5} \times$ volume of hemisphere

$\frac{1}{3} \times \pi \times 35^2 \times h = 1\frac{1}{5} \times \frac{1}{2} \times \frac{4}{3} \times \pi \times 35^3$

$408\frac{1}{3}\pi h = 34\,300\pi$

$\therefore h = 34\,300 \div 408\frac{1}{3}$

$= 84$ cm

The height of the cone is 84 cm.

(ii) Using Pythagoras' Theorem,

$l = \sqrt{84^2 + 35^2}$

$= 91$ cm

Total surface area of the solid

$= \text{Curved surface area of cone} + \text{curved surface area of hemisphere}$

$= \pi \times 35 \times 91 + \frac{1}{2} \times 4 \times \pi \times 35^2$

$= 3185\pi + 2450\pi$

$= 5635\pi \text{ cm}^2$

10. **(i)** Volume of the solid $=$ volume of pyramid $+$ volume of cuboid

$= \frac{1}{3} \times 30 \times 30 \times 28 + 30 \times 30 \times 40$

$= 8400 + 36\,000$

$= 44\,400 \text{ cm}^3$

(ii) Using Pythagoras' Theorem,

slant height of pyramid, $l = \sqrt{28^2 + 15^2}$

$= 31.76$ cm (to 4 s.f.)

Total surface area of the solid

= Total surface area of visible sides of cuboid + total surface area of all triangular faces of pyramid

$= (30 \times 30 + 4 \times 30 \times 40) + \left(4 \times \frac{1}{2} \times 30 \times 31.76\right)$

$= 5700 + 1905.6$

$= 7610$ cm^2 (to 3 s.f.)

Review Exercise 12

1. **(a)** **(i)** Volume of the solid

= Volume of pyramid + volume of cuboid

$= \frac{1}{3} \times 20 \times 20 \times 24 + 20 \times 20 \times 50$

$= 3200 + 20\,000$

$= 25\,200$ cm^3

(ii) Total surface area of the solid

= Total surface area of visible sides of cuboid + total surface area of all triangular faces of pyramid

$= (20 \times 20 + 4 \times 20 \times 50) + \left(4 \times \frac{1}{2} \times 20 \times 26\right)$

$= 4400 + 1040$

$= 5440$ cm^2

(b) **(i)** Volume of the solid

$= 2 \times$ volume of cone + volume of cylinder

$= 2 \times \frac{1}{3} \times \pi \times 0.5^2 \times 1.2 + \pi \times 0.5^2 \times 2.5$

$= \frac{1}{5}\pi + \frac{5}{8}\pi$

$= \frac{33}{40}\pi$

$= 2.59$ m^3 (to 3 s.f.)

(ii) Using Pythagoras' Theorem,

slant height of cone, $l = \sqrt{1.2^2 + 0.5^2}$

$= 1.3$ m

Total surface area of the solid

$= 2 \times$ curved surface area of cone + curved surface area of cylinder

$= 2 \times \pi \times 0.5 \times 1.3 + 2 \times \pi \times 0.5 \times 2.5$

$= 1.3\pi + 2.5\pi$

$= 3.8\pi$

$= 11.9$ m^2 (to 3 s.f.)

(c) **(i)** Volume of the hemisphere

$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 40^3$

$= 42\,666\frac{2}{3}\pi$

$= 134\,000$ cm^3 (to 3 s.f.)

(ii) Total surface area of the hemisphere

$= \frac{1}{2} \times 4 \times \pi \times 40^2 + \pi \times 40^2$

$= 3200\pi + 1600\pi$

$= 4800\pi$

$= 15\,100$ cm^2 (to 3 s.f.)

(d) **(i)** Volume of the solid

$= 2 \times$ volume of hemisphere + volume of cylinder

$= 2 \times \frac{1}{2} \times \frac{4}{3} \times \pi \times 3.5^3 + \pi \times 3.5^2 \times 4$

$= 334$ m^3 (to 3 s.f.)

(ii) Total surface area of the solid

$= 2 \times$ curved surface area of hemisphere + curved surface area of cylinder

$= 2 \times \frac{1}{2} \times 4 \times \pi \times 3.5^2 + 2 \times \pi \times 3.5 \times 4$

$= 49\pi + 28\pi$

$= 77\pi$

$= 242$ m^2 (to 3 s.f.)

2. **(i)** Volume of the structure

= volume of the cone + volume of hemisphere

$= \frac{1}{3} \times \pi \times 5^2 \times 20 + \frac{1}{2} \times \frac{4}{3} \times \pi \times 4^3$

$= 166\frac{2}{3}\pi + 42\frac{2}{3}\pi$

$= 209\frac{1}{3}\pi$

$= 658$ cm^3 (to 3 s.f.)

(ii) Using Pythagoras' Theorem,

slant height, $l = \sqrt{20^2 + 5^2}$

$= 20.62$ cm

Total surface area of the structure

= Total surface area of cone + total surface area of hemisphere

$= (\pi \times 5^2 + \pi \times 5 \times 20.62) + \left(\pi \times 4^2 + \frac{1}{2} \times 4 \times \pi \times 4^2\right)$

$= 128.1\pi + 48\pi$

$= 176.1\pi$

$= 553$ cm^2 (to 3 s.f.)

3. Volume of the rocket

= volume of cone + volume of cylinder

$= \frac{1}{3} \times \pi \times 18^2 \times 49 + \pi \times 18^2 \times 192$

$= 5292\pi + 62\,208\pi$

$= 67\,500\pi$ cm^3

$= 0.0675\pi$ m^3

Density of metal $= \frac{2145}{0.0675\pi}$

$= 10\,115$ kg/m^3 (to the nearest whole number)

4. Surface area of first sphere $= 144\pi$ cm^2

$4\pi r_1^2 = 144\pi$

$r_1^2 = 36$

$\therefore r_1 = \sqrt{36}$ (since $r_1 > 0$)

$= 6$ cm

Volume of first sphere $= \frac{4}{3}\pi r_1^3$

$= \frac{4}{3} \times \pi \times 6^3$

$= 288\pi$ cm^3

Surface area of second sphere = 256π cm^2

$$4\pi r_2^2 = 256\pi$$
$$r_2^2 = 64$$
$$\therefore r_2 = \sqrt{64} \text{ (since } r_2 > 0)$$
$$= 8 \text{ cm}$$

Volume of second sphere $= \frac{4}{3}\pi r_2^3$

$$= \frac{4}{3} \times \pi \times 8^3$$
$$= 682\frac{2}{3}\pi \text{ cm}^3$$

Volume of larger sphere $= 288\pi + 682\frac{2}{3}\pi$

$$\frac{4}{3}\pi R^3 = 970\frac{2}{3}\pi \text{ cm}^3$$
$$R^3 = 728$$
$$\therefore R = \sqrt[3]{728}$$
$$= 8.996 \text{ cm (to 4 s.f.)}$$

Surface area of larger sphere $= 4\pi R^2$

$$= 4 \times \pi \times 8.996^2$$
$$= 1020 \text{ cm}^2 \text{ (to 3 s.f.)}$$

5. **(i)** External radius = $12 \div 2$

= 6 cm

Internal diameter = $12 - 2 - 2$

= 8 cm

Internal radius = $8 \div 2$

= 4 cm

Volume of hollow sphere $= \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \pi \times 6^3 - \frac{4}{3} \times \pi \times 4^3$$
$$= 288\pi - 85\frac{1}{3}\pi$$
$$= 202\frac{2}{3}\pi \text{ cm}^3$$

Mass of hollow sphere $= 202\frac{2}{3}\pi \times 5.4$

= 3438.159 g

= 3.44 kg (to 3 s.f.)

(ii) Volume of solid sphere $= 202\frac{2}{3}\pi$ cm^3

$$\frac{4}{3} \times \pi \times r_s^3 = 202\frac{2}{3}\pi$$
$$r_s^3 = 152$$
$$\therefore r_s = \sqrt[3]{152}$$
$$= 5.34 \text{ cm (to 3 s.f.)}$$

6. **(i)** Number of drops = $5000 \div 12.5$

= 400

(ii) Volume of one drop of oil = 12.5 mm^3

$$\frac{4}{3} \times \pi \times r^3 = 12.5$$
$$r^3 = \frac{75}{8\pi}$$
$$\therefore r = \sqrt[3]{\frac{75}{8\pi}}$$
$$= 1.44 \text{ mm (to 3 s.f.)}$$

7. Let the radius of the cylinder and sphere be r units.

Surface area of the sphere = $4\pi r^2$ units^2

Curved surface area of cylinder = $2\pi rh$

$$= 2 \times \pi \times r \times 2r$$
$$= 4\pi r^2 \text{ units}^2$$

$\therefore$ Surface area of the sphere

= curved surface area of cylinder (shown)

8. Radius of hemispherical roof = $10 \div 2$

= 5 m

Curved surface area of hemispherical roof $= \frac{1}{2} \times 4\pi r^2$

$$= 2 \times \pi \times 5^2$$
$$= 50\pi \text{ m}^2$$

Cost of painting = $50\pi \times \$1.50$

= \$235.62 (to the nearest cent)

9. **(i)** External radius = $50.8 \div 2$

= 25.4 cm

Internal diameter = $50.8 - 2.54 - 2.54$

= 45.72 cm

Internal radius = $45.72 \div 2$

= 22.86 cm

Volume of metal hemispherical bowl

$$= \frac{1}{2} \times \frac{4}{3}\pi R^3 - \frac{1}{2} \times \frac{4}{3}\pi r^3$$
$$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 25.4^3 - \frac{1}{2} \times \frac{4}{3} \times \pi \times 22.86^3$$

= 9300 cm^3 (to 4 s.f.)

= 0.009 300 m^3

Density of metal $= \frac{97.9}{0.009\,300}$

= 10 500 kg/m^3 (to 3 s.f.)

(ii) Volume of liquid in the bowl

$$= \frac{1}{2} \times \frac{4}{3}\pi r^3$$
$$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 22.86^3$$

= 25 020 cm^3 (to 4 s.f.)

= 0.02502 m^3

Mass of the liquid = 31.75×0.02502

= 0.794 kg (to 3 s.f.)

= 794 g

10. (i) Radius of capsule $A = 0.6 \div 2$

$= 0.3$ cm

Surface area of capsule A

$= 2 \times$ curved surface area of hemisphere + curved surface area of cylinder

$= 2 \times \frac{1}{2} \times 4 \times \pi \times 0.3^2 + 2 \times \pi \times 0.3 \times 2.4$

$= 0.36\pi + 1.44\pi$

$= 1.8\pi \text{ cm}^2$

Surface area of capsule $B = 1.8\pi \text{ cm}^2$

$2 \times \pi \times 0.6^2 + 2 \times \pi \times 0.6 \times h = 1.8\pi$

$0.72\pi + 1.2\pi h = 1.8\pi$

$1.2\pi h = 1.08\pi$

$\therefore h = \frac{1.08}{1.2}$

$= 0.9$ cm

(ii) Volume of capsule A

$= 2 \times$ volume of hemisphere + volume of cylinder

$= 2 \times \frac{1}{2} \times \frac{4}{3} \times \pi \times 0.3^3 + \pi \times 0.3^2 \times 2.4$

$= 0.036\pi + 0.216\pi$

$= 0.252\pi$

$= 0.792 \text{ cm}^3$ (to 3 s.f.)

Volume of capsule B

$= \pi \times 0.6^2 \times 0.9$

$= 0.324\pi$

$= 1.02 \text{ cm}^3$ (to 3 s.f.)

11. Radius of pillar $= 40 \div 2$

$= 20$ cm

Since the pillar has the same mass as a solid stone sphere of the same material,

$\therefore$ the pillar has the same volume as the solid stone sphere.

Volume of solid stone sphere $= \frac{4}{3}\pi R^3$

$= \frac{4}{3} \times \pi \times 40^3$

$= \frac{256\ 000}{3}\pi \text{ cm}^3$

Volume of pillar $= \frac{256\ 000}{3}\pi \text{ cm}^3$

$\pi \times 20^2 \times h + \frac{1}{2} \times \frac{4}{3} \times \pi \times 20^3 = \frac{256\ 000}{3}\pi$

$400\pi h + \frac{16\ 000}{3}\pi = \frac{256\ 000}{3}\pi$

$400\pi h = 80\ 000\pi$

$\therefore h = 200$ cm

12. Radius of cylinder and cone $= 2r \div 2$

$= r$ units

Radius of sphere $= 2r \div 2$

$= r$ units

Volume of cylinder $= \pi \times r^2 \times 2r$

$= 2\pi r^3 \text{ units}^3$

Volume of cone $= \frac{1}{3} \times \pi \times r^2 \times 2r$

$= \frac{2}{3}\pi r^3 \text{ units}^3$

Volume of sphere $= \frac{4}{3} \times \pi \times r^3$

$= \frac{4}{3}\pi r^3 \text{ units}^3$

Ratio of volume of cylinder to volume of cone to volume of sphere

$= 2\pi r^3 : \frac{2}{3}\pi r^3 : \frac{4}{3}\pi r^3$

$= 2 : \frac{2}{3} : \frac{4}{3}$

$= 6 : 2 : 4$

$= 3 : 1 : 2$

13. Radius of hemisphere $= 2 \div 2$

$= 1$ cm

Radius of cone $= 6 \div 2$

$= 3$ cm

Volume of hemisphere $= \frac{1}{2} \times \frac{4}{3}\pi r^3$

$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 1^3$

$= \frac{2}{3}\pi \text{ cm}^3$

Volume of cone $= 40 \times \frac{2}{3}\pi$

$\frac{1}{3} \times \pi \times 3^2 \times h = \frac{80}{3}\pi$

$3\pi h = \frac{80}{3}\pi$

$\therefore h = \frac{80}{3} \times \frac{1}{3}$

$= 8\frac{8}{9}$ cm

The height of the chocolate cone is $8\frac{8}{9}$ cm.

14. Radius of cone $= 4.2 \div 2$

$= 2.1$ cm

Volume of hemisphere $= \frac{1}{2} \times \frac{4}{3}\pi r^3$

$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 2.1^3$

$= 6.174\pi \text{ cm}^3$

Volume of cone $= 56 - 6.174\pi$

$\frac{1}{3} \times \pi \times 2.1^2 \times h = 56 - 6.174\pi$

$1.47\pi h = 56 - 6.174\pi$

$\therefore h = \frac{56 - 6.174\pi}{1.47\pi}$

$= 7.93$ cm (to 3 s.f.)

Challenge Yourself

1. Let the side of a face of a tetrahedron which is an equilateral triangle, be x cm,
the slant height of a face of a tetrahedron be l cm, and
the height of the tetrahedron be H cm.
Using Pythagoras' Theorem,

$$l = \sqrt{x^2 - \left(\frac{x}{2}\right)^2} \quad (\text{since } l > 0)$$
$$= \frac{\sqrt{3}}{2}x \text{ cm}$$

The centre of a side of a tetrahedron is $\frac{2}{3}$ of its slant height.

Using Pythagoras' Theorem,

$$H = \sqrt{x^2 - \left(\frac{2}{3} \times \frac{\sqrt{3}}{2}x\right)^2}$$
$$= \sqrt{x^2 - \frac{1}{3}x^2}$$
$$= \sqrt{\frac{2}{3}}x \text{ cm}$$

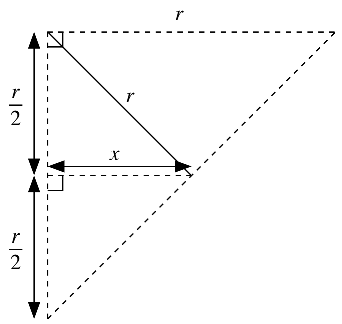
Base area of tetrahedron $= \frac{1}{2} \times x \times \frac{\sqrt{3}}{2}x$
$= \frac{\sqrt{3}}{4}x^2$

Volume of tetrahedron = 500 cm^3

$$\frac{1}{3} \times \frac{\sqrt{3}}{4}x^2 \times \sqrt{\frac{2}{3}}x = 500$$
$$x^3 = 4243 \text{ (to 4 s.f.)}$$
$$\therefore x = \sqrt[3]{4243}$$
$$= 16.19 \text{ (to 4 s.f.)}$$

Total surface area of tetrahedron = 4 × base area
$= 4 \times \frac{\sqrt{3}}{4} \times 16.19^2$
$= 454 \text{ cm}^2$ (to 3 s.f.)

2.



Using Pythagoras' Theorem,

$$x = \sqrt{r^2 - \left(\frac{r}{2}\right)^2}$$
$$= \frac{\sqrt{3}}{2}r$$

Since $x = \frac{\sqrt{3}}{2}r > \frac{r}{2}$, the water in the hemisphere is not a hemisphere on its own.

But the volume of water is **more** than the volume of a hemisphere with $\frac{r}{2}$ as its radius.

Volume of water > volume of hemisphere with radius $\frac{r}{2}$

$$= \frac{1}{2} \times \frac{4}{3} \times \pi \times \left(\frac{r}{2}\right)^3$$
$$= \frac{1}{2} \times \frac{4}{3} \times \pi \times \frac{r^3}{8}$$
$$= \frac{1}{8} \times \frac{1}{2} \times \frac{4}{3} \times \pi \times r^3$$
$$= \frac{1}{8} \times \text{volume of the bowl}$$

∴ The volume of water is **more** than $\frac{1}{8}$ of the volume of the bowl. (shown)

Chapter 13 Symmetry

TEACHING NOTES

Suggested Approach:

Many buildings and objects in our surroundings are symmetrical in shape. Teachers can make use of these real life examples to allow students to appreciate the significance of symmetry in the way things are designed (see Chapter Opener on Page 355). Teachers can also highlight that this is not the first contact students have with symmetry, as most animals and insects, and even our faces, are symmetrical in shape. Students can be encouraged to think about symmetry around them and if there are any animals that are asymmetrical (see Thinking Time on Page 358).

Section 13.1: Line Symmetry

Teachers can point out objects in the classroom with symmetrical designs, such as windows, desks and chairs. Teachers can also ask students to think about other symmetrical objects they come across daily, such as buildings or company logos etc. The link to reflection, which students have learnt in Chapter 9, can be emphasised, whereby symmetry implies that folding along the line of symmetry will give the same exact shape (see Investigation on page 357).

Section 13.2: Rotational Symmetry in Plane Figures

Teachers may bring in plane rectangles, parallelograms or squares to illustrate to students the concept of rotational symmetry. Familiar objects such as the King, Queen and Jack picture cards can be used to enhance association and appreciation of rotational symmetry of order 2.

Section 13.3: Symmetry in Triangles, Quadrilaterals and Polygons

Students can visualise the symmetrical properties of triangles, quadrilaterals and polygons through hands-on interaction by making their own paper cut-outs (see Investigation on Pages 372 to 378).

Section 13.4: Symmetry in Three Dimensions

Students may find it challenging to visualise symmetry in three dimensions, but should be able to relate to figures with infinite lines of symmetry and rotational symmetries. Teachers can bring in objects such as cans and cones to allow students to understand and envision symmetrical planes across the objects better (see Investigation on Page 380).

WORKED SOLUTIONS

Investigation (Line Symmetry in Two Dimensions)

5. The reflected image in the mirror is the same "half-shape" as the original figure on the paper.

Thinking Time (Page 358)

Most human beings and animals are symmetrical.
An example of an asymmetrical animal is the sole fish, which is edible and can be found in markets. The upper side of its body is a dark greyish brown with thicker flesh, while the underside is white with thinner flesh. Some other animals that appear asymmetrical include hermit crabs which have claws of different sizes, flatfish such as flounders which have their eyes on only one side of their bodies and the wrybill, a bird with a beak bent towards the right.

Thinking Time (Page 359)

No. Triangles that have lines of symmetry are equilateral triangles (3 lines) and isosceles triangles (1 line).

Investigation (Rotational Symmetry in Two Dimensions)

2. $\frac{1}{3}$ of a complete turn

3. $\frac{2}{3}$ of a complete turn from the original position

4. 3 thirds will have to be made for the figure to be back in its original position.

Class Discussion (Line and Rotational Symmetry in Circles)

1. Teachers can highlight to students to observe that a circle can be folded in half in an infinite number of ways, and this means that it has infinite lines of symmetry.

2. To determine rotational symmetry, teachers can prompt students to use a pen to hold the circle down in the centre while rotating it on the table. The shape of the circle will always remain, and hence the order of rotational symmetry is infinite.

Investigation (Symmetry in Triangles)

2. The triangle has one line of symmetry, shown by line AD.

3. Yes. Since ABC is an isosceles triangle with line of symmetry AD, the two triangles ABD and ACD are congruent, and thus the sides AB and AC and angles B and C can be deduced to be equal.

4. Yes. The order of rotational symmetry is 1 as it needs to be rotated one full round for the shape to be the same.

5. An equilateral triangle has 3 lines of symmetry and an order of rotational symmetry of order 3. A scalene triangle has no line of symmetry and rotational symmetry of order 1.

Investigation (Symmetry in Special Quadrilaterals)

(a) *The answers are provided in Table 13.2 on Page 376 of the textbook.*

(b) *The answers are provided in Table 13.2 on Page 376 of the textbook.*

(c) The angles directly across each other from the line of symmetry are equal.

Investigation (Symmetry in Regular Polygons)

3.

Types of quadrilateral	Number of lines of symmetry	Order of rotational symmetry
Equilateral triangle	3	$\frac{360°}{120°} = 3$
Square	4	$\frac{360°}{90°} = 4$
Regular pentagon	5	$\frac{360°}{72°} = 5$
Regular hexagon	6	$\frac{360°}{60°} = 6$
Regular heptagon	7	$\frac{360°}{51\frac{3}{7}°} = 7$
Regular octagon	8	$\frac{360°}{45°} = 8$
Regular nonagon	9	$\frac{360°}{40°} = 9$
Regular decagon	10	$\frac{360°}{36°} = 10$

Table 13.3

4. **(a)** Number of lines of symmetry = n
(b) Order of rotational symmetry = n

5. Smallest angle of rotational symmetry = $\frac{360°}{n}$

Thinking Time (Page 379)

Other planes of symmetry are the two planes from the midpoint of the height of the cuboid.

Thinking Time (Page 380)

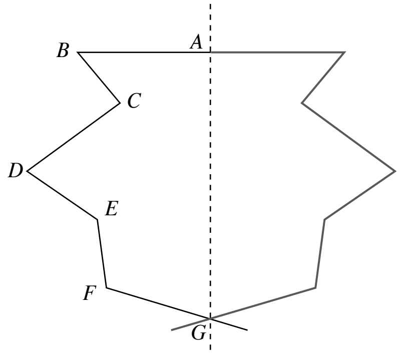
Other axes of rotational symmetry are those that pass through the other two sets of two parallel faces.

Investigation (Symmetry in Cylinders and Cones)

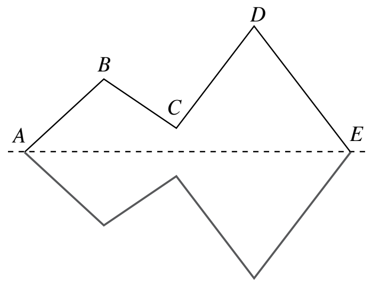
Teachers should help students develop the understanding that three-dimensional objects with a circular cross-section and/or base have infinite planes of symmetry and order of rotational symmetry.

Practise Now 1

(a)

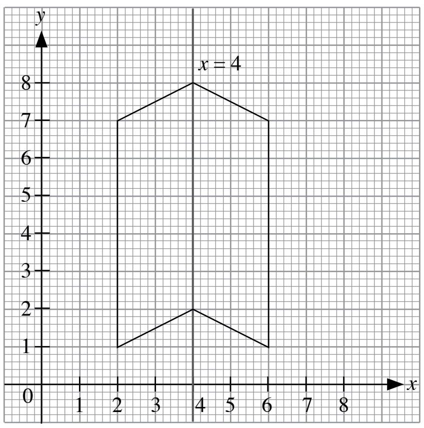


(b)

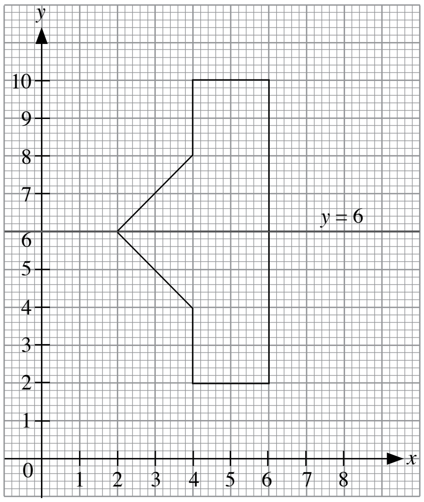


Practise Now 2

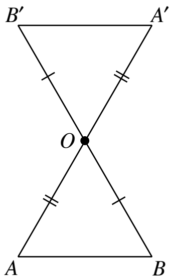
(a)



(b)



Practise Now 3



Practise Now 4

(i) Since the bowl has a circular top, it has infinite number of planes of symmetry.

(ii) It has 1 axis of rotational symmetry passing through centres of the circular top and the base.

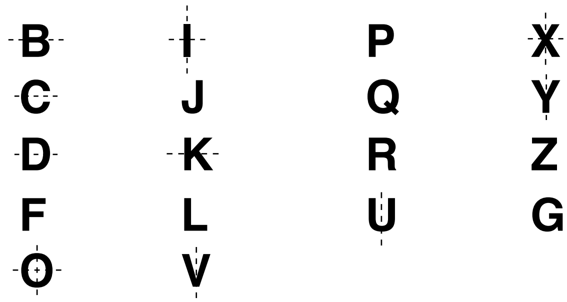
(iii) Since the bowl has a circular top, the order of rotational symmetry is infinite.

Exercise 13A

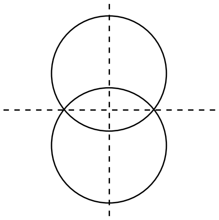
1. (a), (c), (g)

2. **(a)** 1
(b) 1
(c) 1
(d) 3
(e) 2
(f) 2
(g) 2
(h) 2
(i) 4
(j) 2
(k) 1

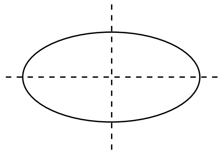
3.



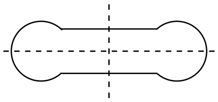
4. **(a)**



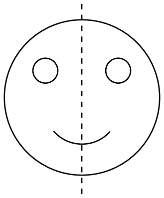
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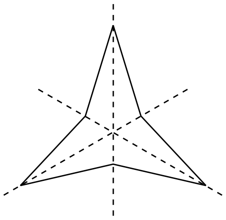
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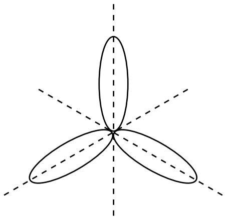
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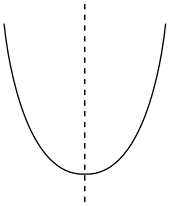
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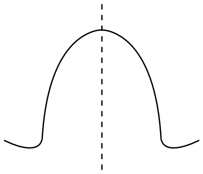
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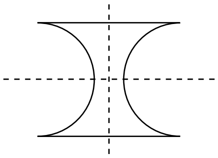
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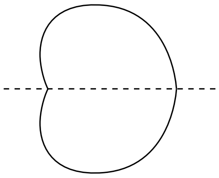
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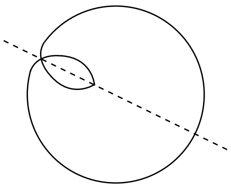
(i)



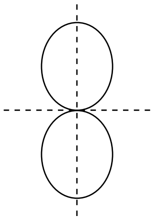
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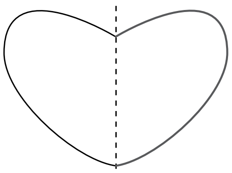
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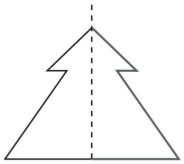
(l)



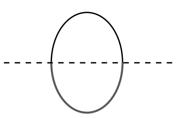
5. (a)



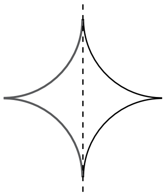
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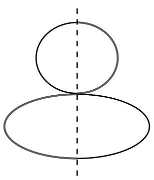
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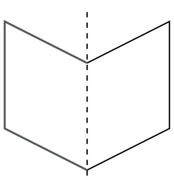
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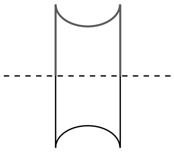
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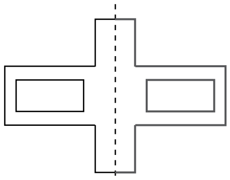
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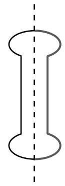
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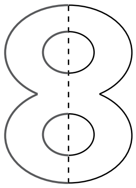
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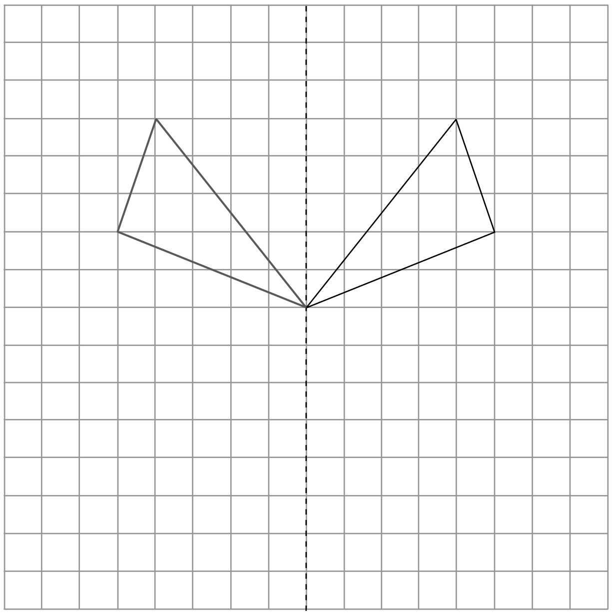
(i)



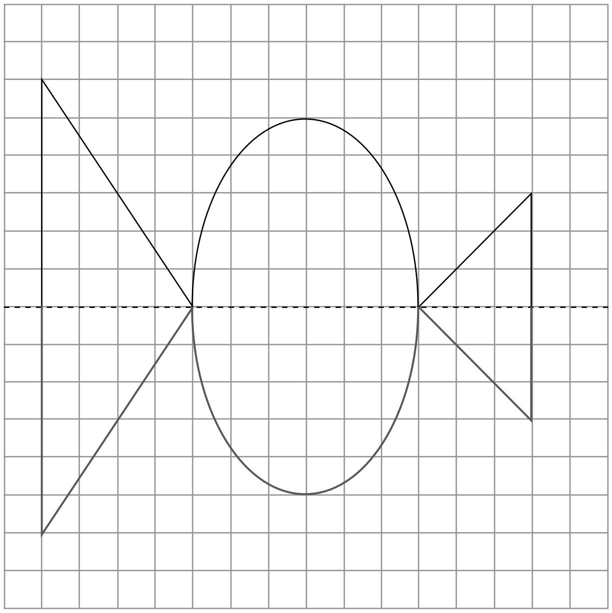
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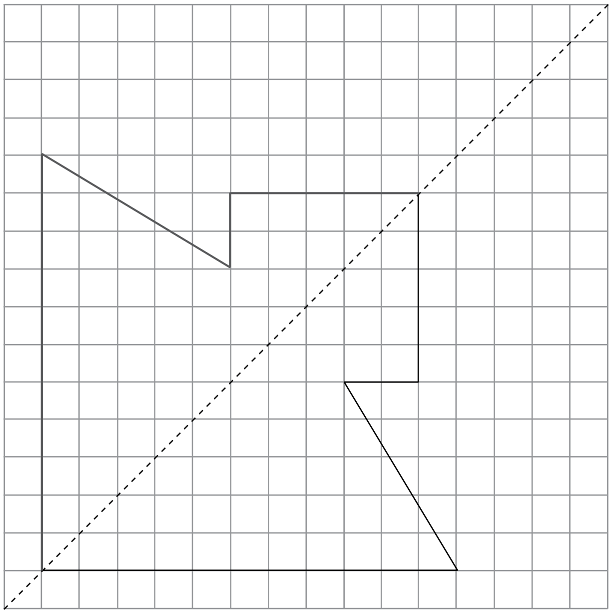
6. (a)



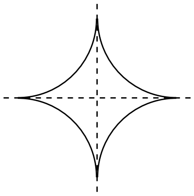
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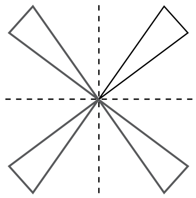
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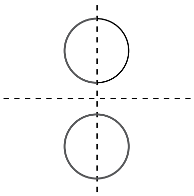
7. **(a)**



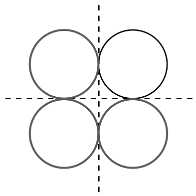
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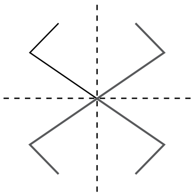
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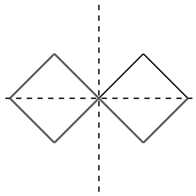
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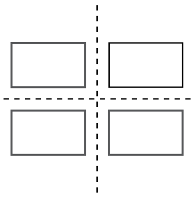
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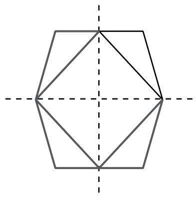
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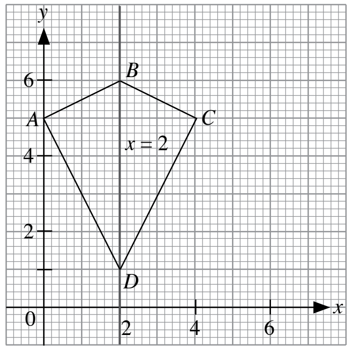
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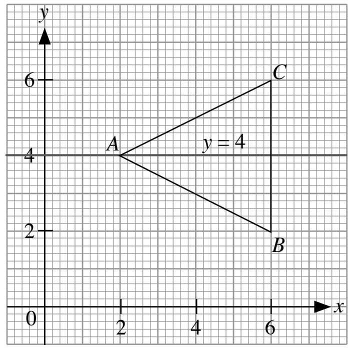
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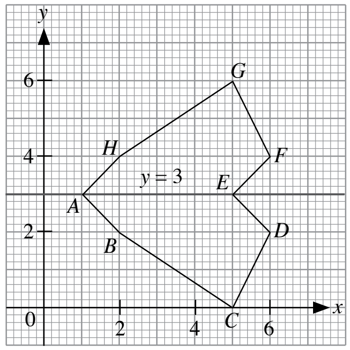
8. **(a)**



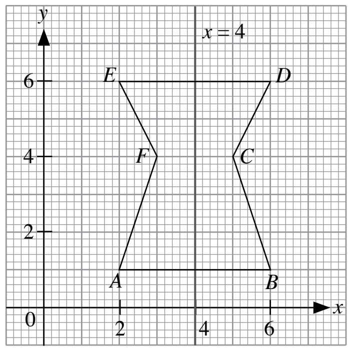
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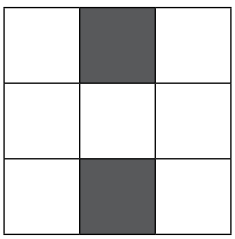
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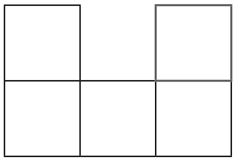


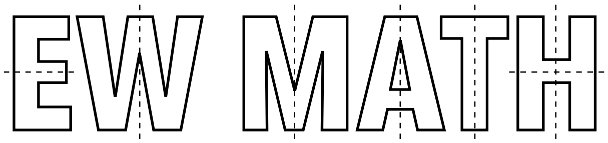
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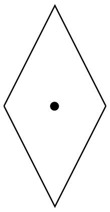
9.

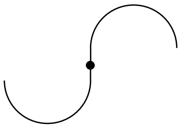


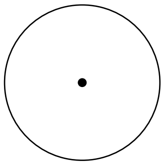
10. 

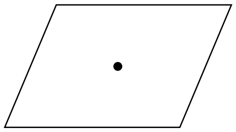
11. **(a)** W, M, A, T, H
(b) E, H
(c) H
(d) N, S
(e) 

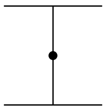
Exercise 13B

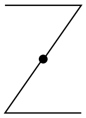
1. **(a)** 

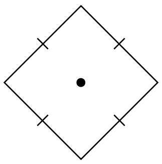
(b) 

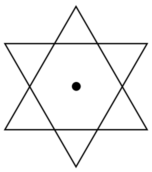
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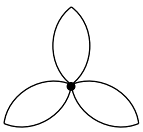
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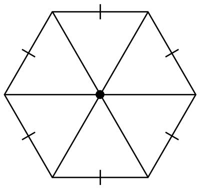
(e) 

(f) 

(g) 

(h) 

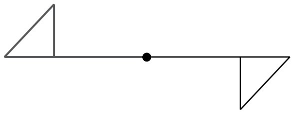
(i) 

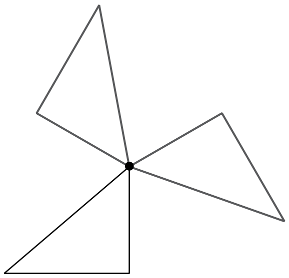
(j) 

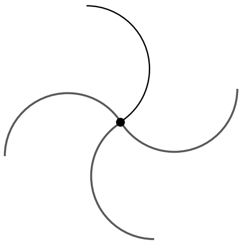
2. Since the figure can be rotated 5 times to fit the original figure, $r = 5$.

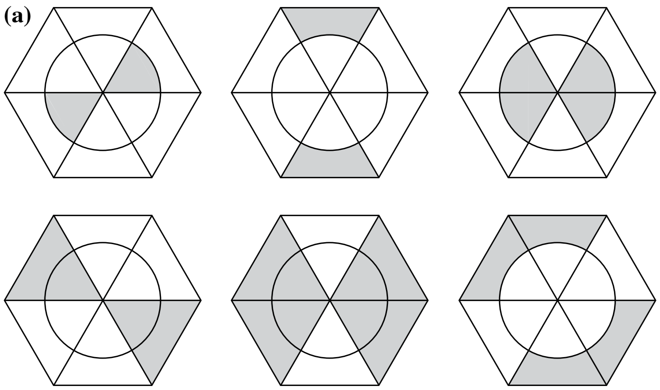
3.

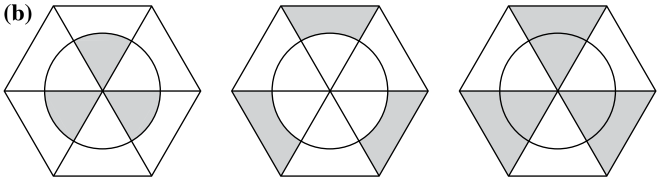
	(i) Lines of symmetry	**(ii)** Order of rotational symmetry
(a)	1	1
(b)	2	2
(c)	0	2
(d)	8	8
(e)	2	2
(f)	2	2
(g)	2	2
(h)	2	2
(i)	0	4

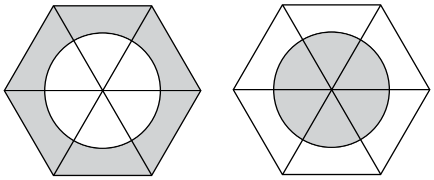
4. **(a)** 

(b) 

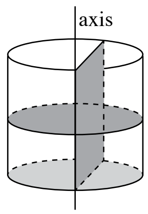
(c) 

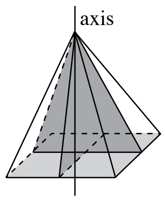
5. **(a)** 

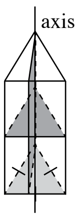
(b)


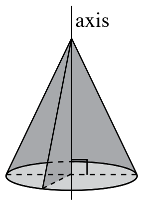
(c)


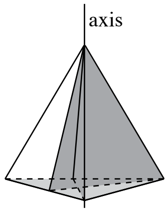
Exercise 13C

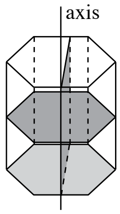
1. **(a)** True
   (b) False
   (c) True
   (d) True
   (e) True
   (f) True
   (g) False
   (h) True
   (i) False
   (j) False
2. **(a)** Since the base is a square with an order of rotational symmetry 4, the order of rotational symmetry for this pyramid is 4.
   (b) Since the base is a regular hexagon with an order of rotational symmetry 6, the order of rotational symmetry for this pyramid is 6.
   (c) Since the base is a circle, the order of rotational symmetry is infinite.
   (d) Since the ends of the axis are at the circular sides, the order of rotational symmetry is infinite.
3. **(a)**


(b)


(c)


(d)


(e)


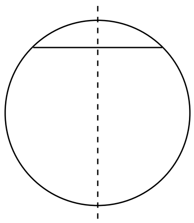
(f)


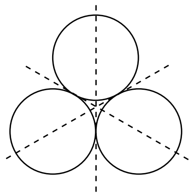
4. A right pyramid with a square base has 4 planes of symmetry.
5.

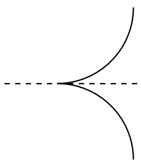
	(i) Planes of symmetry	**(ii)** Number of axes of rotational symmetry
(a)	3	3
(b)	9	13
(c)	4	4
(d)	6	6
(e)	6	4
(f)	Infinite	1
(g)	Infinite	Infinite
(h)	Infinite	1
(i)	4	1
(j)	Infinite	Infinite

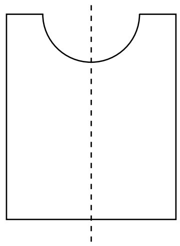
6. There are 9 planes of symmetry in a cube. Yes, a cube has more planes of symmetry than a cuboid (3 planes of symmetry) since all its sides are equal.
7. **(i)** There are 2 planes of symmetry.
   (ii) There is 1 axis of rotational symmetry.
   (iii) The order of rotational symmetry is 2.

Review Exercise 13

1. **(a)**


(b)


(c)


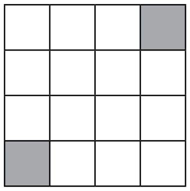
(d)


2.

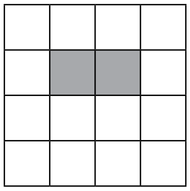
	(i) Lines of symmetry	(ii) Order of rotational symmetry
(a)	2	2
(b)	0	4
(c)	8	8
(d)	3	3
(e)	1	1
(f)	0	2
(g)	2	4

3. **(a)** 1
(b) 2
(c) 1
(d) 2
(e) 2
(f) 1
(g) 2
(h) 2

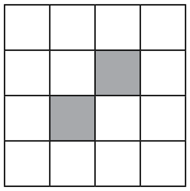
4. **(a)**



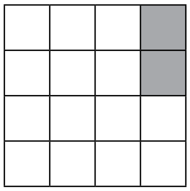
(b)



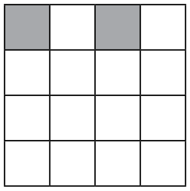
(c)



(d)

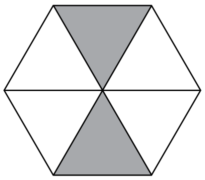


(e)

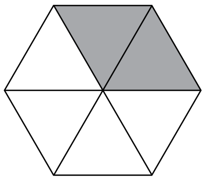


Teachers should note that there are other ways to shade the squares, and these are not the only answers.

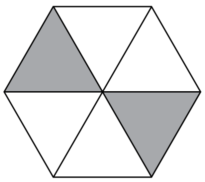
5. **(a)**



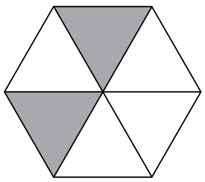
(b)



(c)

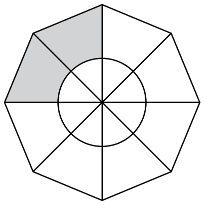


(d)

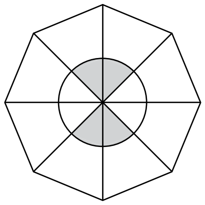


Teachers should note that there are other ways to shade the triangles, and these are not the only answers. However, students should bear in mind that the question states that "exactly two triangles" are to be shaded.

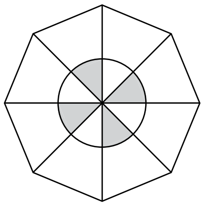
6. **(a)**



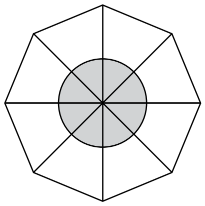
(b)



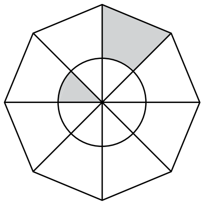
(c)



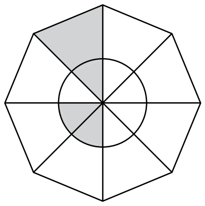
(d)



(e)



(f)

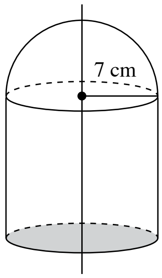


Teachers should note that there are other ways to shade the figure.

Challenge Yourself

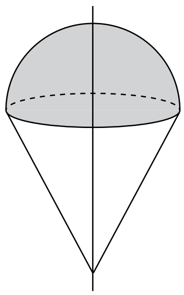
1. **(i)** Since the solid has a circular base, it has an infinite number of planes of symmetry.

(ii)



2. **(i)** Since the cone of ice cream is made up of a hemisphere with a circular base and cone with a circular top, there are infinite planes of symmetry.

(ii)



Revision Exercise C1

1. **(i)** In $\triangle ABC$, $\angle C = 90°$.
Using Pythagoras' Theorem,
$AB^2 = BC^2 + AC^2$
$(2x + 1)^2 = (2x)^2 + (x - 5)^2$

(ii) $(2x + 1)^2 = (2x)^2 + (x - 5)^2$
$4x^2 + 4x + 1 = 4x^2 + x^2 - 10x + 25$
$x^2 - 14x + 24 = 0$
$(x - 12)(x - 2) = 0$
$x - 12 = 0$ or $x - 2 = 0$
$\therefore x = 12$ or $x = 2$ (rejected, since > 5)

(iii) $BC = 2(12) = 24$ cm
$AC = 12 - 5 = 7$ cm
Area of $\triangle ABC = \frac{1}{2} \times BC \times AC$
$= \frac{1}{2} \times 24 \times 7$
$= 84 \text{ cm}^2$

2. In $\triangle LMN$,
$\tan 31° = \frac{26}{LN}$
$\therefore LN = \frac{26}{\tan 31°}$
$= 43.27$ cm (to 4 s.f.)
$HN = 43.27 - 24$
$= 19.27$ cm
In $\triangle HNM$,
$\tan \angle MHN = \frac{26}{19.27}$
$\therefore \angle MHN = \tan^{-1}\left(\frac{26}{19.27}\right)$
$= 53.5°$ (to 1 d.p.)

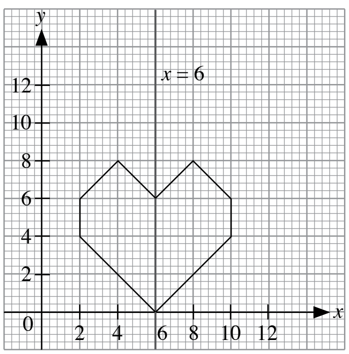
3. Let the diameter of the circular base be x cm.
The height of the cone bisects the vertical angle.
$\therefore \tan \frac{72°}{2} = \frac{\frac{x}{2}}{8.4}$
$\tan 36° = \frac{x}{16.8}$
$\therefore x = 16.8 \tan 36°$
$= 12.2$ (to 3 s.f.)
The diameter of the circular base is 12.2 cm.

4. Volume of fish tank with pyramid and filled with water
$= (0.6 \times 100) \times (0.2 \times 100) \times 27$
$= 32\ 400 \text{ cm}^3$
Volume of pyramid
$= \frac{1}{3} \times \text{base area} \times \text{height}$
$= \frac{1}{3} \times 10 \times 10 \times 27$
$= 900 \text{ cm}^3$
Volume of water in fish tank after pyramid is removed
$= 32\ 400 - 900$
$= 31\ 500 \text{ cm}^3$
Let the height of water in the fish tank after pyramid is removed be h cm.
$\therefore (0.6 \times 100) \times (0.2 \times 100) \times h = 31\ 500$
$1200h = 31\ 500$
$\therefore h = 26.25$ cm
Fall in water level $= 27 - 26.25$
$= 0.75$ cm

5. **(i)** Volume of lip balm in container $= \frac{1}{2} \times \frac{4}{3}\pi r^3$
$30 = \frac{2}{3}\pi r^3$
$r^3 = \frac{45}{\pi}$
$\therefore r = \sqrt[3]{\frac{45}{\pi}}$
$= 2.429$ (to 4 s.f.)
$= 2.43$ cm (to 3 s.f.)
The radius of the container is 2.43 cm.

(ii) Surface area in contact with lip balm
$= \frac{1}{2} \times 4\pi r^2$
$= 2\pi r^2$
$= 2 \times \pi \times 2.429^2$
$= 37.1 \text{ cm}^2$ (to 3 s.f.)

6.



7. Since the solid has a hexagonal top and base, it has an order of rotational symmetry of 6.

Revision Exercise C2

1. **(i)** In $\triangle PQR$, $\angle R = 90°$.

Using Pythagoras' Theorem,

$PQ^2 = QR^2 + PR^2$

$14.7^2 = 9.2^2 + PR^2$

$PR^2 = 14.7^2 - 9.2^2$

$= 216.09 - 84.64$

$= 131.45$

$PR = \sqrt{131.45}$ (since $PR > 0$)

$= 11.47$ cm (to 4 s.f.)

In $\triangle KQR$, $\angle R = 90°$.

Using Pythagoras' Theorem,

$KQ^2 = QR^2 + KR^2$

$11.8^2 = 9.2^2 + KR^2$

$KR^2 = 11.8^2 - 9.2^2$

$= 139.24 - 84.64$

$= 54.6$

$KR = \sqrt{54.6}$ (since $KR > 0$)

$= 7.389$ cm (to 4 s.f.)

$\therefore PK = PR - KR$

$= 11.47 - 7.389$

$= 4.08$ cm (to 3 s.f.)

(ii) In $\triangle PQR$,

$\cos \angle PQR = \frac{9.2}{14.7}$

$\therefore \angle PQR = \cos^{-1}\left(\frac{9.2}{14.7}\right)$

$= 51.26°$ (to 2 d.p.)

In $\triangle KQR$,

$\cos \angle KQR = \frac{9.2}{11.8}$

$\therefore \angle KQR = \cos^{-1}\left(\frac{9.2}{11.8}\right)$

$= 38.77°$ (to 2 d.p.)

$\angle PQK = \angle PQR - \angle KQR$

$= 51.26° - 38.77°$

$= 12.5°$ (to 1 d.p.)

2. Let $TF = x$ m and $YF = y$ m.

In $\triangle TFY$,

$\tan 63° = \frac{x}{y}$ — (1)

In $\triangle TFX$,

$\tan 56° = \frac{x}{y + 20}$ — (2)

From (1), $y = \frac{x}{\tan 63°}$ — (3)

Substitute (3) into (2):

$$\tan 56° = \frac{x}{\frac{x}{\tan 63°} + 20}$$

$$\tan 56° \left(\frac{x}{\tan 63°} + 20\right) = x$$

$$\frac{x \tan 56°}{\tan 63°} + 20 \tan 56° = x$$

$$x - \frac{x \tan 56°}{\tan 63°} = 20 \tan 56°$$

$$x\left(1 - \frac{\tan 56°}{\tan 63°}\right) = 20 \tan 56°$$

$$\therefore x = \frac{20 \tan 56°}{1 - \frac{\tan 56°}{\tan 63°}}$$

$= 121$ (to 3 s.f.)

The height of the tower is 121 m.

3. Let the length of the square base of the pyramid be x cm.

Volume of square pyramid $= 750 \div 8.05$

$= 93.17$ cm^3 (to 4 s.f.)

$\frac{1}{3} \times$ base area $\times$ height $= 93.17$ cm^3

$\frac{1}{3} \times x \times x \times 13.5 = 93.17$

$4.5x^2 = 93.17$

$x^2 = \frac{93.17}{4.5}$

$\therefore x = \sqrt{\frac{93.17}{4.5}}$ (since $x > 0$)

$= 4.55$ (to 3 s.f.)

The length of the square base of the pyramid is 4.55 cm.

4. **(a)** **(i)** Radius of core $= 7 \div 2$

$= 3.5$ cm

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 3.5^2 \times 4$

$= 16\frac{1}{3}\pi$

$= 51.31$ cm^3 (to 4 s.f.)

$= 51.3$ cm^3 (to 3 s.f.)

(ii) Using Pythagoras' Theorem,

Slant height, $l = \sqrt{4^2 + 3.5^2}$

$= 5.315$ cm (to 4 s.f.)

Total surface area of cone $= \pi rl + \pi r^2$

$= \pi \times 3.5 \times 5.315 + \pi \times 3.5^2$

$= 30.8525\pi$

$= 96.9$ cm^2 (to 3 s.f.)

(b) Volume of a sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 0.9^3$

$= 0.972\pi \text{ cm}^3$

Number of spheres that can be obtained $= \dfrac{16\frac{1}{3}\pi}{0.972\pi}$

$= 16.8$ (to 3 s.f.)

The maximum number of spheres that can be obtained is 16.

5. $x + 3y = 5(x - y)$ — (1)

$2y + 11 = 3x - 17$ — (2)

From (1),

$x + 3y = 5(x - y)$

$= 5x - 5y$

$4x = 8y$

$x = 2y$ — (3)

Substitute (3) into (2):

$2y + 11 = 3(2y) - 17$

$= 6y - 17$

$4y = 28$

$y = 7$

Substitute $y = 7$ into (3):

$x = 2(7)$

$= 14$

Length of rectangle = 14 + 3(7)

= 14 + 21

= 35 cm

Breadth of rectangle = 2(7) + 11

= 14 + 11

= 25 cm

The longest length that can be drawn on the rectangle is its diagonal.

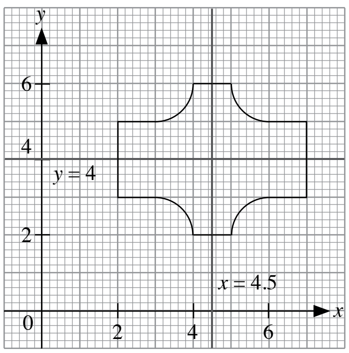
Using Pythagoras' Theorem,

Length of diagonal = $\sqrt{35^2 + 25^2}$

$= \sqrt{1850}$

$= 43.0$ cm (to 3 s.f.)

6.



7. Since the solid has a circular top and base, the order of rotational symmetry is infinite.

Chapter 14 Sets

TEACHING NOTES

Suggested Approach:

Teachers should not take an abstract approach when introducing the basic set notation, the complement of a set, and the union and intersection of sets. Teachers should try to apply the set language to describe things in daily life to arouse students' interest to learn this topic.

Section 14.1: Introduction to Set Notations

It will be a good idea to introduce this chapter by asking the students to think of sentences that relate to the collection of objects before introducing the mathematical term 'set' which is used to describe any collection of well-defined and distinct objects. It is important to engage the students to discuss the meanings of 'well-defined' and 'distinct' objects (see Class Discussion: Well-defined and Distinct Objects in a Set on page 394 of the textbook) before moving on to the different ways of representing sets as shown on page 394 of the textbook.

Teachers should advise the students that when listing, it will be good if they are to arrange the elements either in ascending order for numbers or alphabetical order for letters or according to the given order.

Students will gain a better understanding of equal sets if they are able to think of a counter-example to justify that the statement: if $n(A) = n(B)$, then $A = B$ is not valid (see Thinking Time on page 396).

Teachers should always use a simple example to introduce the different set notations as well as the meaning of equal and empty sets so that students are able to understand them easily.

Section 14.2: Venn Diagrams, Universal Set and Complement of a Set

Teachers may want to introduce Venn diagrams as a way to show the relationship between the set(s) that are under consideration. Teachers should go through some pointers (see Attention on page 399 of the textbook) when drawing the Venn diagram.

When introducing the complement of a set, it will be good if an example can be illustrated using a Venn diagram. By having the students to discuss whether the complement of a set will exist if the universal set is not defined (see Thinking Time on page 401), they can have a better understanding of the meaning of the complement of a set.

Students should be given more opportunities to discuss with each other on proper subsets (see Class Discussion: Understanding Subsets on page 402 of the textbook). It is crucial that the difference between a subset and a proper subset is discussed with the students so that they have a better understanding on proper subsets.

Section 14.3: Intersection of Two Sets

Teachers may wish to use the Chapter Opener to introduce the intersection of two sets and guide the students to think how they can represent the intersection of the two sets on the Venn diagram since all the elements in a set are distinct. Conclude that the intersection of two sets refers to the set of elements which are common to both sets and this is represented by the overlapping region of the two sets.

Teachers could use Practise Now 6, Question 1, to reinforce the meaning of subset and Practise Now 6, Question 2, to introduce disjoint sets.

Section 14.4: Union of Two Sets

Teachers should use Venn diagram to help students to visualise the meaning of the union of two sets, i.e. all the elements which are in either sets.

Section 14.5: Combining Universal Set, Complement of a Set, Subset, Intersection and Union of Sets

By recapping what was covered in the previous section, teachers may guide the students to solve problems involving universal set, intersection and union of sets in this section.

Teachers may use step-by-step approach as shown in Worked Example 9 to identify and shade the required region. It should be reinforced that for a union, shade all the regions with at least one tick; for an intersection, shade all the regions with exactly two ticks and for complement, shade all the regions without any tick.

Challenge Yourself

Question 1 involves the understanding of the terms, 'element' and 'proper subset'. Teachers may advise the students to use a Venn diagram to have a better understanding of each statement.

For Question 2, teachers may wish to use the inductive approach to lead the students to observe a pattern for the number of proper subsets when a set has *n* elements.

Question 3 involves the understanding of the properties of the different types of quadrilaterals and using one of the properties to classify them on a Venn diagram.

WORKED SOLUTIONS

Class Discussion (Well-defined and Distinct Objects in a Set)

1. No, H is not a set as the objects (handsome boys) in the set are not well-defined.
2. $T = \{P_1, P_2\}$ since the 2 identical pens are distinct.
3. $E = \{C, L, E, V, R\}$ since the letter 'E' is not distinct.

Thinking Time (Page 396)

If A and B are two sets such that $n(A) = n(B)$, it may not always be $A = B$. A counter-example is given as follows: Let $A = \{1, 2\}$ and $B = \{3, 4\}$, $n(A) = n(B)$ but $A \neq B$, since the elements in A are different from the elements in B.

Thinking Time (Page 401)

No, since A' is defined to be the set of all elements in the universal set but not in A.

Class Discussion (Understanding Subsets)

1. Yes, since a subset is a collection of well-defined and distinct objects.
2. No, since not every element of P is in Q and vice versa.

Thinking Time (Page 412)

1. $(X \cup Y)' \neq X' \cup Y'$
   $(X \cup Y)' = X' \cap Y'$
2. $(X \cap Y')' \neq X' \cap Y$
   $(X \cap Y')' = X' \cup Y$

Performance Task

1. The universal set will be the students in my class.
2. Yes, G is an empty set. It should be included in the Venn diagram.
3. No, the sets will not be distinct.
4. Yes, the sets should be drawn such that there are overlapping regions between them.
5. Yes, since I am a student of the class which is the universal set ξ.

Practise Now (Page 393)

1. **(a)** $A = \{2, 4, 6, 8\}$
   (b) **(i)** True
   (ii) True
   (iii) False
   (iv) True
   (c) **(i)** $2 \in A$
   (ii) $5 \notin A$
   (iii) $9 \notin A$
   (iv) $6 \in A$
2. $n(B) = 10$

Practise Now 1

(i) $C = \{11, 12, 13, 14, 15, 16, 17\}$
$D = \{10, 11, 12, 13, 14, 15, 16, 17\}$
(ii) No. $10 \in D$ but $10 \notin C$

Practise Now 2

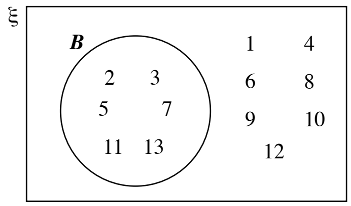
(a) No, a movie may be well-liked by some, but not others.
(b) Yes, it is clear which pupils are fourteen years old.
(c) Yes, it is clear whether someone is an English teacher in the school.

Practise Now 3

(i) $P = \{\ \}$
(ii) P and Q are not equal sets, as P is an empty set while Q consists of an element, 0.

Practise Now 4

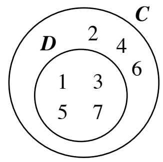
(i) $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$ and
$B = \{2, 3, 5, 7, 11, 13\}$
(ii)



(iii) $B' = \{1, 4, 6, 8, 9, 10, 12\}$
(iv) B' is the set of all integers between 1 and 13 inclusive which are not prime numbers.

Practise Now 5

1. **(i)**



   (ii) Yes, D is a proper subset of C because every element of D is an element of C, and $D \neq C$.
2. **(i)** $P = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$
   $Q = \{2, 3, 5, 7, 11\}$
   (ii) $Q \subset P$ because every element of Q is an element of P, and $Q \neq P$.
   (iii) $R = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$
   (iv) R and P are equal sets because all the elements of R and P are the same, i.e. $R = P$.

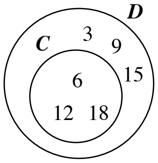
Practise Now (Page 403)

(i) **(a)** $\{7\}, \{8\}, \{7, 8\}$
(b) $\{7\}, \{8\}$

(ii) **(a)** {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}
(b) { }, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}

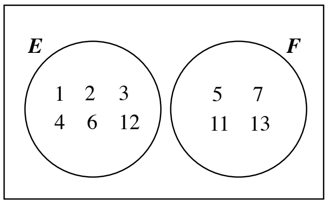
Practise Now 6

1. **(i)** $C = \{6, 12, 18\}$ and $D = \{3, 6, 9, 12, 15, 18\}$
(ii) $C \cap D = \{6, 12, 18\}$
(iii)



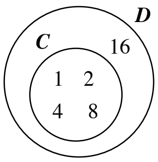
(iv) Yes, since all of the elements of C are also in D.

2. **(i)** $E = \{1, 2, 3, 4, 6, 12\}$ and $F = \{5, 7, 11, 13\}$
(ii) $E \cap F = \varnothing$ since E and F do not share any common elements.
(iii) ξ



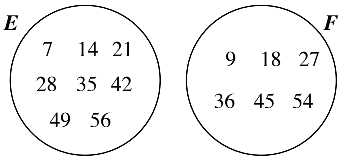
Practise Now 7

1. **(i)** $C = \{1, 2, 4, 8\}$ and $D = \{1, 2, 4, 8, 16\}$
(ii)



(iii) $C \cup D = \{1, 2, 4, 8, 16\}$
(iv) Yes, since all of the elements of C are also in D.

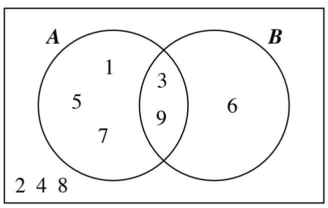
2. **(i)** $E = \{7, 14, 21, 28, 35, 42, 49, 56\}$ and
$F = \{9, 18, 27, 36, 45, 54\}$
(ii)



(iii) $E \cup F = \{7, 9, 14, 18, 21, 27, 28, 35, 36, 42, 45, 49, 54, 56\}$

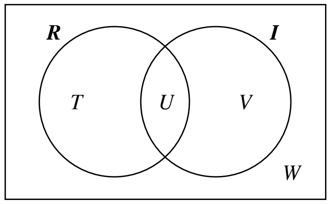
Practise Now 8

1. **(a)** $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 3, 5, 7, 9\}$ and
$B = \{3, 6, 9\}$
(b) ξ



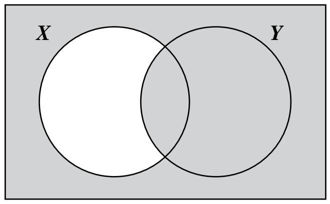
(c) **(i)** $(A \cup B)' = \{2, 4, 8\}$
(ii) $A \cap B' = \{1, 5, 7\}$

2. ξ

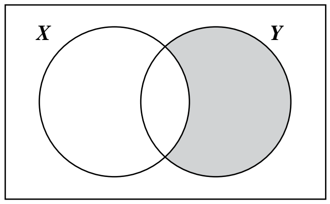


Practise Now 9

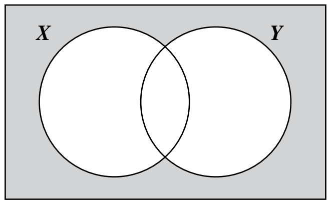
(i) ξ



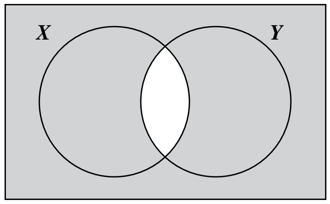
(ii) ξ



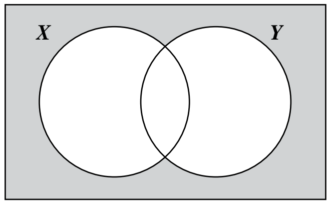
(iii) ξ



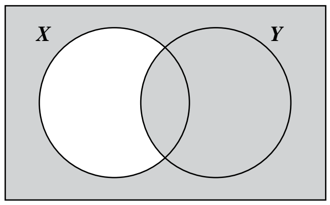
(iv) ξ



(v) ξ



(vi) ξ



Exercise 14A

1. **(a)** $B = \{1, 3, 5, 7, 9\}$
(b) **(i)** True
(ii) True
(iii) False
(iv) True

2. (a) 12
(b) 8
(c) –
(d) –
(e) 7
(f) 12

3. (a) $A = \{2, 3, 4, 5, 6, 7, 8, 9\}$
(b) $B = \{-10, -9, -8, -7, -6, -5, -4, -3, -2, -1\}$
(c) $C = \{2, 4, 6, 8, 10, 12\}$
(d) D = {A}

4. (a) {A, E, I}
(b) {red, orange, yellow, green, blue, indigo, violet}
(c) {9, 18, 27, 36, 45}
(d) –
(e) {12, 14, 16, 18, 20, 22}

5. (a) Yes, it is clear whether someone has two brothers.
(b) No, someone may be considered shy to some, but not to others.
(c) No, an actor may be well-liked by some, but not others.
(d) No, a dish may be well-liked by some, but not others.
(e) Yes, it is clear whether a textbook is used in the school.
(f) No, an actor may be considered most attractive to some, but not others.

6. (a) Yes
(b) Yes
(c) No
(d) Yes
(e) Yes
(f) Yes
(g) No
(h) No
(i) Yes
(j) Yes

7. (a) $E = \{\ \}$. It is an empty set.
(b) $F = \{\ \}$. It is an empty set.
(c) $G = \{\ \}$. It is an empty set.
(d) $H = \{2\}$. It is not an empty set as in contains one element, 2.

8. (a) D = {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}
(b) (i) Tuesday $\in D$
(ii) Sunday $\in D$
(iii) March $\notin D$
(iv) Holiday $\notin D$

9. (i) No, 10 is not a perfect square.
(ii) $P = \{4, 9, 16, 25, 36, 49\}$

10. (a) {red, orange, yellow, green, blue, indigo, violet}
(b) {S, Y, M, T, R}
(c) –
(d) {January, June, July}
(e) {11, 13, 15, 17}
(f) {b, c, d, f, g}
(g) {Tuesday, Thursday}
(h) {2, 4, 6, 8, 10, 12}
(i) {February}

11. (a) $M = \{x: x$ is an even integer$\}$
(b) $N = \{x: x$ is an even integer less than or equal to 8$\}$ or
$N = \{x: x$ is an even integer less than 9$\}$
$N = \{x: x$ is an even integer less than 10$\}$
(c) $O = \{x: x$ is a perfect cube$\}$
(d) $P = \{x: x$ is an integer that is a multiple of 5$\}$
(e) $Q = \{x: x$ is a digit from the first 5 letters of the alphabet$\}$

12. (a) China; the remaining elements are ASEAN countries
(b) Rubber; the remaining elements are edible fruits
(c) 20; the remaining elements are perfect squares
(d) 75; the remaining elements are perfect cubes
(e) Pie chart; the remaining elements are statistical averages

13. (i) $Q = \{\ \}, R = \{1\}$
(ii) $Q = \varnothing$ but $R \neq \varnothing$ as it contains an element, 1.

14. (i) False, as 'c' is an element of the set.
(ii) False, as the word 'car' is not an element of the set.
(iii) False, as {c} is a set, not an element.
(iv) False, as {c, a, r} is a set, not the number of elements in the set.
(v) True, as '5' is an element of the set.
(vi) False, as '4' is not an element of the set.
(vii) False, as the word 'bus' is not an element of the set, only the individual letters are.
(viii) True, as 'b' is an element of the set.

15. (a) True
(b) True
(c) False, as 4 is an even number.
(d) False, as {S, C, O, H, L} is a set, not an element.
(e) False, as 5 is not an even number.
(f) False, as {3} is a set, not an element.

16. (a) $S = \{x: x$ is a girl in my current class wearing spectacles$\}$
(b) $T = \{x: x$ is a prime number$\}$
(c) $U = \{x: x$ is a multiple of 4$\}$
(d) $V = \{x: x$ is a multiple of 4 between –8 and 12 inclusive$\}$

17. (i) False, as 0 is an element.
(ii) True
(iii) False, as $\varnothing$ is an element.
(iv) True

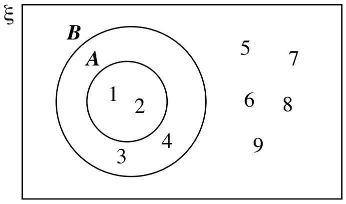
Exercise 14B

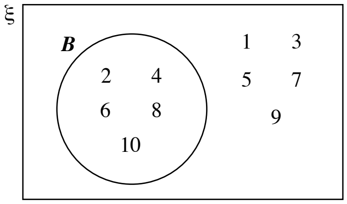
1. (a) {2, 4, 6, 8, 10, …, 20}
(b) {4, 8, 12, 16, 20}
(c) {3, 6, 9, 12, 15, 18}
(d) {1, 3, 5, 7, 9, 11, 13, 15, 17, 19}
(e) {1, 2, 3, 5, 6, 7, 9, 10, 11, 13, 14, 15, 17, 18, 19}
(f) {1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20}

2. (a) {30, 31, 32, 34, 35, 36, 38, 40, 41, 43, 45}
(b) {35, 43, 44}
(c) {31, 37, 41, 43}
(d) {30, 31, 33, 34, 35, 37, 38, 39, 41, 42, 43, 45}
(e) {31, 32, 34, 35, 37, 38, 40, 41, 43, 44}

3. **(a)** {2, 3, 5, 7}
(b) {2, 4, 6, 8, 10}
(c) {5, 10}
(d) {1, 4, 6, 8, 9,10}
(e) {1, 3, 5, 7, 9}
(f) {1, 2, 3, 4, 6, 7, 8, 9}

4. **(i)** A = {cat, dog, mouse}
ξ = {cat, dog, mouse, lion, tiger}
(ii) A' = {tiger, lion}

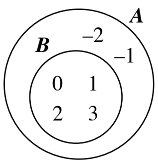
5. 

6. **(i)** ξ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
B = {2, 4, 6, 8, 10}
(ii) 

(iii) B' = {1, 3, 5, 7, 9}
(iv) B' is the set of all integers between 1 and 10 inclusive which are odd numbers.

7. **(a)** {20, 40, 60, 80}
(b) {60}
(c) {40, 80}
(d) ∅

8. **(i)** A = {s, t, u}
B = {s, t, u, v, w, x, y, z}
(ii) Yes, A is a proper subset of B because every element of A is an element of B, and $A \neq B$.

9. **(i)** 

(ii) Yes, B is a proper subset of A because every element of B is an element of A, and $B \neq A$.

10. **(i)** ξ = {1, 2, 3, 4, 5, 6, 7, 8, 9},
C = {1, 4, 6, 8, 9},
C' = {2, 3, 5, 7}
(ii) C' is a set of all integers between 0 and 10 which are prime numbers.

11. **(i)** ξ = {a, b, c, d, e, f, g, h, i, j},
D = {b, c, d, f, g, h, j},
D' = {a, e, i}
(ii) D' is a set of the first 10 letters of the English alphabet which are vowels.

12. **(i)** E = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}
F = {4, 8, 12, 16}
(ii) $F \subset E$ because every element of F is an element of E, and $F \neq E$.
(iii) G = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}
(iv) E and G are equal sets because all the elements of E and G are the same, i.e. $E = G$.

13. Yes, because every element of I is an element of H, and $I \neq H$.

14. **(a)** True
(b) True
(c) True
(d) False
(e) True

15. **(a)** { }, {1}, {2}, {1, 2}
(b) { }, {pen}, {ink}, {ruler}, {pen, ink}, {pen, ruler}, (ink, ruler}, {pen, ink, ruler}
(c) { }, {Thailand}, {Vietnam}, {Thailand, Vietnam}
(d) { }, {a}, {e}, {i}, {o}, {a, e}, {a, i}, {a, o}, {e, i}, {e, o}, {i, o}, {a, e, i}, {e, i, o}, {a, e, o}, {a, i, o}, {a, e, i, o}

16. **(a)** { }, {x}, {y}
(b) { }, {Singapore}, {Malaysia}
(c) { }, {3}, {4}, {5}, {3, 4}, {3, 5}, {4, 5}
(d) { }, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}

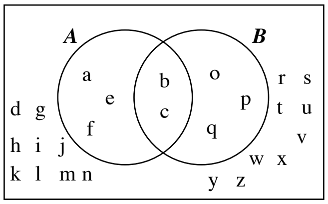
17. **(i)** O' = {1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20}
(ii) O' = {x : x is a positive integer less than 21 which is not divisible by 3}

18. **(a)** ≠
(b) ⊃
(c) =
(d) ≠
(e) ≠
(f) =
(g) ⊂
(h) =
(i) ≠
(j) =
(k) ⊂
(l) ≠

19. **(a)** True
(b) True
(c) False
(d) False
(e) False
(f) True
(g) True
(h) False
(i) True
(j) True
(k) False
(l) True
(m) False

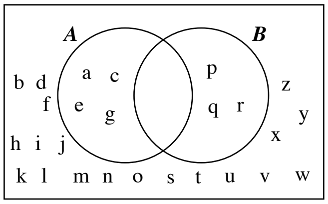
Exercise 14C

1. (i) ξ



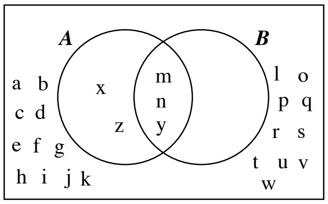
$A \cap B = \{b, c\}$

(ii) ξ



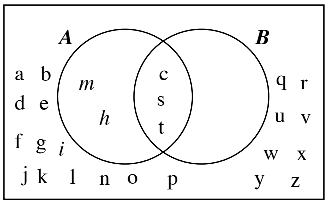
$A \cap B = \varnothing$

(iii) ξ



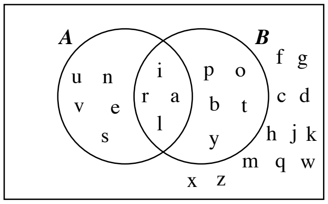
$A \cap B = \{m, n, y\}$

(iv) ξ



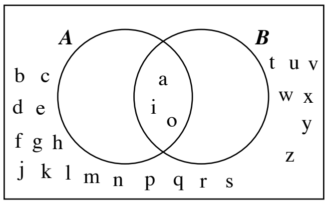
$A \cap B = \{c, s, t\}$

(v) ξ



$A \cap B = \{a, i, r, l\}$

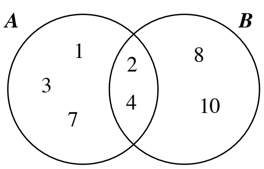
(vi) ξ



$A \cap B = \{a, i, o\}$

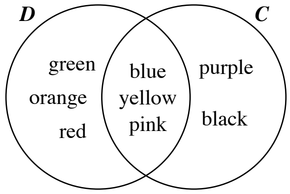
2. (i) $A \cap B = \{2, 4\}$

(ii)

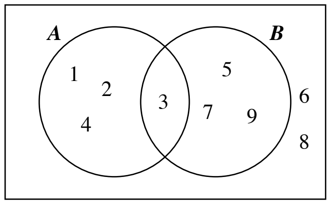


3. (i) $C \cap D = \{\text{blue, yellow, pink}\}$

(ii)

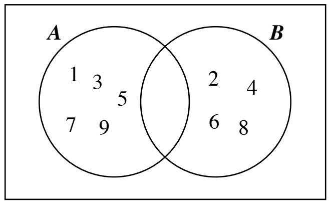


4. (i) ξ



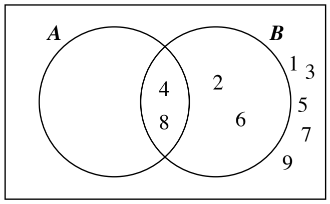
$A \cup B = \{1, 2, 3, 4, 5, 7, 9\}$

(ii) ξ



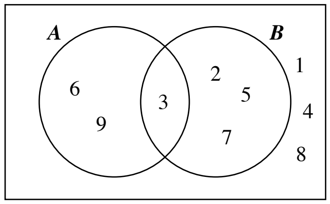
$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(iii) ξ



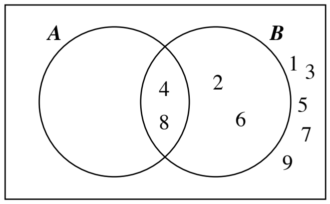
$A \cup B = \{2, 4, 6, 8\}$

(iv) ξ



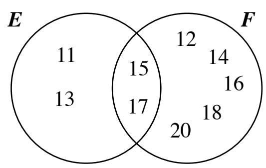
$A \cup B = \{2, 3, 5, 6, 7, 9\}$

(v) ξ



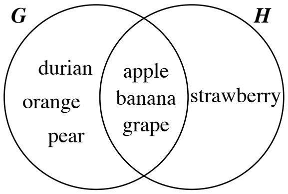
$A \cup B = \{2, 4, 6, 8\}$

5. (i)



(ii) $E \cup F = \{11, 12, 13, 14, 15, 16, 18, 20\}$

6. **(i)**



(ii) $G \cup H$ = {apple, banana, durian, grape, orange, pear, strawberry}

7. **(i)** $A \cup B$ = {durian, mango, pineapple, rambutan, soursop}

(ii) $A \cap B$ = {durian, mango}

8. **(a)** $A \cup B = \{3, 6, 8, 9, 12\}$

$A \cap B = \{6, 9\}$

(c) $C \cup D$ ={a, b, x, y, m, n, o, p}

$C \cap D = \varnothing$

(c) $E \cup F$ = {monkey, goat, lion, tiger}

$E \cap F$ = {goat}

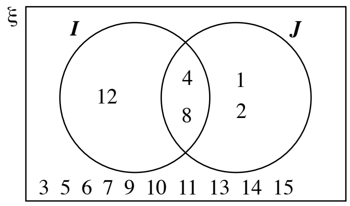
(d) $G \cup H$ = {a, m, k, y}

$G \cap H = \varnothing$

9. **(a)** $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$,

$I = \{4, 8, 12\}$ and $J = \{1, 2, 4, 8\}$

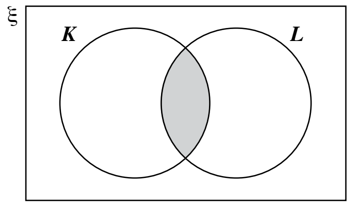
(b)



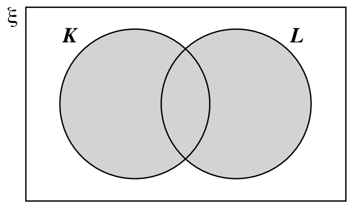
(c) **(i)** $(I \cup J)' = \{3, 5, 6, 7, 9, 10, 11, 13, 14\ 15\}$

(ii) $I \cap J' = \{12\}$

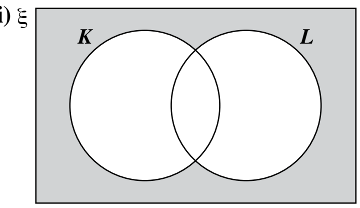
10. **(i)**



(ii)

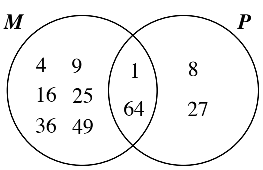


(iii)



11. **(i)** $M = \{1, 4, 9, 16, 25, 36, 49, 64\}$ and $P = \{1, 8, 27, 64\}$

(ii)

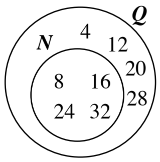


(iii) $M \cap P = \{1, 64\}$

12. **(i)** $N = \{8, 16, 24, 32\}$ and $Q = \{4, 8, 12, 16, 20, 24, 28, 32\}$

(ii) $N \cap Q = \{8, 16, 24, 32\}$

(iii)

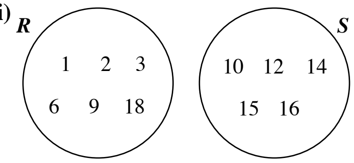


(iv) Yes, since all the elements of N are also in Q.

13. **(i)** $R = \{1, 2, 3, 6, 9, 18\}$ and $S = \{10, 12, 14, 15, 16\}$

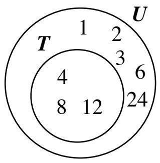
(ii) $R \cap S = \varnothing$ since R and S do not share any common elements.

(iii)



14. **(i)** $T = \{4, 8, 12\}$ and $U = \{1, 2, 3, 4, 6, 8, 12, 14\}$

(ii)

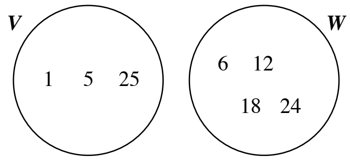


(iii) $T \cup U = \{1, 2, 3, 4, 6, 8, 12, 24\}$

(iv) Yes, since all of the elements of T are also in U.

15. **(i)** $V = \{1, 5, 25\}$ and $W = \{6, 12, 18, 24\}$

(ii)

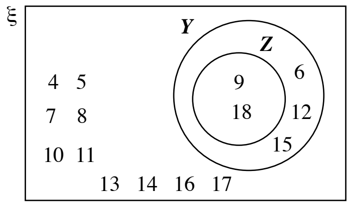


(iii) $V \cup W = \{1, 5, 6, 12, 18, 24, 25\}$

16. **(a)** $\xi = \{4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18\}$,

$Y = \{6, 9, 12, 15, 18\}$ and $Z = \{9, 18\}$

(b)

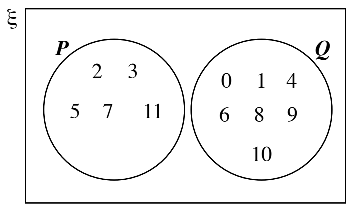


(c) **(i)** $(Y \cup Z)' = \{4, 5, 7, 8, 10, 11, 13, 14, 16, 17\}$

(ii) $Y \cap Z' = \{6, 12, 15\}$

17. **(a)** $\xi = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$, $P = \{2, 3, 5, 7, 11\}$ and $Q = \{0, 1, 4, 6, 8, 9, 10\}$

(b)

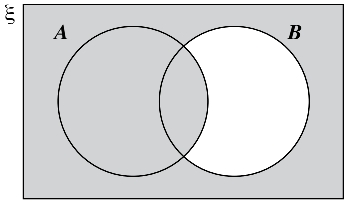


(c) **(i)** $P \cup Q = \; = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

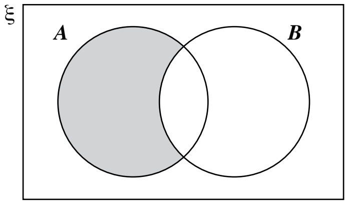
(ii) $(P \cup Q)' = \varnothing$

(iii) $P' \cap Q = Q = \{0, 1, 4, 6, 8, 9, 10\}$

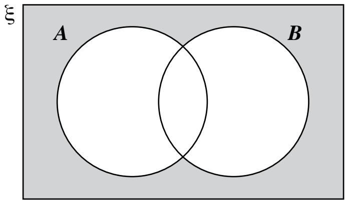
18. (a) (i)



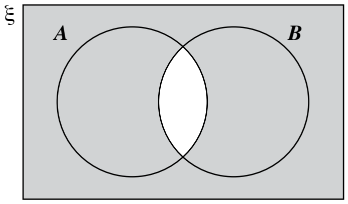
(ii)



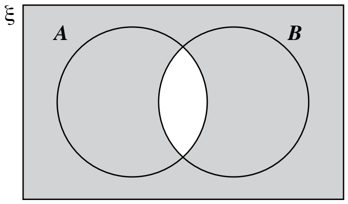
(iii)



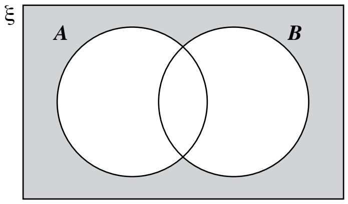
(iv)



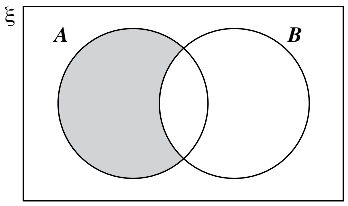
(v)



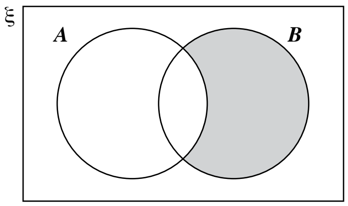
(vi)



(vii)



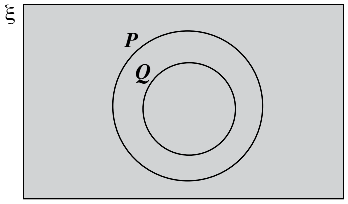
(viii)



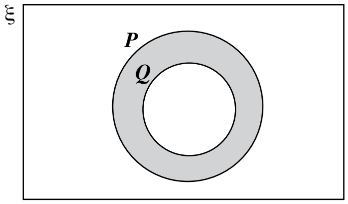
(b) (i) They are the same.

(ii) They are the same.

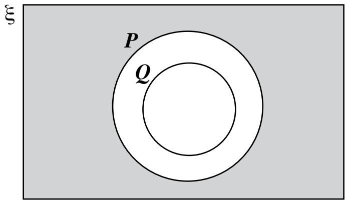
19. (i)



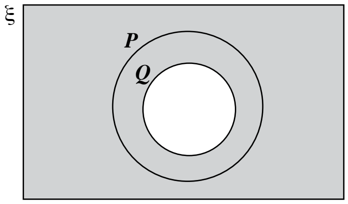
(ii)



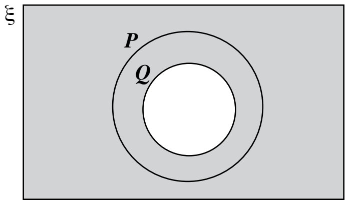
(iii)



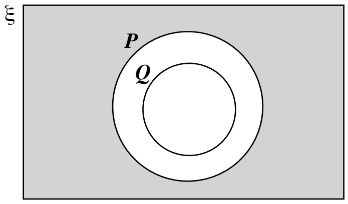
(iv)



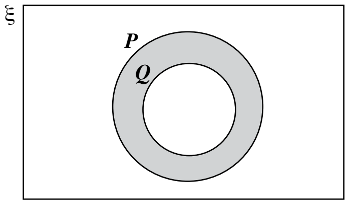
(v)



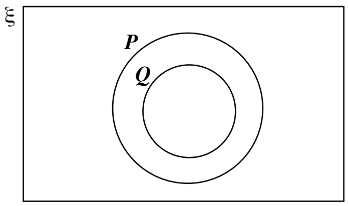
(vi)



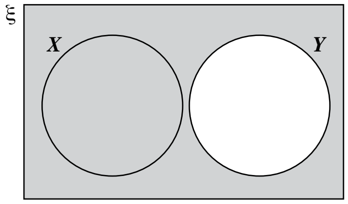
(vii)



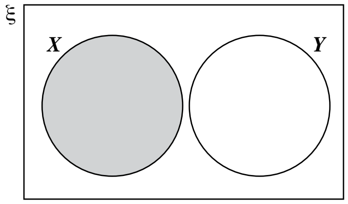
(viii)



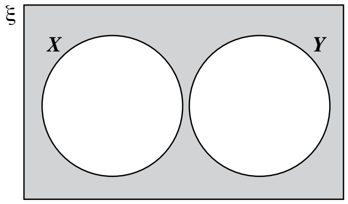
20. (i)



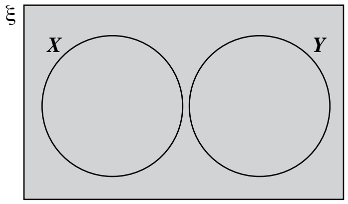
(ii)



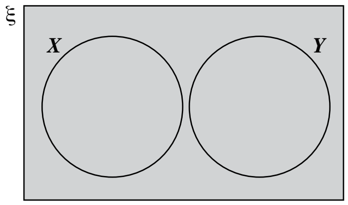
(iii)



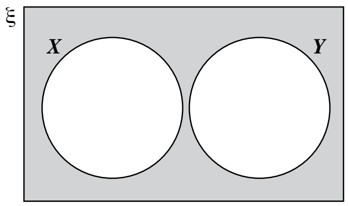
(iv)



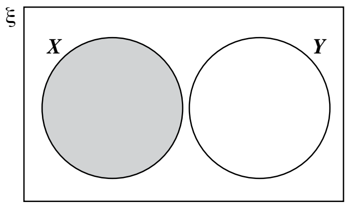
(v)



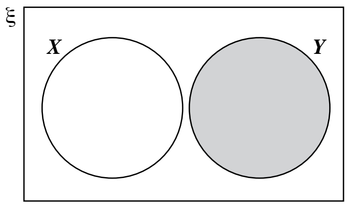
(vi)



(vii)



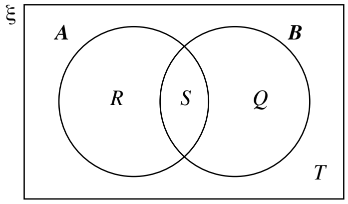
(viii)



21. (i) $A \cap \xi = A$
(ii) $A \cup \xi = \xi$
(iii) $A \cap \varnothing = \varnothing$
(iv) $A \cup \varnothing = A$

22. (i) $A \cap B = A$
(ii) $A \cup B = B$
(iii) Not possible to simplify further
(iv) Not possible to simplify further
(v) $(B \cup C) \cap A = A$
(vi) $(B \cap C) \cap A = \varnothing$
(vii) $(A \cup C) \cap B = A \cup C$
(viii) $(A \cap C) \cup B = B$

23.



Review Exercise 14

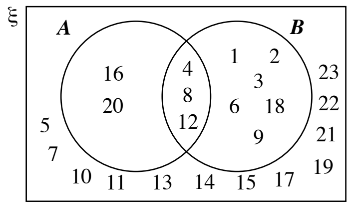
1. (a) $A = \{1, 3, 5, 7, 9\}$
(b) (i) True
(ii) True
(iii) False
(iv) False
(c) (i) $-3 \notin A$
(ii) $3 \in A$
(iii) $0 \notin A$
(iv) $9 \in A$

2. (a) $B = \{2\}$. It is not an empty set.
(b) C = {Saturday, Sunday}. It is not an empty set.
(c) $D = \varnothing$. It is an empty set.
(d) $E = \varnothing$. It is an empty set.

3. (a) $A' = \{-4, -2, 0, 1, 3\}$
(b) $B' = \varnothing$
(c) $C' = \{-5, -4, -3, -2, -1, 0, 1\}$
(d) $D' = \{-5, -4, -3, -2, -1, 0, 1, 2$

4. (a) True
(b) False
(c) True
(d) True

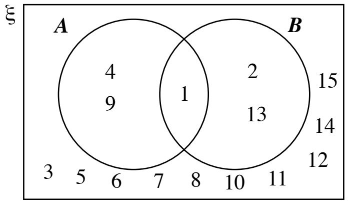
5. (i) $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23\}$, $A = \{4, 8, 12, 16, 20\}$ and $B = \{1, 2, 3, 4, 6, 8, 12, 18\}$



(ii) $A \cup B' = \{4, 5, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23\}$

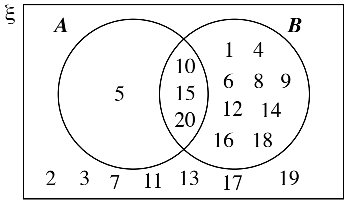
6. **(i)** $A' = \{-7, 7\}$
(ii) $A \cap B = \{1, 2, 3, 4, 5, 6\}$
(iii) $A \cup B' = \{-7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$

7. **(i)** $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$, $A = \{1, 4, 9\}$ and $B = \{1, 2, 13\}$



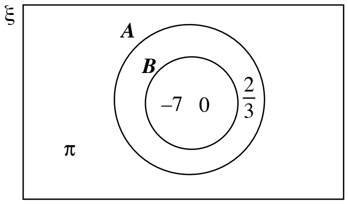
(ii) $A' \cap B' = \{3, 5, 6, 7, 8, 10, 11, 12, 14, 15\}$

8. **(i)** $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$, $A = \{5, 10, 15, 20\}$ and $B = \{1, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20\}$



(ii) $A \cap B' = \{5\}$

9. **(i)** { }, {s}, {i}, {t}, {s, i}, {s, t}, {i, t}, {s, i, t}
(ii) { }, {s}, {i}, {t}, {s, i}, {s, t}, {i, t}

10. 

Challenge Yourself

1. **(i)** True, since a is an element of the set, *S*.
(ii) True, since {a} is an element of the set, *S*.
(iii) True, since {a} is a proper subset of the set, *S*.
(iv) True, since {{a}} is a proper subset of the set, *S*.

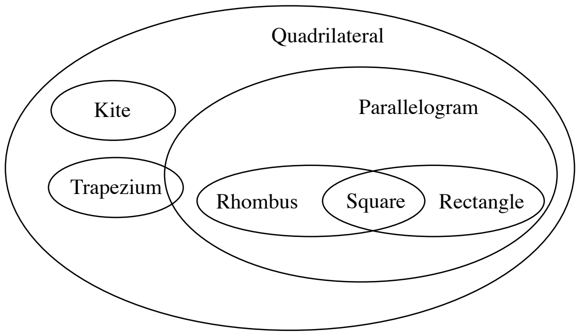
2. If the set *S* has 2 elements, e.g. x and y then there are 4 subsets: ∅, {x}, {y} and {x, y}.
If the set *S* has 3 elements, e.g. 3, 4 and 5, then there are 8 subsets: ∅, {3}, {4}, {5}, {3, 4}, {3, 5}, {4, 5} and {3, 4, 5}.
If the set *S* has 4 elements, e.g. a, b, c and d, then there are 16 subsets: ∅, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d} and {a, b, c, d}.

No. of elements in a set S	No. of subsets
2	$4 = 2^2$
3	$8 = 2^3$
4	$16 = 2^4$

Note that the total number of subsets doubles each time. Hence if the set *S* has *n* elements, then there are 2^n subsets. So, if the set *S* has *n* elements, then there are $2^n - 1$ proper subsets, taking away the original set.

3. **(a)** A trapezium has at least two parallel sides.
(b) A kite has at least 2 equal adjacent sides.
(c) A parallelogram has 2 pairs of parallel sides and 2 pairs of equal sides.
(d) A rectangle has 2 pairs of parallel sides, 2 pairs of equal sides and 4 equal angles.
(e) A rhombus has 2 pairs of parallel sides and 4 equal sides.
(f) A square has 2 pairs of parallel sides, 4 equal sides and 4 equal angles.

The Venn diagram based on the number of pairs of parallel sides a quadrilateral has is shown below.



Chapter 15 Probability of Single Events

TEACHING NOTES

Suggested Approach

Students would not have learnt probability in primary school so the concept will be entirely new for them. Teachers may begin the lesson by first arousing the students' interest in this topic. This new topic can be introduced by using the chapter opener on Page 419 and discussing statements that are often used in our daily life (see Thinking Time on page 421).

Section 15.1: Introduction to Probability

From the statements discussed in our daily life (see Thinking Time on page 421), teachers can build upon them and guide the students to determine the chance of each event happening. This measure of chance is the definition of probability.

This can then lead to relating the chance of any event happening to a number line taking on values between 0 and 1 inclusive. Teachers should further discuss the reason for a 'certain' event to take the value of 1 as well as the reason for an 'impossible' event to take the value of 0.

At the end of this section, a simple class activity can be carried out by encouraging students to form statements to describe an event. Words such as 'unlikely', 'likely', 'impossible' or 'certain' are encouraged to be used in the statements. Students can then mark the chance of the event occurring on a number line. At the end of this section, students should be able to define probability as a measure of chance.

Section 15.2: Sample Space

Teachers can do a simple experiment such as tossing a coin to introduce the words 'outcomes' and 'sample space'. Going through the different experiments listed in the table on Page 422 can help to reinforce the meanings of 'outcomes' and 'sample space'.

For the last experiment, teachers are to let the students know that there is a difference between drawing the first and second black ball. Thus, there is a need to differentiate the two black balls. Similarly, there is a need to differentiate the three white balls. Hence, the outcomes are two individual black balls and three individual white balls.

Teachers can assess students' understanding of the terms using the Practise Now exercises available.

Section 15.3: Probability of Single Events

To start off this section, students should attempt the two activities (see Investigation: Tossing a Coin and Investigation: Rolling a Die). The terms 'experimental probability' and 'theoretical probability' and their relationship should be highlighted and explained to students. Students are expected to be able to conclude that as the number of trials increase, the experimental probability of an outcome occurring tends towards the theoretical probability of the outcome happening.

At the same time, teachers can illustrate the meanings of 'fair' and 'unbiased'. If any of the two terms are used, then the chance of any outcome happening in an event is exactly equal.

In the previous sections, students would have learnt events with a probability of 0 or 1. Here, they are to grasp that for any event E, $0 \leqslant P(E) \leqslant 1$ and later $P(\text{not } E) = 1 - P(E)$.

As a recommended technique for solving problems involving probability, teachers should encourage the students to always list all the possible outcomes in a simple chance situation to calculate the probability. Doing so will allow better visualisation of the outcomes for a particular event to happen.

Section 15.4 Further Examples on Probability of Single Events

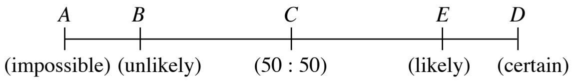
In this section, there are more calculations of probability using real-life examples. Teachers should use this section to reinforce the concept of probability.

Challenge Yourself

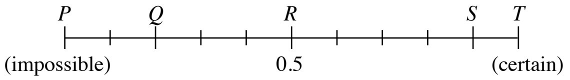
For Question 1, teachers can provide a hint to students to focus solely on the picture cards by recalling the number of it in a standard pack. By listing the possible outcomes in Question 2, students should be able to answer all three parts.

WORKED SOLUTIONS

Thinking Time (Page 421)

1. 

2. It takes a value of 0.5.
3. Suggested answers:
   P: The sun will rise from the west at least once a year.
   Q: There is a 20 percent chance that it will rain tomorrow.
   R: A lady will give birth to a boy if she is pregnant with a child.
   S: A red ball is drawn from a bag which contains 9 red balls and 1 black ball.
   T: A die will land on a number between 1 to 6 inclusive.



Investigation (Tossing a Coin)

1. No.
2. **(i)** *Teachers may guide the students to fill in the necessary information in the table.*
   (ii) The fraction should be close to $\frac{1}{2}$.
   (iii) The results each classmate got are likely to be different. The results obtained are those of a random experiment.
3. **(a)** *Teachers may guide the students to fill in the necessary information in the table.*
   (b) *Teachers may guide the students to fill in the necessary information in the table.*
4. Yes, the probabilities are approaching $\frac{1}{2}$ when there are more tosses.
5. No, we can only expect to get close to it as every outcome cannot be predicted with certainty before a coin is tossed.

Investigation (Rolling a Dice)

1. *Teacher may ask students to visit the website:* <u>*http://www.shinglee.com.sg/StudentResources*</u>/*and open the spreadsheet 'Rolling a Die' to obtain the outcomes so as to fill in the necessary information for the table.*
2. *Teacher may guide the students to fill in the necessary information for the table.*
3. Yes, the probabilities will approach $\frac{1}{6}$ when there are more rolls.
4. No, we can only expect to get close to it as every outcome cannot be predicted with certainty before a die is rolled.

Thinking Time (Page 429)

1. The probability of *D* occurring is 1. Event *D* will definitely occur.
2. The probability of *A* occurring is 0. Event *A* will never occur.
3. No.

Performance Task (Page 429)

Suggested answers of other real-life applications of probability theory are in

- risk assessment,
- trades on financial markets,
- reliability of consumer products etc.

Teachers may wish to group the students for this task. Each group can be responsible to do research on one of the above applications and then present their findings to the class.

Practise Now 1

1. The spinner has the colours Red, Orange, Purple, Yellow and Green. i.e. the sample space consists of the colours Red, Orange, Purple, Yellow and Green.
   Total number of possible outcomes = 5

Practise Now 2

(a) Let B_1, B_2, B_3, B_4, B_5 represent the 5 blue marbles;
R_1, R_2, R_3, R_4 represent the 4 red marbles.
The sample space consists of $B_1, B_2, B_3, B_4, B_5, R_1, R_2, R_3$ and R_4.
Total number of possible outcomes = 9

(b) The sample space consists of N_1, A_1, T, I, O, N_2, A_2 and L.
Total number of possible outcomes = 8

(c) The sample space consists of the serial numbers 357, 358, 359, ..., 389.
Total number of possible outcomes
= first 389 receipts – first 356 receipts
= 389 – 356
= 33

Practise Now 3

Total number of possible outcomes $= 24 - 9$
$= 15$

(i) P(drawing a '21') $= \frac{1}{15}$

(ii) There are 7 odd numbers from 10 to 24, i.e. 11, 13, 15, 17, 19, 21 and 23.
P(drawing an odd number) $= \frac{7}{15}$

(iii) There are 10 composite numbers from 10 to 24, i.e. 10, 12, 14, 15, 16, 18, 20, 21, 22 and 24.
P(drawing a composite number) $= \frac{10}{15}$
$= \frac{2}{3}$

(iv) There are no perfect cubes from 10 to 24.
P(drawing a perfect cube) $= \frac{0}{15}$
$= 0$

Practise Now 4

Total number of possible outcomes = 52

(i) There are 26 red cards in the pack.

P(drawing a red card) $= \frac{26}{52}$

$= \frac{1}{2}$

(ii) There are 4 aces in the pack, i.e. the ace of clubs, the ace of diamonds, the ace of hearts and the ace of spade.

P(drawing an ace) $= \frac{4}{52}$

$= \frac{1}{13}$

(iii) There is only one three of clubs in the pack.

P(drawing the three of clubs) $= \frac{1}{52}$

(iv) Since there is only one three of clubs in the pack, there are $52 - 1 = 51$ cards which are not the three of clubs.

P(drawing a card which is not the three of clubs) $= \frac{51}{52}$

Practise Now 5

1. Total number of letters = 8

(i) There is 1 'D'.

P(a 'D' is chosen) $= \frac{1}{8}$

(ii) There are 6 consonants, i.e. 1 'C', 1 'H', 1 'L', 1 'D', 1 'R', and 1 'N',

P(letter chosen is a consonant) $= \frac{6}{8}$

$= \frac{3}{4}$

(iii) Method 1:

There are 2 vowels, i.e. 1 'I' and 1 'E'.

P(letter chosen is not a consonant) $= \frac{2}{8}$

$= \frac{1}{4}$

Method 2:

P(letter chosen is a consonant)

= 1 – P(letter chosen is a consonant)

$= 1 - \frac{3}{4}$

$= \frac{1}{4}$

2. Total number of possible outcomes $= 9 + 6 + 4 + 5$

$= 24$

(i) There are 4 purple marbles.

P(drawing a purple marble) $= \frac{4}{24}$

$= \frac{1}{6}$

(ii) There are $9 + 5 = 14$ red or blue marbles.

P(drawing a red or a blue marble) $= \frac{14}{24}$

$= \frac{7}{12}$

(iii) There are no white marbles.

P(drawing a white marble) = 0

(iv) Method 1:

There are 24 marbles that are not white.

P(drawing a marble that is not white) $= \frac{24}{24}$

= 1

Method 2:

P(drawing a marble that is not white)

= 1 – P(drawing a marble that is white)

= 1 – 0

= 1

3. Method 1:

P(drawing a red ball) $= \frac{1}{3}$

$\frac{\text{Number of red balls}}{\text{Total number of balls}} = \frac{1}{3}$

$\therefore$ Number of red balls $= \frac{1}{3} \times 24$

$= 8$

P(drawing a green ball) $= \frac{1}{6}$

$\frac{\text{Number of green balls}}{\text{Total number of balls}} = \frac{1}{6}$

$\therefore$ Number of green balls $= \frac{1}{6} \times 24$

$= 4$

$\therefore$ Number of blue balls $= 24 - 8 - 4$

$= 12$

Method 2:

P(drawing a blue ball)

= 1 – P(drawing a red ball) – P(drawing a green ball)

$= 1 - \frac{1}{3} - \frac{1}{6}$

$= \frac{1}{2}$

$\frac{\text{Number of blue balls}}{\text{Total number of balls}} = \frac{1}{2}$

$\therefore$ Number of blue balls $= \frac{1}{2} \times 24$

$= 12$

Practise Now 6

(i) Number of girls $= 40 - 24$

$= 16$

P(student chosen is a girl) $= \frac{16}{40}$

$= \frac{2}{5}$

(ii) Number of boys who are not short-sighted = 16

Number of girls who are not short-sighted = 16 – 4

= 12

Number of students who are not short-sighted = 16 + 12

= 28

P(student is not short-sighted) $= \frac{28}{40}$

$= \frac{7}{10}$

Practise Now 7

(i) P(point selected lies in the red sector)

$= \frac{\text{Area of the red sector}}{\text{Area of the circle}}$

$= \frac{\text{Area of the red sector}}{360°}$

$= \frac{135°}{360°}$

$= \frac{3}{8}$

(ii) Angle of the blue sector = 360° – 90° – 135° – 45° – 30°

= 60°

P(point selected lies in the blue sector)

$= \frac{\text{Area of the blue sector}}{\text{Area of the circle}}$

$= \frac{\text{Area of the blue sector}}{360°}$

$= \frac{60°}{360°}$

$= \frac{1}{6}$

(iii) P(point selected lies in the purple sector)

$= \frac{\text{Area of the purple sector}}{\text{Area of the circle}}$

$= \frac{0}{\text{Area of the circle}}$

= 0

(iv) P(point selected lies in the green or white sector)

$= \frac{\text{Area of the green sector + Area of the white sector}}{\text{Area of the circle}}$

$= \frac{\text{Area of the green sector + Area of the white sector}}{360°}$

$= \frac{45° + 30°}{360°}$

$= \frac{75°}{360°}$

$= \frac{5}{24}$

Practise Now 8

1. **(i)** Total number of balls = 12 + (*x* + 2)

$= x + 14$

(ii) P(drawing a yellow ball) $= \frac{x+2}{x+14}$

(iii) Given that $\frac{x+2}{x+14} = \frac{2}{5}$,

$5(x + 2) = 2(x + 14)$

$5x + 10 = 2x + 28$

$3x = 18$

$x = 6$

2. Total number of boys and girls = 28 + 25

= 53

After *y* girls leave the hall,

P(selecting a girl at random) $= \frac{25 - y}{53 - y}$

Given that $\frac{25 - y}{53 - y} = \frac{3}{7}$,

$7(25 - y) = 3(53 - y)$

$175 - 7y = 159 - 3y$

$4y = 16$

$y = 4$

Exercise 15A

1. The dart board has the numbers 1, 2, 3, 4, 5 and 6.

i.e. the sample space consists of the numbers 1, 2, 3, 4, 5 and 6.

Total number of possible outcomes = 6

2. **(a)** The die has the labels 2, 3, 4 and 5.

i.e. the sample space consists of the labels 2, 3, 4, 5 and 6.

Total number of possible outcomes = 5

(b) The box has the cards A, B, C, D, E, F, G, H, I and J.

i.e. the sample space consists of cards A, B, C, D, E, F, G, H, I and J.

Total number of possible outcomes = 10

(c) Let R_1, R_2, R_3, R_4, R_5 represents the 5 red discs;

B_1, B_2, B_3 represent the 3 blue discs;

G_1, G_2 represent the 2 green discs.

The sample space consists of $R_1, R_2, R_3, R_4, R_5, B_1, B_2, B_3, G_1$ and G_2.

Total number of possible outcomes = 10

(d) The sample space consists of T, E_1, A, C, H, E_2 and R.

Total number of possible outcomes = 7

(e) The sample space consists of the numbers 100, 101, 102, ..., 999.

Total number of possible outcomes

= first 999 numbers – first 99 numbers

= 999 – 99

= 900

3. Total number of possible outcomes = 8

(i) There are 3 '7's.

P(getting a '7') $= \frac{3}{8}$

(ii) There are 2 '3's and 1 '4'.

P(getting a '3' or '4') $= \frac{3}{8}$

(iii) There are 8 numbers less than 10, i.e. 2, 3, 3, 4, 7, 7, 7 and 9.

P(getting a number less than 10) $= \frac{8}{8}$

= 1

(iv) Method 1:

There are 7 numbers that are not ‘2’, i.e. 3, 3, 4, 7, 7, 7 and 9.

P(getting a number which is not ‘2’) = $\frac{7}{8}$

Method 2:

There is 1 ‘2’.

P(getting a ‘2’) = $\frac{1}{8}$

P(getting a number which is not ‘2’)

= 1 – P(getting a ‘2’)

= $1 - \frac{1}{8}$

= $\frac{7}{8}$

4. Total number of possible outcomes = 22 – 9

= 13

(i) There are 7 even numbers from 10 to 22, i.e. 10, 12, 14, 16, 18, 20 and 22.

P(drawing an even number) = $\frac{7}{13}$

(ii) There are 7 numbers between 13 and 19 inclusive, i.e. 13, 14, 15, 16, 17, 18 and 19.

P(drawing a number between 13 and 19 inclusive) = $\frac{7}{13}$

(iii) There are 3 prime numbers less than 18, i.e. 11, 13 and 17.

P(drawing a prime number that is less than 18) = $\frac{3}{13}$

(iv) There are no numbers greater than 22.

P(drawing a number greater than 22) = 0

(v) There are 3 numbers divisible by 4, i.e. 12, 16 and 20.

P(drawing a number that is divisible by 4) = $\frac{3}{13}$

5. Total number of possible outcomes = 52

(i) There is only one ace of spades in the pack.

P(drawing the ace of spades) = $\frac{1}{52}$

(ii) P(drawing a heart or a club) = $\frac{26}{52}$

= $\frac{1}{2}$

(iii) There are $3 \times 4 = 12$ picture cards in the pack.

P(drawing a picture card) = $\frac{12}{52}$

= $\frac{3}{13}$

(iv) Method 1:

There are $10 \times 4 = 40$ non-picture cards in the pack.

P(drawing a non-picture card) = $\frac{40}{52}$

= $\frac{10}{13}$

Method 2:

P(getting a non-picture card)

= 1 – P(getting a picture card)

= $1 - \frac{3}{13}$

= $\frac{10}{13}$

6. Total number of possible outcomes = 11

(i) There is 1 ‘A’ in the cards.

P(card shows the letter ‘A’) = $\frac{1}{11}$

(ii) There are 2 ‘B’s in the cards.

P(card shows the letter ‘B’) = $\frac{2}{11}$

(iii) There are 4 vowels in the cards, i.e. 1 ‘O’, 1 ‘A’ and 2 ‘I’s.

P(card shows a vowel) = $\frac{4}{11}$

(iv) Method 1:

There are 7 consonants, i.e. 1 ‘P’, 1 ‘R’, 2 ‘B’s, 1 ‘L’, 1 ‘T’ and 1 ‘Y’.

P(cards shows a consonant) = $\frac{7}{11}$

Method 2:

P(card shows a consonant)

= 1 – P(card shows a vowel)

= $1 - \frac{4}{11}$

= $\frac{7}{11}$

7. Total number of possible outcomes = 5

(i) There is one sector with the label ♥.

P(stops at a sector whose label is ♥) = $\frac{1}{5}$

(ii) There are 3 sectors with a letter of the English alphabet, i.e. A, V and F.

P(stops at a sector whose label is a letter of the English alphabet)

= $\frac{3}{5}$

(iii) There is one sector with a vowel, i.e. A.

P(stops at a sector whose label is a vowel) = $\frac{1}{5}$

(iv) There are 2 sectors with a consonant, i.e. V and F.

P(stops at a sector whose label is a consonant) = $\frac{2}{5}$

8. Total number of possible outcomes = 4

(i) There is one caramel in the bag.

P(candy is a caramel) = $\frac{1}{4}$

(ii) There are 2 pieces that are either a chocolate or gummy in the bag.

P(candy is either a chocolate or a gummy) = $\frac{2}{4}$

= $\frac{1}{2}$

(iii) Method 1:

There are 3 pieces that are not a licorice, i.e. caramel, chocolate and gummies.

P(candy is not a licorice) = $\frac{3}{4}$

Method 2:

There is one licorice.

P(candy is a licorice) = $\frac{1}{4}$

P(candy is not licorice)

= 1 – P(candy is a licorice)

= $1 - \frac{1}{4}$

= $\frac{3}{4}$

9. Total number of possible outcomes = 40

There are 15 vouchers with a value of \$100 each.

P(voucher has a value of \$100) = $\frac{15}{40}$

= $\frac{3}{8}$

10. (i) Total number of possible outcomes = 30

There are 9 + 12 = 21 males in the group.

P(person is a male) = $\frac{21}{30}$

= $\frac{7}{10}$

(ii) Method 1:

There are 6 + 12 + 3 = 21 people in the group that is either a woman, a boy or a girl.

P(person is either a woman, a boy or a girl) = $\frac{21}{30}$

= $\frac{7}{10}$

Method 2:

There are 9 men in the group.

P(person is a man) = $\frac{9}{30}$

= $\frac{3}{10}$

P(person is either a woman, a boy or a girl)

= P(person is not a man)

= 1 – P(person is a man)

= $1 - \frac{3}{10}$

= $\frac{7}{10}$

11. The sample space consists of the numbers, 10, 11, 12, ..., 99.

Total number of possible outcomes = 99 – 9

= 90

(i) There are 10 numbers less than 20, i.e. 10, 11, 12, ..., 19.

P(number is less than 20) = $\frac{10}{90}$

= $\frac{1}{9}$

(ii) There are 6 perfect squares that are two-digit numbers, i.e. 16, 25, 36, 49, 64, 81.

P(number is a perfect square) = $\frac{6}{90}$

= $\frac{1}{15}$

12. Total number of possible outcomes = 54

(i) There are 26 red cards in the pack.

P(drawing a red card) = $\frac{26}{54}$

= $\frac{13}{27}$

(ii) There are 4 ‘2’ cards in the pack.

P(drawing a two) = $\frac{4}{54}$

= $\frac{2}{27}$

(iii) There are 2 joker cards in the pack.

P(drawing a joker) = $\frac{2}{54}$

= $\frac{1}{27}$

(iv) There are 4 + 4 = 8 cards that are either a queen or a king in the pack.

P(drawing a queen or a king) = $\frac{8}{54}$

= $\frac{4}{27}$

13. Total number of possible outcomes = 52 – 13

= 39

(i) There are 13 black cards in the pack, i.e. 13 spades.

P(drawing black card) = $\frac{13}{39}$

= $\frac{1}{3}$

(ii) There are 13 diamonds in the pack.

P(drawing a diamond) = $\frac{13}{39}$

= $\frac{1}{3}$

(iii) There are 3 × 3 = 9 picture cards in the pack.

P(drawing a picture card) = $\frac{9}{39}$

= $\frac{3}{13}$

(iv) There are 3 aces in the pack, i.e. the ace of hearts, the ace of diamonds and the ace of spades.

P(drawing an ace) = $\frac{3}{39}$

= $\frac{1}{13}$

P(drawing a card which is not an ace) = 1 – P(drawing an ace)

= $1 - \frac{1}{13}$

= $\frac{12}{13}$

14. (i) The sample space in column A consists of the integers 0, 1, 2, 3, 4 and 5.

Total number of possible outcomes = 6

P(number in colum A is a 4) = $\frac{1}{6}$

(ii) The sample space in column B consists of the integers 0, 1, 2, ..., 9.

Total number of possible outcomes = 10

P(number in column B is an 8) = $\frac{1}{10}$

(iii) There are 6 numbers that are less than 6 in column A.

P(number in column A is less than 6) = $\frac{6}{6}$

= 1

(iv) There are 4 numbers that are greater than 5 in column B, i.e. 6, 7, 8 and 9.

P(number in column B is greater than 5) = $\frac{4}{10}$

= $\frac{2}{5}$

15. Total number of possible outcomes = 2×12

= 24

P(draw a pair of tinted lenses) = $\frac{8}{24}$

= $\frac{1}{3}$

P(draw a pair of non-tinted lenses)

= 1 – P(draw a pair of tinted lenses)

= $1 - \frac{1}{3}$

= $\frac{2}{3}$

16. (a) Total number of possible outcomes = 26 + 62 + 8 + 9 + 12

= 117

(i) There are 62 teachers in the school.

P(school personnel is a teacher) = $\frac{62}{117}$

(ii) There are 26 management staff in the school.

P(school personnel is a management staff) = $\frac{26}{117}$

= $\frac{2}{9}$

(iii) There are 9 + 12 = 21 staff that are either an administrative or maintenance staff in the school.

P(school personnel is an administrative or maintenance staff)

= $\frac{21}{117}$

= $\frac{7}{39}$

(b) Total number of possible outcomes = 117 – 2 – 1

= 114

(i) There are 9 – 1 = 8 administrative staff in the school.

P(school personnel is an administrative staff) = $\frac{8}{114}$

= $\frac{4}{57}$

(ii) There are 8 laboratory staff in the school.

P(school personnel is a laboratory staff) = $\frac{8}{114}$

= $\frac{4}{57}$

P(school personnel is not a laboratory staff)

= 1 – P(school personnel is a laboratory staff)

= $1 - \frac{4}{57}$

= $\frac{53}{57}$

17. Total number of possible outcomes = 117

(i) P(pair of socks is yellow) = $\frac{2}{9}$

$\frac{\text{Number of yellow pairs of socks}}{\text{Total number of pairs of socks}} = \frac{2}{9}$

$\therefore$ Number of yellow pairs of socks = $\frac{2}{9} \times 117$

= 26

(ii) P(pair of socks is neither yellow nor grey)

= 1 – P(pair of socks is yellow) – P(pair of socks is grey)

= $1 - \frac{2}{9} - \frac{3}{13}$

= $\frac{64}{117}$

$\frac{\text{Number of pairs of socks neither yellow nor grey}}{\text{Total number of pairs of socks}} = \frac{64}{117}$

$\therefore$ Number of pairs of sock neither yellow nor grey

= $\frac{64}{117} \times 117$

= 64

18. The sample space consists of the question numbers 1, 2, 3, ..., 80.

Total number of questions = 80

(i) There are 9 question numbers containing only a single digit, i.e. 1, 2, 3, ..., 9.

P(question number contains only a single digit) = $\frac{9}{80}$

(ii) There are 13 question numbers greater than 67, i.e. 68,69, 70, ..., 80.

P(question number is greater than 67) = $\frac{13}{80}$

(iii) There are 7 + 9 = 16 question numbers containing exactly one '7',

i.e. 7, 17, 27, 37, 47, 57, 67, 70, 71, 72, 73, 74, 75, 76, 78 and 79.

P(question number contains exactly one '7') = $\frac{16}{80}$

= $\frac{1}{5}$

(iv) There are 8 question numbers divisible by both 2 and 5, i.e. 10, 20, 30, 40, 50, 60, 70 and 80.

P(question number is divisible by both 2 and 5) $= \frac{8}{80}$

$= \frac{1}{10}$

19. The sample space consists of the two-digit numbers 23, 25, 27, 32, 35, 37, 52, 53, 57, 72, 73 and 75.

Total number of possible outcomes = 12

(i) There are 3 numbers divisible by 4, i.e. 32, 52 and 72.

P(two-digit number is divisible by 4) $= \frac{3}{12}$

$= \frac{1}{4}$

(ii) There are 4 prime numbers, i.e. 23, 37, 53 and 73.

P(two-digit number is a prime number) $= \frac{4}{12}$

$= \frac{1}{3}$

20. Let the probability of getting a '1' be x.

P(getting a '3') $= 2x$

P(getting a '2') $= 3(2x)$

$= 6x$

P(getting a '4') $= 6x$

$\therefore x + 2x + 6x + 6x = 1$

$15x = 1$

$\therefore x = \frac{1}{15}$

2 and 3 are prime numbers.

P(getting a prime number) $= 6x + 2x$

$= 8x$

$= 8\left(\frac{1}{15}\right)$

$= \frac{8}{15}$

Exercise 15B

1. (i) Total number of girls $= 8 + 3 + 1$

$= 12$

P(student is a girl) $= \frac{12}{30}$

$= \frac{2}{5}$

(ii) Total number of non-Chinese students $= 3 + 1 + 4 + 3$

$= 11$

P(student is not a Chinese) $= \frac{11}{30}$

(iii) P(student is an Indian boy) $= \frac{3}{30}$

$= \frac{1}{10}$

P(student is not an Indian boy) = 1 – P(student is an Indian boy)

$= 1 - \frac{1}{10}$

$= \frac{9}{10}$

(iv) Number of Eurasian students = 0

P(student is a Eurasian) $= \frac{0}{30}$

$= 0$

2. (i) Total number of books $= 5 + 5$

$= 10$

P(book chosen is in Japanese) $= \frac{3}{10}$

(ii) Number of novels in Japanese $= 3 - 2$

$= 1$

Number of novels in English $= 10 - 5 - 1$

$= 4$

P(book chosen is a novel in English) $= \frac{4}{10}$

$= \frac{2}{5}$

3. (i) P(student prefers apple)

$= \frac{\text{Area of the sector of denoting apple}}{\text{Area of the circle}}$

$= \frac{\text{Area of the sector of denoting apple}}{360°}$

$= \frac{150°}{360°}$

$= \frac{5}{12}$

(ii) Angle of the sector denoting mango

$= 360° - 150° - 90° - 45°$

$= 75°$

P(student prefers mango)

$= \frac{\text{Area of the sector denoting mango}}{\text{Area of the circle}}$

$= \frac{\text{Area of the sector denoting mango}}{360°}$

$= \frac{75°}{360°}$

$= \frac{5}{24}$

(iii) Angle of sector denoting papaya or guava

$= 90° + 45°$

$= 135°$

P(student prefers papaya or guava)

$= \dfrac{\text{Area of the sector denoting papaya or guava}}{\text{Area of the circle}}$

$= \dfrac{\text{Area of the sector denoting papaya or guava}}{360°}$

$= \dfrac{135°}{360°}$

$= \dfrac{3}{8}$

4. **(i)** Total number of sides in an octagon = 8

P(point lies in region R) = $\dfrac{1}{8}$

(ii) P(points lies in region S) = $\dfrac{3}{8}$

(iii) P(point lies in region P or Q) = $\dfrac{2+2}{8}$

$= \dfrac{4}{8}$

$= \dfrac{1}{2}$

5. **(i)** Total number of students = $15 + x$

$= x + 15$

(ii) P(student is a girl) = $\dfrac{15}{x+15}$

(iii) Given that $\dfrac{15}{x+15} = \dfrac{1}{5}$,

$5(15) = x + 15$

$75 = x + 15$

$\therefore x = 60$

6. **(i)** P(student is a girl who did not check in her luggage) = $\dfrac{8}{38}$

$= \dfrac{4}{19}$

(ii) Total number of girls = 38 – 18

= 20

Total number of girls who checked in their luggage = 20 – 8

= 12

Total number of students who checked in their luggage

= 12 + 6

= 18

P(student checked in his/her luggage) = $\dfrac{18}{38}$

$= \dfrac{9}{19}$

7. **(a)** **(i)** Total number of students = 16 + 24

= 40

P(student is a boy) = $\dfrac{16}{40}$

$= \dfrac{2}{5}$

(ii) Total number of left-handed students = 3 + 2

= 5

P(student is left-handed) = $\dfrac{5}{40}$

$= \dfrac{1}{8}$

(b) **(i)** Total number of students who can borrow the visualiser

= 40 – 1

= 39

P(students is a boy who is left-handed) = $\dfrac{3}{39}$

$= \dfrac{1}{13}$

(ii) Number of girls who are not left-handed = 24 – 2 – 1

= 21

P(student is a girl who is not left-handed) = $\dfrac{21}{39}$

$= \dfrac{7}{13}$

8. Total number of presents = $(3h + 11) + (h + 5)$

$= 4h + 16$

P(Ethan obtains a red present) = $\dfrac{3h+11}{4h+16}$

Given that $\dfrac{3h+11}{4h+16} = \dfrac{19}{26}$,

$26(3h + 11) = 19(4h + 16)$

$78h + 286 = 76h + 304$

$2h = 18$

$h = 9$

9. $\dfrac{7}{13} + \dfrac{1}{k} + \dfrac{1}{2k} = 1$

$14k + 26 + 13 = 26k$

$12k = 39$

$\therefore k = \dfrac{39}{12}$

$= 3\dfrac{1}{4}$

The value of k is $3\dfrac{1}{4}$.

10. Total number of toothbrushes = 15 + 5

= 20

P(draw a toothbrush with soft bristles) = $\dfrac{p+5}{20}$

Given that $\dfrac{p+5}{20} = \dfrac{3}{4}$,

$4(p + 5) = 20(3)$

$4p + 20 = 60$

$4p = 40$

$p = 10$

11. Total number of boys and girls remaining after graduation

$= 23 + 35 - q - (q + 4)$

$= 54 - 2q$

Total number of boys remaining after graduation

$= 23 - q$

P(boy is selected for event) $= \dfrac{23 - q}{54 - 2q}$

Given that $\dfrac{23 - q}{54 - 2q} = \dfrac{2}{5}$,

$5(23 - q) = 2(54 - 2q)$

$115 - 5q = 108 - 4q$

$q = 7$

12. **(i)** P(drawing a black ball)

$= 1 -$ P(drawing a red ball) $-$ P(drawing a yellow ball)

$= 1 - \dfrac{1}{4} - \dfrac{2}{5}$

$= \dfrac{7}{20}$

(ii) Total number of balls in the bag now

$= 40 + (2x + 1) + (x + 2) - (x - 3)$

$= 40 + 2x + 1 + x + 2 - x + 3$

$= 2x + 46$

(iii) Before the balls were added,

P(drawing a yellow ball) $= \dfrac{2}{5}$

$\dfrac{\text{Number of yellow balls}}{\text{Total number of balls}} = \dfrac{2}{5}$

$\therefore$ Number of yellow balls $= \dfrac{2}{5} \times 40$

$= 16$

After the balls were added,

Total number of yellow balls $= 16 + x + 2$

$= x + 18$

P(drawing a yellow ball) $= \dfrac{x + 18}{2x + 46}$

Given that $\dfrac{x + 18}{2x + 46} = \dfrac{3}{7}$

$7(x + 18) = 3(2x + 46)$

$7x + 126 = 6x + 138$

$x = 12$

Number of yellow balls in the bag now $= 12 + 18$

$= 30$

13. Total number of students = 50

$2x + y = 50$ — (1)

After some boys left and some girls entered the auditorium,

Total number of students $= 50 - (y - 6) + (2x - 5)$

$= 50 - y + 6 + 2x - 5$

$= 2x - y + 51$

Total number of girls $= y + 2x - 5$

$= 2x + y - 5$

P(girl is selected at random) $= \dfrac{2x + y - 5}{2x - y + 51}$

Given that $\dfrac{2x + y - 5}{2x - y + 51} = \dfrac{9}{13}$,

$13(2x + y - 5) = 9(2x - y + 51)$

$26x + 13y - 65 = 18x - 9y + 459$

$8x + 22y = 524$

$4x + 11y = 262$ — (2)

From (1),

$y = 50 - 2x$ — (3)

Substitute (3) into (2):

$4x + 11(50 - 2x) = 262$

$4x + 550 - 22x = 262$

$18x = 288$

$\therefore x = 16$

Substitute $x = 16$ into (3):

$\therefore y = 50 - 2(16)$

$= 18$

The value of x and of y are 16 and 18 respectively.

Review Exercise 15

1. **(a)** **(i)** The sample space consists of the numbers 5, 6, 8, 56, 58, 65, 68, 85, 86, 568, 586, 658, 685, 856, 865.

(ii) Total number of possible outcomes = 15

(b) **(i)** There are 6 two-digit numbers, i.e. 56, 58, 65, 68, 85 and 86.

P(number consists of two digits) $= \dfrac{6}{15}$

$= \dfrac{2}{5}$

(ii) There are 5 numbers that are a multiple of 5, i.e. 5, 65, 85, 685 and 865.

P(number is a multiple of 5) $= \dfrac{5}{15}$

$= \dfrac{1}{3}$

2. The sample space consists of 1, 2, 3, 4, 5 and 6.

Total number of possible outcomes = 6

(i) There are 3 even numbers, i.e. 2, 4 and 6.

P(rolls an even number) $= \dfrac{3}{6}$

$= \dfrac{1}{2}$

(ii) There are 2 composite numbers, i.e. 4 and 6.

P(rolls a composite number) $= \dfrac{2}{6}$

$= \dfrac{1}{3}$

(iii) There is only one number divisible by 4, i.e. 4.

P(rolls a number divisible by 4) $= \dfrac{1}{6}$

3. Total number of possible outcomes = 26

(i) There is only one queen of hearts in the pack.

P(drawing the queen of hearts) = $\frac{1}{26}$

(ii) There is no jack of clubs in the pack.

P(drawing the jack or clubs) = $\frac{0}{26}$
= 0

(iii) There are 2 cards that are either the six of hearts or the seven of diamonds.

P(drawing either the six of hearts or the seven of diamonds)
= $\frac{2}{26}$
= $\frac{1}{13}$

(iv) There are 2 cards that are a nine in the pack, namely, the nine of hearts and the nine of diamonds.

P(drawing a card that is a nine) = $\frac{2}{26}$
= $\frac{1}{13}$

P(drawing a card that is not a nine)
= 1 – P(drawing a card that is a nine)
= $1 - \frac{1}{13}$
= $\frac{12}{13}$

4. Total number of possible outcomes = 6

(i) There is only one sector that has an umbrella as a prize.

P(wins an umbrella) = $\frac{1}{6}$

(ii) There are 3 sectors that has a voucher as a prize, namely a \$30 shopping voucher, a \$50 supermarket vourcher and a \$40 dining voucher.

P(wins a voucher) = $\frac{3}{6}$
= $\frac{1}{2}$

(iii) There are no sectors with a prize of \$100 cash.

P(wins \$100 cash) = $\frac{0}{6}$
= 0

5. (i) There are 7 'orange' symbols.

P(shows the symbol 'orange') = $\frac{7}{22}$

(ii) Number of 'grape' symbols = 22 – 4 – 7 – 9
= 2

P(shows the symbol 'grape') = $\frac{2}{22}$
= $\frac{1}{11}$

(iii) There are no 'pineapple' symbol.

P(shows the symbol 'pineapple') = $\frac{0}{22}$
= 0

(iv) There are 4 + 9 = 13 symbols that are either 'cherry' or 'peach'.

P(shows either the symbol 'cherry' or the symbol 'peach') = $\frac{13}{22}$

6. (i) P(sweet is a mint wrapped in red paper) = $\frac{4}{20}$
= $\frac{1}{5}$

(ii) There are 7 + 3 = 10 toffees in the bag.

P(sweet is a toffee) = $\frac{10}{20}$
= $\frac{1}{2}$

(iii) There are 7 + 6 = 13 sweets wrapped in green paper.

P(sweet is wrapped in green paper) = $\frac{13}{20}$

7. (a) (i) Total number of staplers = 7 + 11
= 18

There are no green staplers in the bag.

P(stapler is green) = $\frac{0}{18}$
= 0

(ii) There are 7 + 11 = 18 staplers that are either white or orange in the bag.

P(stapler is either white or orange) = $\frac{18}{18}$
= 1

(b) (i) Total number of staplers = 18 + 12
= 30

There are 12 red staplers in the bag.

P(stapler is red) = $\frac{12}{30}$
= $\frac{2}{5}$

(ii) There are 11 orange staplers in the bag.

P(stapler is orange) = $\frac{11}{30}$

P(stapler is not orange) = $1 - \frac{11}{30}$
= $\frac{19}{30}$

8. (i) Angle of the sector denoting travel by car
= 360° – 90° – 110° – 60°
= 100°

P(student travels to school by car)
= $\frac{\text{Area of the sector denoting travel by car}}{\text{Area of the circle}}$
= $\frac{\text{Area of the sector denoting travel by car}}{360°}$
= $\frac{100°}{360°}$
= $\frac{5}{18}$

(ii) Angle of the sector denoting travel by train or on foot

$= 110° + 60°$

$= 170°$

P(students travels to school by train or on foot)

$= \dfrac{\text{Area of the sector denoting travel by train or on foot}}{\text{Area of the circle}}$

$= \dfrac{\text{Area of the sector denoting travel by train or on foot}}{360°}$

$= \dfrac{170°}{360°}$

$= \dfrac{17}{36}$

(iii) Angle of sector denoting travel by bicycle = 0°

P(student travels to school by bicycle)

$= \dfrac{\text{Area of the sector denoting travel by bicycle}}{\text{Area of the circle}}$

$= \dfrac{\text{Area of the sector denoting travel by bicycle}}{360°}$

$= \dfrac{0°}{360°}$

$= 0$

9. **(i)** Number of tulips $= 100 - 20 - h$

$= 80 - h$

P(picking a stalk of tulip) $= \dfrac{80 - h}{100}$

Given that $\dfrac{80 - h}{100} = \dfrac{1}{4}$,

$4(80 - h) = 100$

$320 - 4h = 100$

$4h = 220$

$h = 55$

(ii) Number of flowers remaining $= 100 - 10$

$= 90$

Number of roses = 55

P(picking a stalk of rose) $= \dfrac{55}{90}$

$= \dfrac{11}{18}$

10. **(i)** Total number of vehicles $= 125 + 3p + 2q + 20$

$= 3p + 2q + 145$

P(vehicle is a motorcycle) $= \dfrac{3p}{3p + 2q + 145}$

Given that $\dfrac{3p}{3p + 2q + 145} = \dfrac{3}{40}$,

$40(3p) = 3(3p + 2q + 145)$

$120p = 9p + 6q + 435$

$111p - 6q - 435 = 0$

$37p - 2q - 145 = 0$

(ii) P(vehicle is a bus) $= \dfrac{20}{3p + 2q + 145}$

Given that $\dfrac{20}{3p + 2q + 145} = \dfrac{1}{10}$,

$10(20) = 3p + 2q + 145$

$200 = 3p + 2q + 145$

$3p + 2q - 55 = 0$

(iii) $37p - 2q - 145 = 0$ — (1)

$3p + 2q - 55 = 0$ — (2)

(1) + (2):

$(37p - 2q - 145) + (3p + 2q - 55) = 0 + 0$

$40p - 200 = 0$

$40p = 200$

$\therefore p = 5$

Substitute $p = 5$ into (2):

$3(5) + 2q - 55 = 0$

$15 + 2q - 55 = 0$

$2q = 40$

$\therefore q = 20$

The value of p and of q are 5 and 20 respectively.

Challenge Yourself

1. Let the number of cards in the smaller pile be x and number of cards in the bigger pile be $52 - x$.

Total number of picture cards in a standard pack of 52 playing cards = 12

$\therefore \dfrac{4}{11}x + \dfrac{2}{15}(52 - x) = 12$

$60x + 22(52 - x) = 1980$

$60x + 1144 - 22x = 1980$

$38x = 836$

$\therefore x = 22$

Number of cards in the bigger pile $= 52 - 22$

$= 30$

There are 22 cards in the smaller pile and 30 cards in the bigger pile.

2. **(i)** Let the 3 friends Farhan, Vishal and Michael be A, B and C, and the letters be 1, 2 and 3.

i.e. Farhan should receive letter 1, Vishal should receive letter 2 and Michael should receive letter 3. $A \leftrightarrow 1$, $B \leftrightarrow 2$, $C \leftrightarrow 3$.

The sample space consists of {$A1$, $B2$, $C3$}, {$A1$, $B3$, $C2$}, {$A2$, $B1$, $C3$}, {$A2$, $B3$, $C1$}, {$A3$, $B1$, $C2$}, {$A3$, $B2$, $C1$}.

Total number of possible outcomes = 6

There are 3 ways where exactly one of his friends receive the correct letter,

i.e. {$A1$, $B3$, $C2$}, {$A2$, $B1$, $C3$} and {$A3$, $B2$, $C1$}.

P(exactly one of his friends receive the correct letter) $= \dfrac{3}{6}$

$= \dfrac{1}{2}$

(ii) There are no ways where exactly two of his friends receive the correct letters.

P(exactly two of his friends receive the correct letter) $= \dfrac{0}{6}$

$= 0$

(iii) There is only one way where all three of his friends receive the correct letters, i.e. {$A1$, $B2$, $C3$}.

P(all three of his friends receive the correct letters) $= \dfrac{1}{6}$

Chapter 16 Statistical Diagrams

TEACHING NOTES

Suggested Approach

In Secondary One, students would have explored the usage of some statistical diagrams like pictograms, bar graphs, pie charts and line graphs.

In this chapter, students will learn about other statistical diagrams, namely dot diagrams, stem-and-leaf diagrams, scatter diagrams and histograms.

Other than illustrating how each type of diagram is used, teachers may either provide more examples than shown in the textbook, or instruct students to develop their own examples. Students are expected to be familiar with the similarities and differences, advantages and disadvantages among various statistical diagrams and decide the most suitable statistical diagram to use.

Section 16.1: Statistical Diagrams

Students will recap what are pictograms, bar graphs, pie charts and line graphs here. Teachers may revise examples from Book 1 to allow recollection and application of the diagrams.

Section 16.2: Dot Diagrams

Teachers should go through the worked examples clearly with the students and introduce the dot diagram. Students should know how to draw dot diagrams, extract relevant information from the diagram and describe the distribution of the data.

It is important that the values on the number line are spaced evenly and one dot is used to represent one data value.

Section 16.3: Stem-and-Leaf Diagrams

The features of a stem-and-leaf diagram should be explained clearly to students. Teachers can state and explain the points listed on the right (see Attention on page 448). Using the worked examples and questions available, students should be familiar with the different types of stem-and-leaf diagrams, namely the stem-and-leaf diagram, stem-and-leaf diagram with split stems and back-to-back stem-and-leaf diagram.

Students should also be given opportunities to compare between two stem-and-leaf diagrams, interpret and explain the findings according to the context of the data.

Section 16.4: Scatter Diagrams

Teachers can introduce the concept of scatter diagrams by establishing everyday situations with correlations that can be positive or negative, and others that have no correlations at all. Teachers can illustrate how the use of scatter diagrams can verify claims as to the type of correlation two variables have, which might not be as evident when using other statistical diagrams.

Section 16.5 Histograms for Ungrouped Data

Teachers may want to introduce histograms by revising the drawing of a dot diagram on Page 465. It is important that students realise that histograms and bar graphs, though they look similar, are different representations of sets of data.

Teachers may want to group students, discuss and present the similarities, differences, advantages and disadvantages between a dot diagram and a histogram (see Journal Writing on page 466). The same may be done for the Class Discussion (see Class Discussion: Evaluation of Statistical Diagrams)

Section 16.6 Histograms for Grouped Data

It is crucial that the differences between ungrouped data and grouped data are stated at the beginning. The use of class intervals for grouped data is what differentiates both types of data, as the interval ‘groups’ similar data together. Students are also required to make frequency tables in this section.

Similar to the previous section, students can be grouped together to discuss and present the similarities, differences, advantages and disadvantages between a stem-and-leaf diagram and a histogram for grouped data (see Journal Writing on page 470).

To assess and reinforce students’ understanding, teachers can get the entire class to measure the length of their right shoe and present the results in the form of a histogram (see Performance Task on page 472).

The last activity in this section is to expose students to the usage of histograms for grouped data with unequal class intervals (see Class Discussion: Histograms for Grouped Data with Unequal Class Intervals).

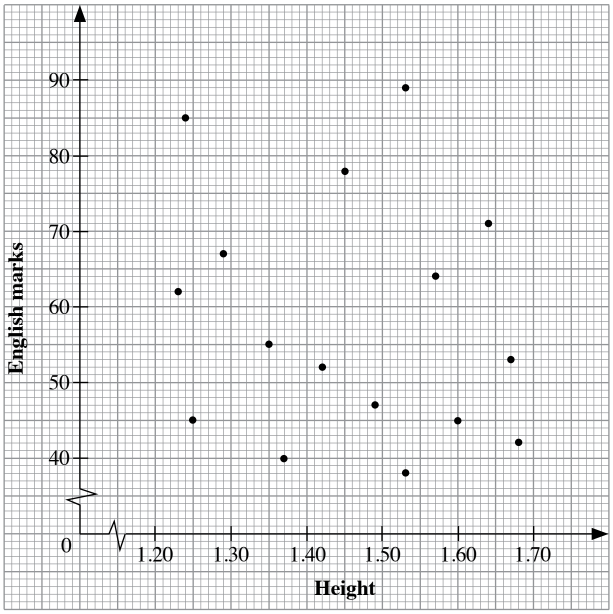
WORKED SOLUTIONS

Thinking Time (Page 454)

Yes, the equation of the line of best fit can similarly be found by obtaining two points on the line to find the gradient and the y-intercept, which was covered in Chapter 2.

Class Discussion (Scatter Diagram with No Correlation)

1.



2. There is no correlation between the height and English marks of the pupils, as no clear trend can be observed.

Thinking Time (Page 459)

1. Positive. This is true in most cases for students preparing for an examination.
2. Positive. More cars on the road generally will cause more traffic accidents.
3. Zero. A person's IQ will not be affected by everyday activities.
4. Zero. There is no relationship between the house number and weight of an occupant.
5. Positive. The amount of interest earned depends on the principal you have.
6. Zero. The price of a shirt usually varies by design, and shirts of the same design can also have small variations, such as colour.
7. Negative. The amount of food one can consume depends on its accumulated quantity; the smaller the cupcake, the higher the number of cupcakes that can be eaten.
8. Zero. Human beings generally have 10 fingers and height does not affect this.
9. Positive. It is assumed that the sprinter competes in the same league.

Journal Writing (Page 459)

Teachers can get students to come up with a list of variables, and assess their understanding. A few of these answers can be used for a class discussion to further consolidate what was covered in this section.

Journal Writing (Page 466)

1. Both the dot diagram and histogram can be used to show the frequency of a certain variable. The horizontal axis in both diagrams is the same and the values in the axis have to be arranged in a certain order. The range, clusters, extreme data or symmetry can be identified from the dot diagram and histogram.

   There is only one axis in a dot diagram while there are two axes in a histogram. In a dot diagram, dots are used to represent the values in the data, while in a histogram, the frequency of a value is collated and represented as a rectangle. There are spaces between vertical rows of dots in a dot diagram while there are no spaces between the rectangles in a histogram.
2. **(a)** A dot diagram is more appropriate than a histogram when there are extreme values and gaps in between values. Also, a dot diagram is more suitable when the number of data and the data set is small.

   (b) A histogram is more appropriate than a dot diagram when there are no gaps in between values and the data set is large.

Class Discussion (Evaluation of Statistical Diagrams)

1. The favourite ice-cream flavours of Nora's classmates is a form of categorical data. Pictograms, bar charts or pie charts are suitable for showing categorical data.
   Dot diagrams are more suited for numerical data, where the horizontal axis is the number line.
   Therefore, the choice of a dot diagram to present Nora's survey data is not suitable.
2. Jun Wei's first statement, that there are three times as many households which own 2 smartphones as that which own 0 smartphones and that which own 1 smartphone, is incorrect.

   The numbers of households who own 0, 1 and 2 smartphones are 100, 100 and 200 respectively. Hence, the number of households who own 2 smartphones is actually twice, and not three times that of households which own 0 smartphones and that which own 1 smartphone.

   His second statement, that the number of households which own 3 smartphones, is half of that which own 2 smartphones is incorrect as well.

   The numbers of households who own 2 and 3 smartphones are 200 and 125 respectively. Hence, the number of households who own 3 smartphones is more than half that of households which own 2 smartphones.

   The most probable reason why Jun Wei was mistaken with his statements was that he took the height of the rectangle to be proportional to the frequency. However, the vertical axis starts from 50, and not 0, in this histogram, which contributed to his error.

Journal Writing (Page 470)

(a) A stem-and-leaf diagram is more appropriate than a histogram when the range of the data is small. It is better to use a stem-and-leaf diagram when we want to know the exact values in the data set.

(b) A histogram is more appropriate than a stem-and-leaf diagram when the range of the data is large and we are not concerned with the exact values in the data set. A histogram also provides a better graphical representation of a data set compared to a stem-and-leaf diagram, as we can simply identify the data set with the lowest and highest frequencies.

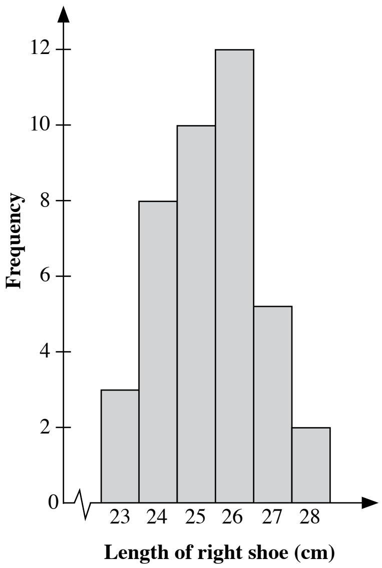
Performance Task (Page 472)

1. The length of the right shoe for most students, correct to the nearest cm, should range from 23 cm to 28 cm.

2. Sample frequency table for a class of 40 students:

Length of right shoe (cm)	23	24	25	26	27	28
Frequency	3	8	10	12	5	2

3.



4. The length of the right shoe, correct to the nearest cm, for a class of 40 students ranges from 23 cm to 28 cm. The length is clustered around 24 cm to 28 cm.

Class Discussion (Histograms for Grouped Data with Unequal Class Intervals)

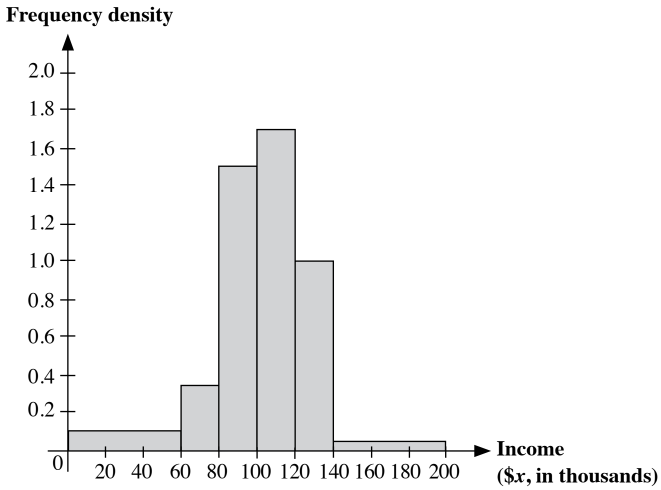
Fig. 16.12 gives a more accurate representative of the data.
The heights of the rectangles are obtained by dividing the frequency by the class intervals multiplied by 20.

1.

Income ($\$x$, in thousands)	Frequency	Frequency density $= \dfrac{\text{Frequency}}{\text{Size of class interval}}$
$0 \leqslant x < 60$	6	$\frac{6}{60} = 0.1$
$60 \leqslant x < 80$	7	$\frac{7}{20} = 0.35$
$80 \leqslant x < 100$	30	$\frac{30}{20} = 1.5$
$100 \leqslant x < 120$	34	$\frac{34}{20} = 1.7$
$120 \leqslant x < 140$	20	$\frac{20}{20} = 1$
$140 \leqslant x < 200$	3	$\frac{3}{60} = 0.05$

Table 16.7

2.



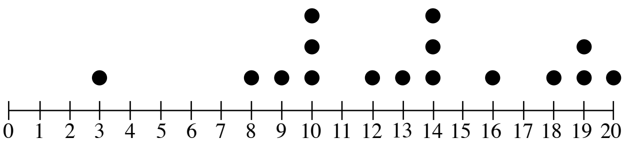
3. Area of rectangle for $0 \leqslant x < 60$
$= 0.1 \times 60$
$= 6$ units^2
Area of rectangle for $60 \leqslant x < 80$
$= 0.35 \times 20$
$= 7$ units^2
Area of rectangle for $80 \leqslant x < 100$
$= 1.5 \times 20$
$= 30$ units^2
Area of rectangle for $100 \leqslant x < 120$
$= 1.7 \times 20$
$= 24$ units^2
Area of rectangle for $120 \leqslant x < 140$
$= 1 \times 20$
$= 20$ units^2
Area of rectangle for $140 \leqslant x < 200$
$= 0.05 \times 60$
$= 3$ units^2
The frequencies in Table 16.7 are the same as the areas of the rectangles.

4. The histogram with unequal class intervals is preferred over the histogram with equal class intervals because it has no gaps.

5. Some examples where unequal class intervals are used in histograms, beside income, are mass, height, temperature, age etc, where there are extreme values and gaps in between the data.

Practise Now 1

(i) **Step 1:** Identify the range of values, i.e. 3 marks to 20 marks.
Step 2: Draw a horizontal number line with equal intervals to represent the range of values identified in Step 1.
Step 3: Using the data from the table, plot each datum with a dot over its value on the number line.



(ii) Number of students who passed the test $= 50\% \times 16$

$$= \frac{50}{100} \times 16$$
$$= 8$$

From the dot diagram, passing mark = 14

(iii) The marks obtained by 16 students in a class for a Mathematics test ranges from 3 marks to 20 marks. There is an extreme value of 3 marks.

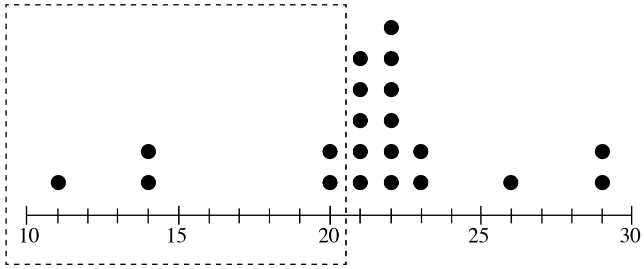
Practise Now 2

(i) The youngest contestant is 11 years old.

(ii) The total number of contestants who join the karaoke contest is 20.

(iii) Number of contestants less than or equal to x years

$= \frac{1}{4} \times 20$
$= 5$
Since $1 + 2 + 2 = 5$,



$\therefore$ Value of $x = 20$

(iv) The age, in years, of 20 contestants who join a karaoke contest ranges from 11 years to 29 years. The age clustered around 20 to 23 years. The extreme ages of 11 years, 14 years and 29 years deviate considerably from the other ages recorded.

Practise Now 3

Stem	Leaf
1	2 4
2	0 3
3	0 3 5 7
4	1 4
5	6 9

Key: 1 | 2 means 12 text messages

Practise Now 4

(i) To construct a stem-and-leaf diagram with spilt stems, we separate the stems into smaller number of equal-sized units.

Stem	Leaf
8	0 3 4
8	6 6 6 7 8 8 9 9
9	0 0 0 0 1 1 2 2
9	5 6 7 7 9
10	0 1 2 4 4
10	6 8 8

Key: 8 | 3 means 83 kg

(ii) The most common mass is 90 kg.

(iii) Required percentage $= \frac{8}{32} \times 100\%$
$= 25\%$

Practise Now 5

(i) A back-to-back stem-and-leaf diagram consists of a stem and leaves on both sides of the stem.

Step 1: Construct a stem-and-leaf diagram for Michael.

Step 2: Using the same stem, construct the stem-and-leaf diagram for Khairul on the left side of the stem.

Leaves for Khairul	Stem	Leaves for Michael
9 9 6 6 3 2 1	4	7
3 2 2	5	8 9
	6	5 5
9 8 8	7	0 2 7
5 4 2 2	8	1 1 3 3 3 4 8 9 9
5 2 0	9	1 8 9

Key: 4 | 7 means 47 minutes

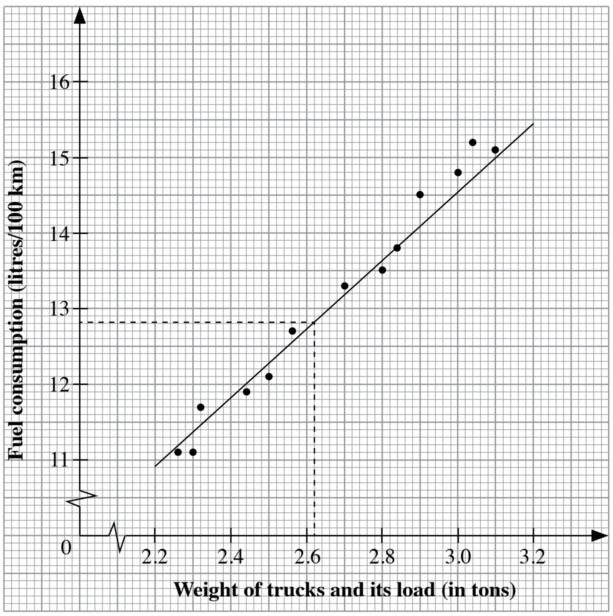
(ii) Michael used his tablet computer for the longest time of 99 minutes in a day.

(iii) Khairul used his tablet computer for the shortest time of 41 minutes in a day.

(iii) Michael. The times for Michael clustered from 80 to 89 minutes whereas the times for Khairul clustered from 40 to 49 minutes.

Practise Now 6

(i)



(ii) A strong positive correlation is shown.

(iii) From the graph, 12.8 litres of petrol will be needed for a truck with a total mass of 2.62 tons to travel 100 km.

(iv) Take two points on the line (2.4, 11.8) and (2.5, 12.3).

$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{12.3 - 11.8}{2.5 - 2.4} = \frac{0.5}{0.1} = 5$$

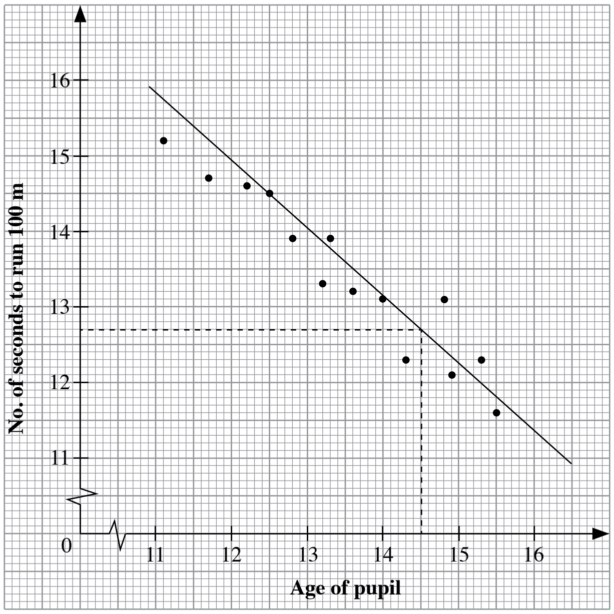
From graph, y-intercept = 9.8

$\therefore$ The equation of the best fit line is $y = 5x + 9.8$.

(v) No, it would not be reliable as a total mass of 4.8 tons is out of the range.

Practise Now 7

(i)



(ii) A strong negative correlation is shown.

(iii) From the graph, a pupil aged 14.5 years is expected to complete the 100 m race in 12.7 seconds.

(iv) Take two points on the line (14.5, 12.7) and (15.3, 12).

$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{12.7 - 12}{14.5 - 15.3} = -\frac{0.7}{0.8} = -\frac{7}{8}$$

From graph, y-intercept = 16.3

$\therefore$ The equation of the best fit line is $y = -\frac{7}{8}x + 16.3$.

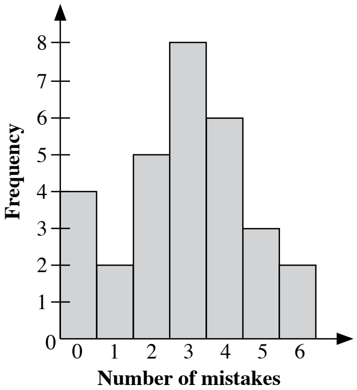
(v) No, the above data should not be used as the age 17 years is outside the range of the study.

Practise Now 8

(i)

Number of mistakes	0	1	2	3	4	5	6
Frequency	4	2	5	8	6	3	2

(ii)



(iii) The most common number of mistakes made is 3.

(iv) Fraction of students who made at most 3 mistakes in the test

$= \dfrac{4+2+5+8}{30}$

$= \dfrac{19}{30}$

Practise Now 9

(i)

Waiting time (x minutes)	Tally	Frequency
$5 \leqslant x < 10$	//	2
$10 \leqslant x < 15$	卌 /	6
$15 \leqslant x < 20$	卌 //	7
$20 \leqslant x < 25$	///	3
$25 \leqslant x < 30$	////	4
$30 \leqslant x < 35$	////	4
$35 \leqslant x < 40$	///	3
$40 \leqslant x < 45$	//	2
$45 \leqslant x < 50$	///	3
$50 \leqslant x < 55$	//	2
Total frequency		36

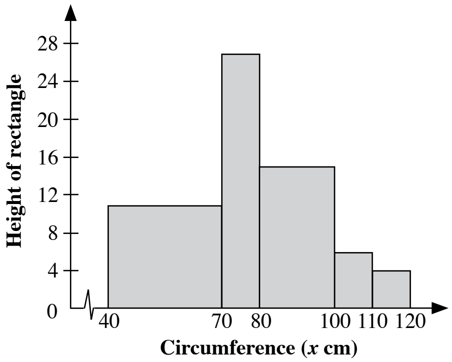
(ii)



Practise Now 10

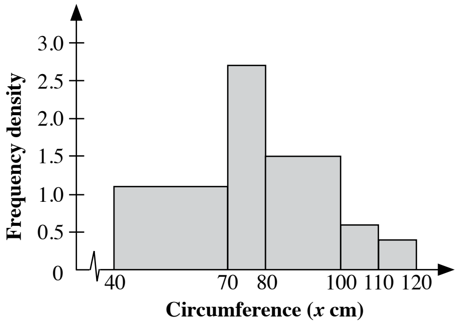
Method 1 (Using the heights of rectangles)

Circumference (x cm)	Class width		Frequency	Rectangle's height
$40 < x \leqslant 70$	30	$3 \times$ standard	33	$33 \div 3 = 11$
$70 < x \leqslant 80$	10	$1 \times$ standard	27	$27 \div 1 = 27$
$80 < x \leqslant 100$	20	$2 \times$ standard	30	$30 \div 2 = 15$
$100 < x \leqslant 110$	10	$1 \times$ standard	6	$6 \div 1 = 6$
$110 < x \leqslant 120$	10	$1 \times$ standard	4	$4 \div 1 = 4$



Method 2 (Using frequency densities)

Circumference (x cm)	Frequency	Class width	Frequency density $= \dfrac{\text{Frequency}}{\text{Class width}}$
$40 < x \leqslant 70$	33	30	$33 \div 30 = 1.1$
$70 < x \leqslant 80$	27	10	$27 \div 10 = 2.7$
$80 < x \leqslant 100$	30	20	$30 \div 20 = 1.5$
$100 < x \leqslant 110$	6	10	$6 \div 10 = 0.6$
$110 < x \leqslant 120$	4	10	$4 \div 10 = 0.4$

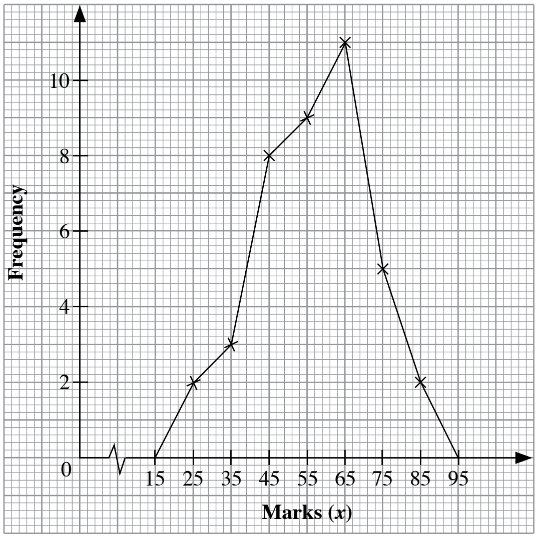


Practise Now 11

(a) Mid-value = 45

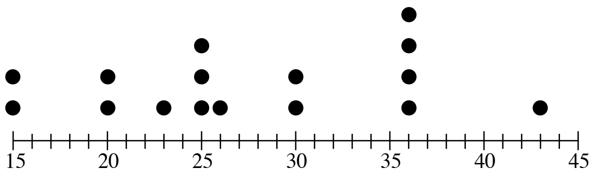
(b)

Marks (x)	Mid-value	Number of students
$20 < x \leqslant 30$	25	2
$30 < x \leqslant 40$	35	3
$40 < x \leqslant 50$	45	8
$50 < x \leqslant 60$	55	9
$60 < x \leqslant 70$	65	11
$70 < x \leqslant 80$	75	5
$80 < x \leqslant 90$	85	2



Exercise 16A

1. **(i)**



(ii) The mass of the lightest bag of rice is 15 kg.

(iii) The most common mass is 36 kg.

(iv) Number of bags with a mass of less than 30 kg = 9

2. **(i)** The most common time taken was 1 minute.

(ii) Shortest time = 1 minute

Longest time = 14 minutes

Difference = 14 – 1

= 13

(iii) Total number of people who completed the questionnaire

= 5 + 2 + 2 + 2 + 1 + 2 + 2 + 2 + 2 + 3 + 3 + 2 + 2 + 1

= 31

(iv) Number of people who took 10 minutes or longer

= 3 + 3 + 2 + 2 + 1

= 11

Required ratio = 11 : 31

3. **(i)**

Stem	Leaf
2	5 8 8
3	3 5 5 6
4	0 5 8 8
5	5 5
6	4 5 5 6 8
7	0 5

Key: 2 | 5 means 2.5 hours

(ii) Percentage of students who spent less than 3 hours

$= \frac{3}{20} \times 100\%$

= 15%

(iii) Fraction of students who had 2 marks deducted

$= \frac{7}{20}$

4. **(a)** Strong, negative correlation

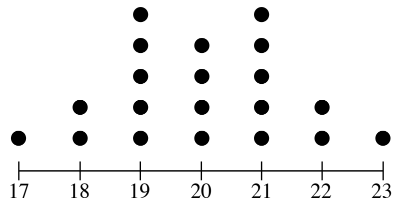
(b) Moderate, positive correlation

(c) Strong, positive correlation

(d) No correlation observed

5. From the graph, the monthly income of an individual with 18 years of experience is estimated to be $7000.

6. **(i)**



(ii) Percentage of temperatures recorded that are at most 20 °C

$= \frac{12}{20} \times 100\%$

= 60%

(iii) The average daily temperature of a city in autumn for 20 days ranges from 17 °C to 23 °C. The temperature are symmetrical about 20 °C.

7. **(i)** The most common attention span is 5.4 minutes.

(ii) P(attention span of pre-schooler not more than 5.5 minutes)

$= \frac{12}{22}$

$= \frac{6}{11}$

(iii) The attention span, in minutes, of 22 pre-schoolers, ranges from 4.0 minutes to 7.0 minutes. The time is clustered around 5.4 minutes to 5.8 minutes.

8. **(a)** Number of boys who failed the test = 2

(b) **(i)** Percentage of girls who scored distinctions $= \frac{5}{12} \times 100\%$

$= 41\frac{2}{3}\%$

(ii) Percentage of boys who scored disinctions $= \frac{2}{12} \times 100\%$

$= 16\frac{2}{3}\%$

(c) The girls performed better in the test, as they had a higher percentage of students who scored distinctions.

9. (i)

Stem	Leaf
2	0 2 4 4 4
2	5 6 6 6 6 6 7 8 9 9 9
3	1 1 1 2 2 3 4 4
3	5 5 7 7 8 8
4	0 1 2 3 3 3
4	5 5 6 8

Key: 2 | 2 means 22

(ii) The most common number of chocolates is 26.

(iii) Required percentage $= \frac{14}{40} \times 100\%$

$= 35\%$

10. (i)

Stem	Leaf
7	0 4 4
7	5 7 8 9
8	0 1 1 3 3 3 3
8	5 5 5
9	4 4
9	8

Key: 7 | 4 means 7.4 minutes

(ii) The fastest time taken was 7 minutes. The slowest time taken was 9.8 minutes.

(iii) Percentage of sportsmen who took at least 8.5 minutes

$= \frac{6}{20} \times 100\%$

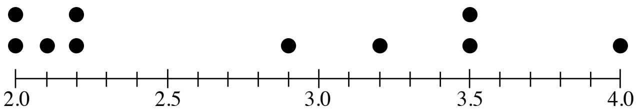
$= 30\%$

(iv) P(sportsman failed the test) $= \frac{6}{20}$

$= \frac{3}{10}$

11. (a) 'Good years: 2007, 2008, 2010, 2011, 2012

(b) (i)



(ii)

Stem	Leaf
2	0 0 1 2 2
2	9
3	2
3	5 5
4	0

Key: 2 | 2 means 22

(c) (i) An advantage of using the dot diagram is that it is easy to display the data.

(ii) An advantage of using the stem-and-leaf diagram is that the shape of the data distribution can be easily observed.

12. (i) School Q had the highest score of 100 marks.

(ii) School P has the lowest score of 50 marks.

(iii) Fraction of students in school P who scored 80 or above marks

$= \frac{18}{29}$

Fraction of students in school Q who scored 80 or above marks

$= \frac{13}{29}$

School P performed better in the quiz as it has a larger fraction of students who scored 80 marks or above.

13. (i)

Leaves for Class *B*	Stem	Leaves for Class *A*
	3	1 9
2	4	5 5 7
5	5	0 0 5 9
9 9 8 7 6 5	6	0 3 6 8 8 9
9 8 5 2 0	7	0 0 7
8 7 2 0	8	5
4 2 0	9	8

Key: 4 | 1 means 41 marks

(ii) Percentage of students who scored distinctions in Class A

$= \frac{5}{20} \times 100\%$

$= 25\%$

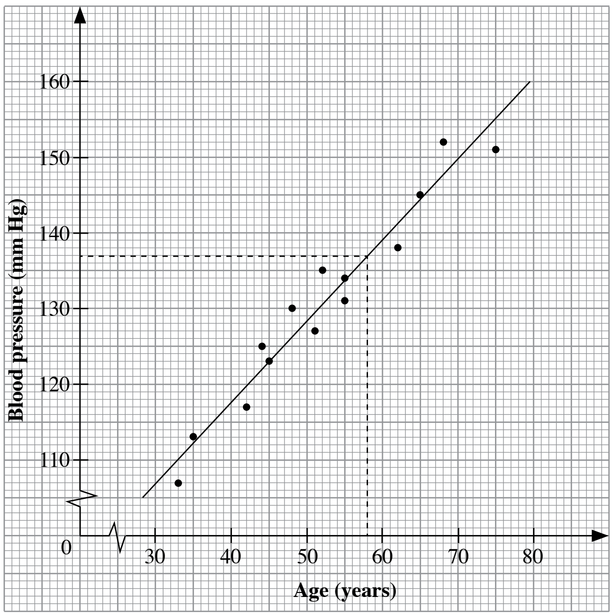
Percentage of students who scored distinctions in Class B

$= \frac{12}{20} \times 100\%$

$= 60\%$

(iii) Class B performed better in the examination as it had a larger percentage of students who scored distinctions.

14. (i)

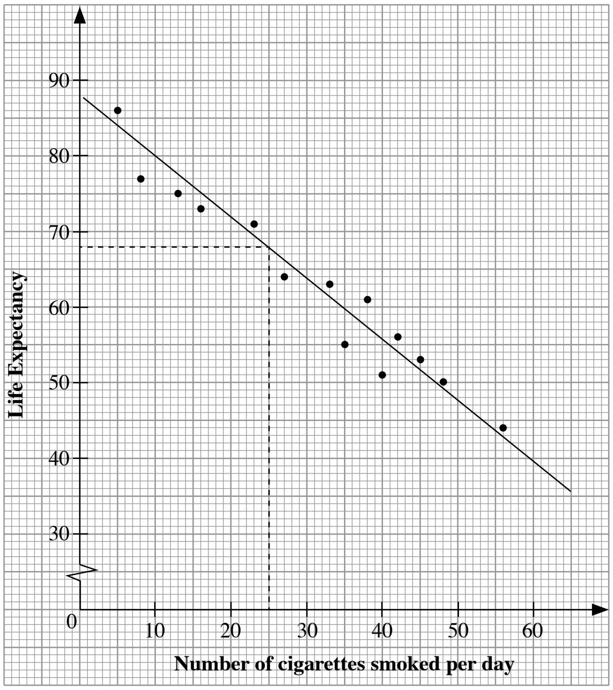


(ii) A positive correlation is observed i.e. older women tend to have higher blood pressure.

(iv) From the graph, Mary's blood pressure would be 137 mm Hg, given her age of 58 years.

(v) No, it would not be reliable as the study was done on women only and gender differences are not accounted for. Moreover, the age of 86 years is out of range.

15. (i)



(ii) A strong negative correlation exists i.e. the more cigarettes smoked, the shorter the life expectancy of the individual.

(iv) From graph, Johnny's life expectancy is expected to be 68 years.

(v) Take two points on the line (30, 63) and (40, 55).

$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$

$$= \frac{80 - 39}{10 - 60}$$

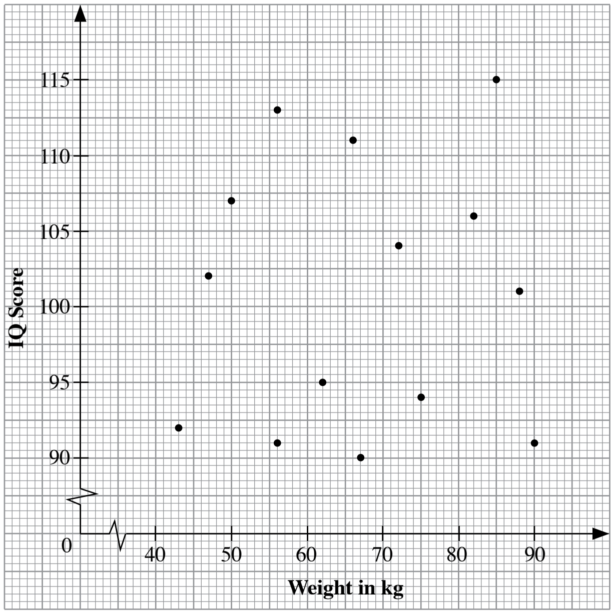
$$= -\frac{41}{50}$$

From graph, y-intercept = 88

$\therefore$ The equation of the best fit line is $y = -\frac{41}{50}x + 88$.

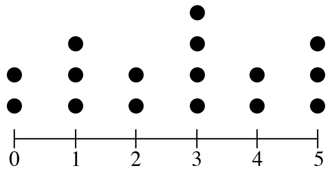
(vi) No, it would not be reliable as the data provided are based on a smoking lifestyle.

16. (i)



(ii) There is no correlation between the weight of a pupil and his/her IQ score.

17. (i)



(ii) The most common number of goals scored is 3.

(iii) Percentage of number of matches that have at least 3 goals scored

$$= \frac{9}{16} \times 100\%$$

$$= 56\frac{1}{4}\%$$

(iv) In 2010, number of matches with not more than one goal at full time = 5

$100\% - 37.5\% = 62.5\% \leftrightarrow 5$ matches

$100\% \leftrightarrow \frac{5}{62.5\%} \times 100\% = 8$ matches

In 2006, number of matches with not more than one goal at full time = 8

$\therefore$ In 2010, number of matches with more than one goal at full time = 16 – 8
= 8

18. The heights, in cm, of 14 plants after 8 weeks range from 4.4 cm to 8.6 cm.

The only repeating heights are 7.2 cm and 8.1 cm. Since the range is rather wide, with many distinct values and few repeating values, a dot diagram is not very useful in this case.

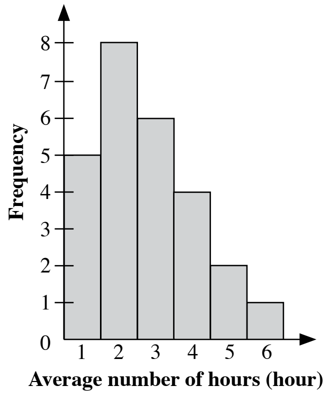
A stem-and-leaf diagram should be used.

Exercise 16B

1. (i)

Average number of hours (hour)	1	2	3	4	5	6
Frequency	5	8	6	4	2	1

(ii)



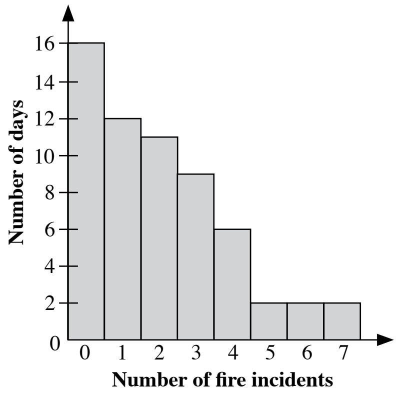
(iii) The most common average number of hours is 2 hours.

(iv) Number of teachers who participated in the survey
= 5 + 8 + 6 + 4 + 2 + 1
= 26

(v) Fraction of teachers who spent an average of 3 hours = $\frac{6}{26}$

$= \frac{3}{13}$

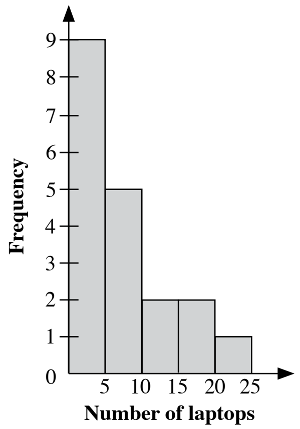
2. (i)



(ii) Required ratio = 16 : (2 + 2)
= 16 : 4
= 4 : 1

(iii) No. The most common number of fire incidents is 0, of which 16 days out of 60 days had 0 fire incidents.

3. (i)



(ii) Number of days number of laptops sold are more than or equal to 15
= 2 + 1
= 3 days

4. (i)

Number of hour (x hours)	**Frequency**
$0 \leqslant x < 2$	0
$2 \leqslant x < 4$	30
$4 \leqslant x < 6$	40
$6 \leqslant x < 8$	60
$8 \leqslant x < 10$	20
Total frequency	150

(ii) Number of students who spent more than or equal to 4, but less than 6 hours
= 40

(iii) Total number of Secondary Two students
= 30 + 40 + 60 + 20
= 150

(iv) Percentage of students who spent less than 6 hours

$= \frac{70}{150} \times 100\%$

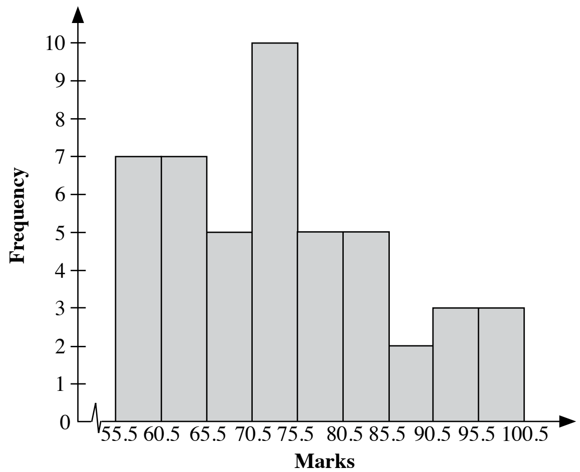
$= 46\frac{2}{3}\%$

5. (a)

Marks	Tally	Lower class boundary	Upper class boundary	Frequency
56 – 60	### //	55.5	60.5	7
61 – 65	### //	60.5	65.5	7
66 – 70	###	65.5	70.5	5
71 – 75	### ###	70.5	75.5	10
76 – 80	###	75.5	80.5	5
81 – 85	###	80.5	85.5	5
86 – 90	//	85.5	90.5	2
91 – 95	///	90.5	95.5	3
96 – 100	///	95.5	100.5	3

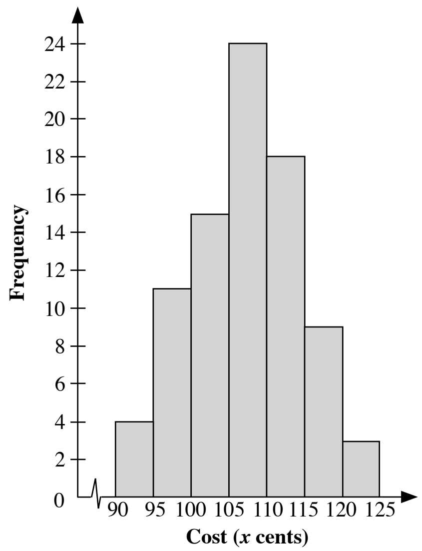
(b) 90.9: 91 – 95
66.2: 66 – 70
81.5: 81 – 85

(c)



6. (a) Total number of shops = 4 + 11 + 15 + 24 + 18 + 9 + 3
= 84

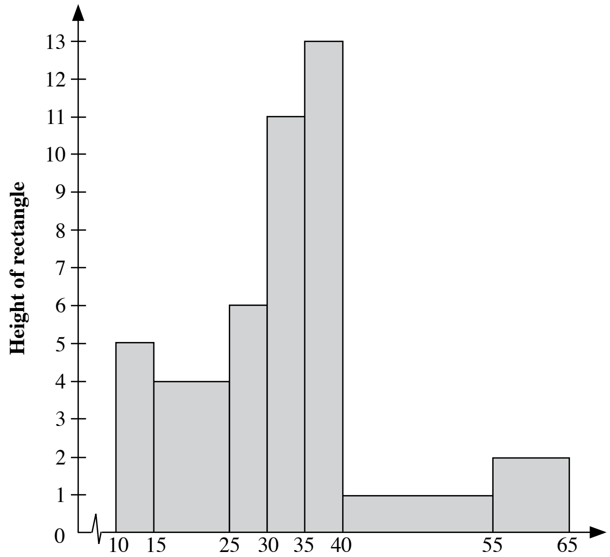
(b)



7. (a)

Class interval	Class width		Frequency	Height of rectangle
10 – 14	5	1 × standard	5	5 ÷ 1 = 5
15 – 24	10	2 × standard	8	8 ÷ 2 = 4
25 – 29	5	1 × standard	6	6 ÷ 1 = 6
30 – 34	5	1 × standard	11	11 ÷ 1 = 11
35 – 39	5	1 × standard	13	13 ÷ 1 = 13
40 – 54	15	3 × standard	3	3 ÷ 3 = 1
55 – 64	10	2 × standard	4	4 ÷ 2 = 2

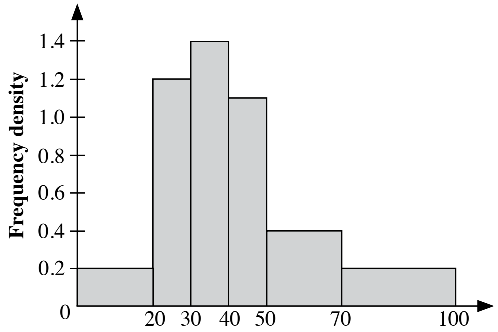
(b)



8. (a)

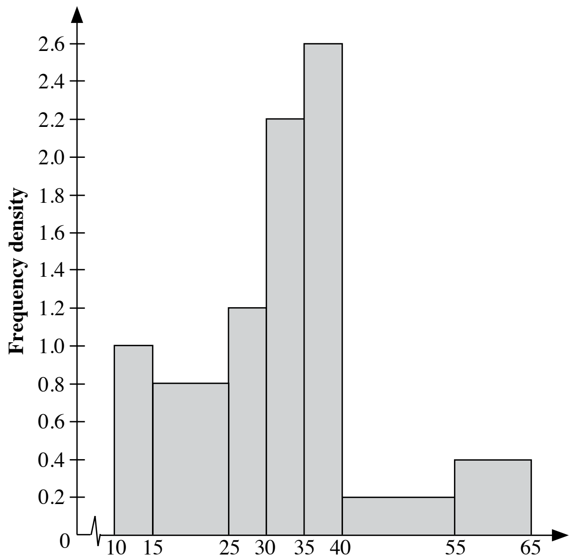
Class interval	Class width	Frequency	Frequency density $= \frac{\text{Frequency}}{\text{Class width}}$
$0 < x \leqslant 20$	20	4	4 ÷ 20 = 0.2
$20 < x \leqslant 30$	10	12	12 ÷ 10 = 1.2
$30 < x \leqslant 40$	10	14	14 ÷ 10 = 1.4
$40 < x \leqslant 50$	10	11	11 ÷ 10 = 1.1
$50 < x \leqslant 70$	20	8	8 ÷ 20 = 0.4
$70 < x \leqslant 100$	30	6	6 ÷ 30 = 0.2

(b)



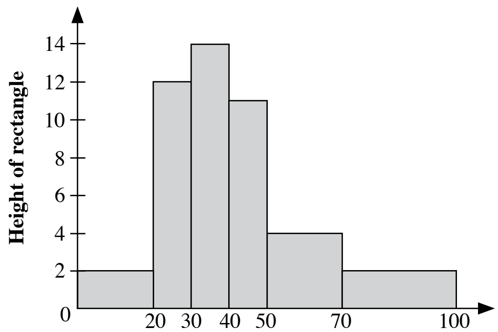
9. (a)

Class interval	Class width	Frequency	Frequency density $= \frac{\text{Frequency}}{\text{Class width}}$
$10 \leqslant x < 15$	5	5	$5 \div 5 = 1.0$
$15 \leqslant x < 25$	10	8	$8 \div 10 = 0.8$
$25 \leqslant x < 29$	5	6	$6 \div 5 = 1.2$
$30 \leqslant x < 35$	5	11	$11 \div 5 = 2.2$
$35 \leqslant x < 40$	5	13	$13 \div 5 = 2.6$
$40 \leqslant x < 55$	15	3	$3 \div 15 = 0.2$
$55 \leqslant x < 65$	10	4	$4 \div 10 = 0.4$



(b)

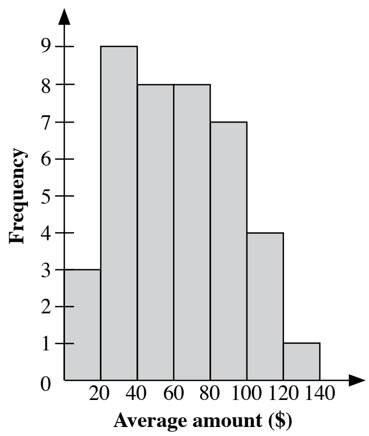
Class interval	Class width		Frequency	Height of rectangle
$0 < x \leqslant 20$	20	$2 \times$ standard	4	$4 \div 2 = 2$
$20 < x \leqslant 30$	10	$1 \times$ standard	12	$12 \div 1 = 12$
$30 < x \leqslant 40$	10	$1 \times$ standard	14	$14 \div 1 = 14$
$40 < x \leqslant 50$	10	$1 \times$ standard	11	$11 \div 1 = 11$
$50 < x \leqslant 70$	20	$2 \times$ standard	8	$8 \div 2 = 4$
$70 < x \leqslant 100$	30	$3 \times$ standard	6	$6 \div 3 = 2$



10. (i)

Average amount ($$x$)	Tally	Frequency
$0 \leqslant x < 20$	///	3
$20 \leqslant x < 40$	~~////~~ ////	9
$40 \leqslant x < 60$	~~////~~ ///	8
$60 \leqslant x < 80$	~~////~~ ///	8
$80 \leqslant x < 100$	~~////~~ //	7
$100 \leqslant x < 120$	////	4
$120 \leqslant x < 140$	/	1
Total frequency		40

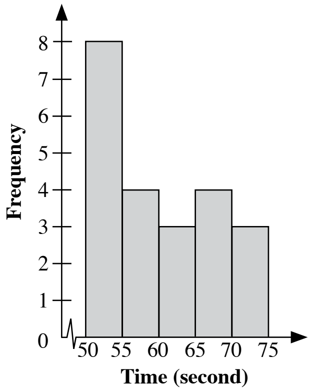
(ii)



11. (i)

Time (x seconds)	Frequency
$50 \leqslant x < 55$	8
$55 \leqslant x < 60$	4
$60 \leqslant x < 65$	3
$65 \leqslant x < 70$	4
$70 \leqslant x < 75$	3

(ii)



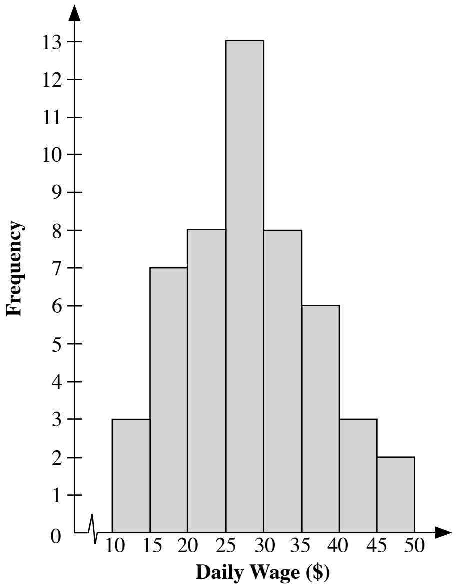
(iii) Required percentage $= \frac{8}{7} \times 100\%$

$= 114\frac{2}{7}\%$

(iv) Advancements in nutrition, fitness and training regiments lead to fitter and better swimmers over the years, thus resulting in the faster times from 1920 to 2012.

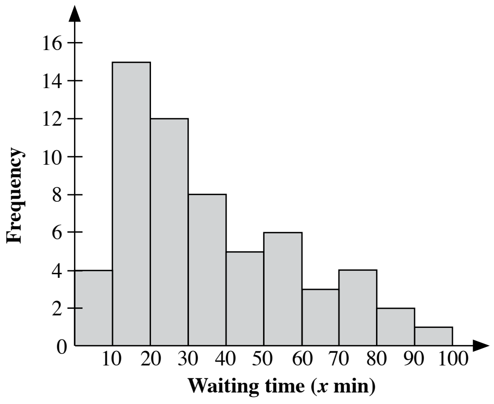
12.

Daily wage ($)	Frequency
10 – 14	3
15 – 19	7
20 – 24	8
25 – 29	13
30 – 34	8
35 – 39	6
40 – 44	3
45 – 49	2



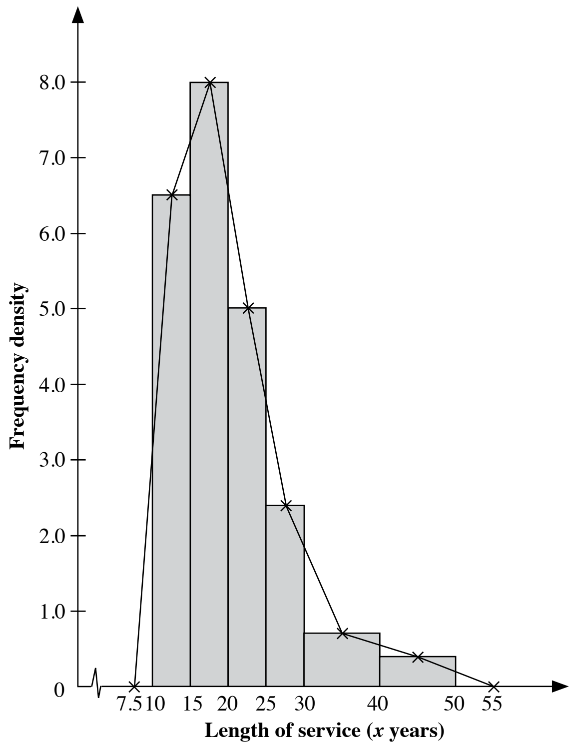
13.

Waiting time (x minutes)	Frequency
$0 < x \leqslant 10$	4
$10 < x \leqslant 20$	15
$20 < x \leqslant 30$	12
$30 < x \leqslant 40$	8
$40 < x \leqslant 50$	5
$50 < x \leqslant 60$	6
$60 < x \leqslant 70$	3
$70 < x \leqslant 80$	4
$80 < x \leqslant 90$	2
$90 < x \leqslant 100$	1



14. Since the class intervals are unequal, the histogram is to be drawn using either height of rectangle or frequency density.

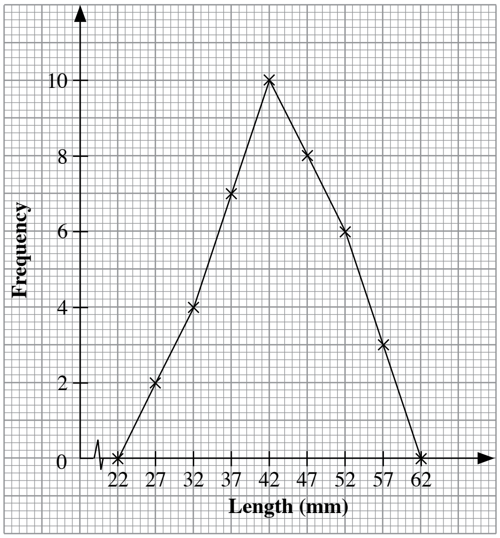
Class interval	Class width	Frequency	Frequency density $= \dfrac{\text{Frequency}}{\text{Class width}}$
$10 \leqslant x < 15$	5	32	$32 \div 5 = 6.4$
$15 \leqslant x < 20$	5	40	$40 \div 5 = 8$
$20 \leqslant x < 25$	5	25	$25 \div 5 = 5$
$25 \leqslant x < 30$	5	12	$12 \div 5 = 2.4$
$30 \leqslant x < 40$	10	7	$7 \div 10 = 0.7$
$40 \leqslant x < 50$	10	4	$4 \div 10 = 0.4$



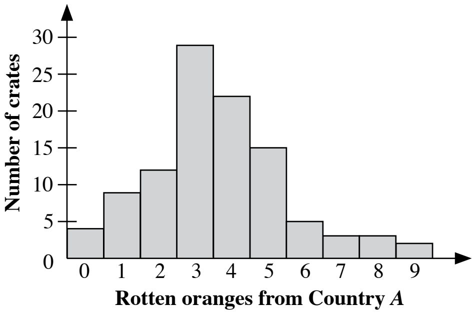
15.

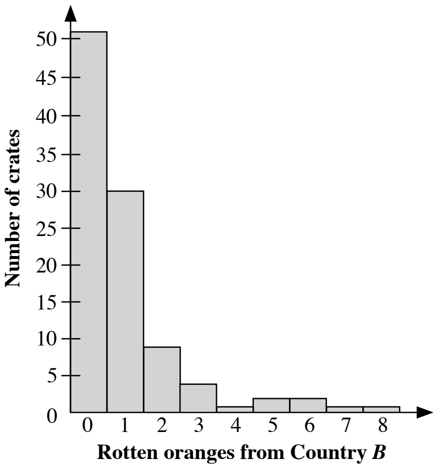
Length (mm)	Mid-value	Frequency
25 – 29	27	2
30 – 34	32	4
35 – 39	37	7
40 – 44	42	10
45 – 49	47	8
50 – 54	52	6
55 – 59	57	3

The points to be plotted are (22, 0), (27, 2), (32, 4), (37, 7), (42, 10), (47, 8), (52, 6), (57, 3) and (62, 0).



16. (i)





(ii) Largest number of rotten oranges found in a crate from country $A = 9$

Largest number of rotten oranges found in a crate from country $B = 8$

(iii) Total number of rotten oranges from country A

$= (4 \times 0) + (9 \times 1) + (12 \times 2) + (28 \times 3) + (22 \times 4) + (15 \times 5) + (5 \times 6) + (2 \times 7) + (2 \times 8) + (1 \times 9)$

$= 0 + 9 + 24 + 84 + 88 + 75 + 30 + 14 + 16 + 9$

$= 349$

Total number of rotten oranges from country B

$= (51 \times 0) + (30 \times 1) + (8 \times 2) + (4 \times 3) + (1 \times 4) + (2 \times 5) + (2 \times 6) + (1 \times 7) + (1 \times 8)$

$= 0 + 30 + 16 + 12 + 4 + 10 + 12 + 7 + 16$

$= 99$

(iv) P(crate contains no fewer than p rotten oranges) $= \frac{3}{4}$

Number of crates with no fewer than p rotten oranges

$= \frac{3}{4} \times 100$

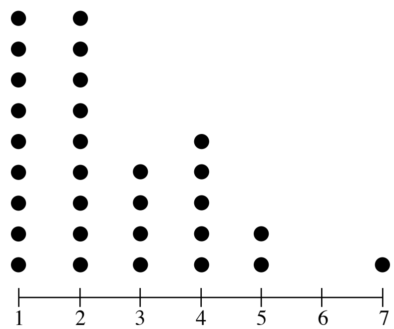
$= 75$

Since $1 + 2 + 2 + 5 + 15 + 22 + 28 = 75$,

$\therefore p = 3$

Review Exercise 16

1. (i)



(ii) The common time taken were 1 minute and 2 minutes.

(iii) Percentage of students who took at most 3 minutes to read

$= \frac{22}{30} \times 100\%$

$= 73\frac{1}{3}\%$

(iv) The time taken, measured to the nearest minute, for 30 students to read a passage ranges from 1 minute to 7 minutes. The time is clustered around 1 minute to 2 minutes. There is one extreme value of 7 minutes.

2. (i) Most children eat 4 sweets in a week.

(ii) The greatest number of sweets the children eat in a week is 6.

(iii) Fraction of children who do not eat sweets in a week $= \frac{4}{25}$

3. (i)

Stem	Leaf
13	0 0 1 2 3
14	1 1 5 5 5 6 6 7
15	0 1 2 3 8
16	5 5

Key: 13 | 1 means 131 cm

(ii) The length of the longest metal rod is 165 cm.

(iii) The most common length is 145 cm.

(iv) Required ratio $= 10 : 5$

$= 2 : 1$

4. (i) The most common speed is 82 km/h.

(ii) Percentage of vehicles that exceeded the speed limit

$= \frac{3}{60} \times 100\%$

$= 5\%$

(iii) P(speed exceeds x km/h) $= \frac{3}{10}$

Number of vehicles that exceeded the speed limit $= \frac{3}{10} \times 60$

$= 18$

Counting from the fastest speed, $1 + 4 + 13 = 18$,

$\therefore x = 84$

5. **(i)**

Leaves for Class B	Stem	Leaves for Class A
7	2	8
9 8 6 5 2	3	0 0 6
6 5 2	4	2
9 9	5	1 5
2 1 0	6	4 6 8 8
3	7	0 3 4
0	8	5
	9	6

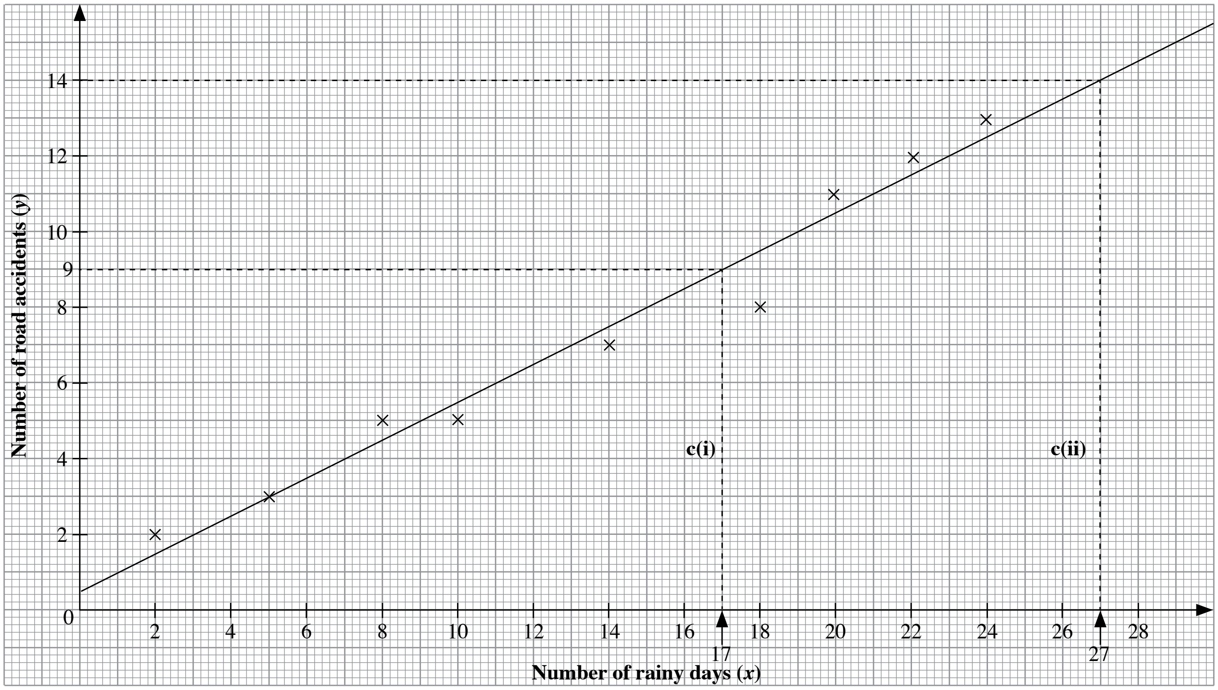
Key: 2 | 8 means $28

(ii) Greatest average amount saved by students in group A = $96

Greatest average amount saved by students in group B = $80

(iii) The students from group A save more money in a month. The average amount saved by students in group A cluster around $60 to $70, while the average amount saved by students in group B cluster around $30 to $40.

6.



(c) From graph,

(i) When $x = 17$, $y = 9$

$\therefore$ It is estimated that there will be 9 road accidents in a month with 17 rainy days.

(ii) When $x = 27$, $y = 14$

$\therefore$ It is estimated that there will be 14 road accidents in a month with 27 rainy days.

(d) From the graph, two points are (15, 8) and (17, 9).

$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{9-8}{17-15} = \frac{1}{2}$$

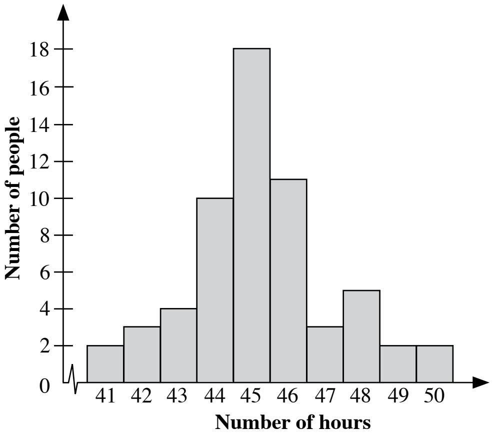
$$y\text{-intercept} = \frac{2}{5}$$

$\therefore$ The equation of the best fit line is $y = \frac{1}{2}x + \frac{2}{5}$.

(e) There is a strong, positive correlation between rain and road accidents.

7. There is no correlation between the variables in the scatter diagram. A possible scenario would the height of students and their test marks.

8. (i)



(ii) Fraction of workers who worked no more than 45 hours in that week = $\frac{37}{60}$

(iii) Total salary to workers who worked 49 hours in that week

$= 2 \times [(42 \times \$6) + (7 \times \$9)]$

$= (\$252 + \$63)$

$= 2(\$315)$

$= \$630$

Total salary to workers who worked 50 hours in that week

$= 2 \times [(42 \times \$6) + (8 \times \$9)$

$= 2(\$252 + \$72)$

$= 2(\$324)$

$= \$648$

Total salary paid to workers who worked more than 48 hours in that week

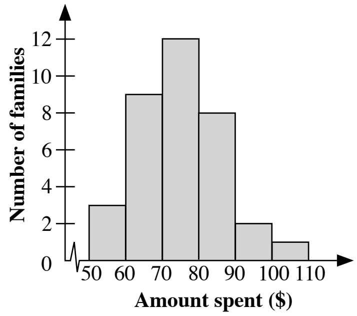
$= \$630 + \$648$

$= \$1278$

9. (i)

Amount spent ($\$x$)	Tally	Frequency
$50 \leqslant x < 60$	///	3
$60 \leqslant x < 70$	### ////	9
$70 \leqslant x < 80$	### ### //	12
$80 \leqslant x < 90$	### ///	8
$90 \leqslant x < 100$	//	2
$100 \leqslant x < 110$	/	1
Total frequency		35

(ii)



(iii) P(family selected is eligible for draw) = $\frac{6}{35}$

10. (a)

Height (x m)	Number of flats
$10 < x \leqslant 15$	2
$15 < x \leqslant 20$	5
$20 < x \leqslant 25$	6
$25 < x \leqslant 30$	12
$30 < x \leqslant 35$	7
$35 < x \leqslant 40$	4
$40 < x \leqslant 45$	3

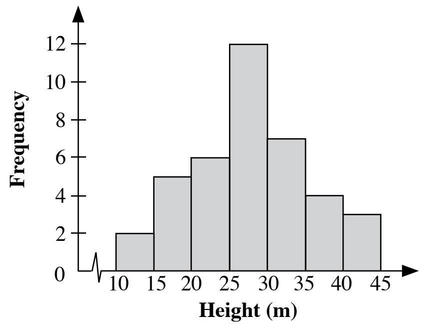
(b) Total number of flats = 2 + 5 + 6 + 12 + 7 + 4 + 3

= 39

(c)

Height (x m)	Mid-value	Frequency
$10 < x \leqslant 15$	12.5	2
$15 < x \leqslant 20$	17.5	5
$20 < x \leqslant 25$	22.5	6
$25 < x \leqslant 30$	25.5	12
$30 < x \leqslant 35$	32.5	7
$35 < x \leqslant 40$	35.5	4
$40 < x \leqslant 45$	42.5	3

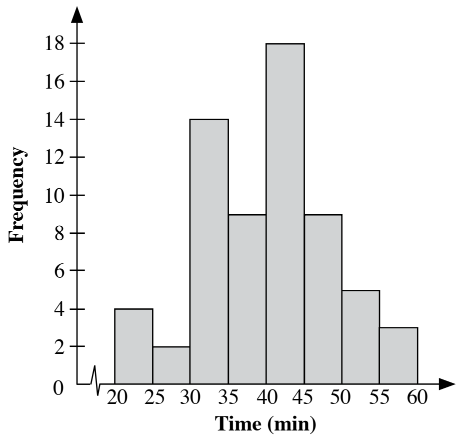
(d)



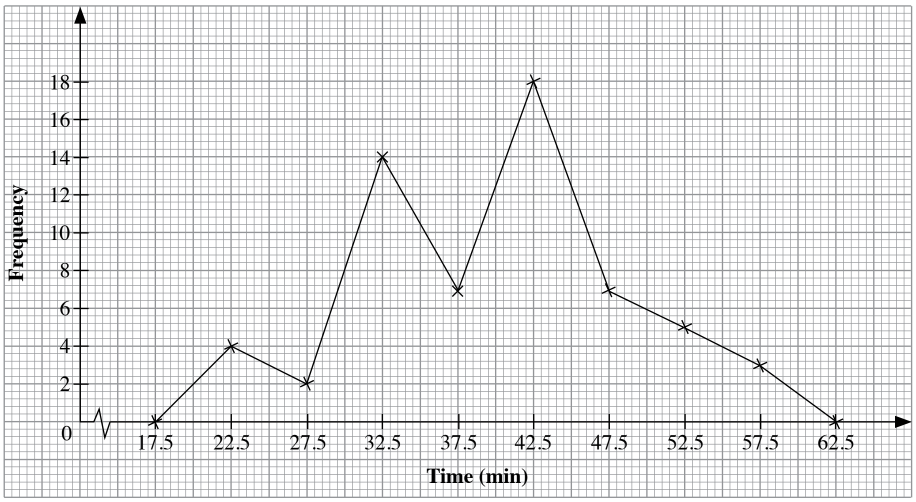
11. (a)

Time (x minutes)	Tally	Frequency
$20 < x \leqslant 25$	////	4
$25 < x \leqslant 30$	//	2
$30 < x \leqslant 35$	~~////~~ ~~////~~ ////	14
$35 < x \leqslant 40$	~~////~~ //	7
$40 < x \leqslant 45$	~~////~~ ~~////~~ ~~////~~ ///	18
$45 < x \leqslant 50$	~~////~~ //	7
$50 < x \leqslant 55$	~~////~~	5
$55 < x \leqslant 60$	///	3

(b)



(c)



Challenge Yourself

Let the number of students who obtained 3 marks be a,
the number of students who obtained 4 marks be b,
the number of students who obtained 9 marks be c.

$$0 + 0 + 1 + a + b + x - 1 + x + x + 3 + x + 5 + c + 4 = 36$$
$$a + b + c + 4x + 12 = 36$$
$$a + b + c + 4x = 24 \quad \text{— (1)}$$

Since 25% of the students scored at least 9 marks,

$$c + 4 = 25\% \times 36$$
$$= \frac{25}{100} \times 36$$
$$= 9$$
$$\therefore c = 5$$

Since P(student obtained more than 4 marks but not more than 9 marks) $= \frac{2}{3}$,

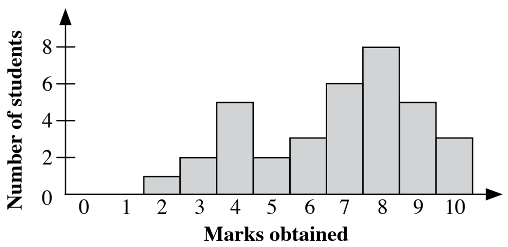
$$x - 1 + x + x + 3 + x + 5 + 5 = \frac{2}{3} \times 36$$
$$4x + 12 = 24$$
$$4x = 12$$
$$\therefore x = 3$$

Since the average marks by students who scored more than or equal to 4 marks but less than or equal to 6 marks is 4.8,

$$\frac{(b \times 4) + [(3 - 1) \times 5] \times (3 \times 6)}{b + (3 - 1) + 3} = 48$$
$$\frac{4b + 10 + 18}{b + 5} = 4.8$$
$$4b + 28 = 4.8b + 24$$
$$0.8b = 4$$
$$\therefore b = 5$$

Substitute $b = 5$, $c = 5$ and $x = 3$ into (1):

$$a + 5 + 5 + 4(3) = 24$$
$$a + 22 = 24$$
$$\therefore a = 2$$



Chapter 17 Averages of Statistical Data

TEACHING NOTES

Suggested Approach

In primary school, students have learnt that the average of a set of data is the sum of all the data divided by the number of data. Teachers can further explain that in statistics, there are other types of 'averages'. The average that students are familiar with is also known as the mean. In this chapter, students are to know and learn the properties of median and mode as well.

By the end of the chapter, students should know how to calculate mean, median and mode from the various statistical diagrams and be aware of the situations where one numerical measure is preferred over another.

Section 17.1: Mean

Teachers can guide students through the worked examples to show how the mean is calculated. Students should be reminded to be careful not to miss out any values or use any wrong values in the calculation.

Teachers should note calculating the mean from a frequency table as well as estimating the mean of a set of grouped data are new to students. More practice and guidance may be required for some students here.

Section 17.2: Median

The definition and purpose of a median should be well-explained to the students. The example on page 501 is a good example why the median is preferred over the mean. Students may need to be reminded that the numerical average is to give the best representation of any data.

The main features of finding the median, namely whether the number of data is even or odd and that the data must be arranged in order, are important and must be emphasised to students. The activities are meant to test and reinforce students' understanding (see Thinking Time on page 502 and Class Discussion: Creating Sets of Data with Given Conditions).

Section 17.3: Mode

The mode is arguably the easiest numerical average that students will need to learn, as it involves identifying the most frequent data without any calculations involved. Teachers ought to be able to quickly go through the examples of finding the mode from the various statistical diagrams.

Students should be reminded that a data has to be the most frequent, meaning it occurred at least two or more times, otherwise the set of data may not have a mode (see Exercise 17B, question 4(e)).

Section 17.4 Mean, Median and Mode

Questions involving all three numerical averages will be covered in this section. Students may need to recall the algebraic skills they have picked up at the first half of the textbook.

In this section, teachers should use the activities that compare the mean, median mode and question students on the most suitable numerical average depending on the set of data provided (see Thinking Time on page 510 and Class Discussion: Comparison of Mean, Median and Mode).

WORKED SOLUTIONS

Thinking Time (Page 502)

Rearranging the data in descending order instead, we have,
30, 21, 19, 14, 12, 9, 8, 5

$\therefore$ Median = mean of the data in the 4th and the 5th position

$$= \frac{14 + 12}{2}$$

$$= 13$$

Hence, the median remains the same if the data is arranged in descending order instead.

Class Discussion (Creating Sets of Data with Given Conditions)

Some sets of data are shown as follows.

(i) 1, 1, 1, 1, 2, 4, 11
Difference between minimum and maximum value = 10
Mean = 3
Median = 1

(ii) 1, 1, 1, 2, 2, 3, 11
Difference between minimum and maximum value = 10
Mean = 3
Median = 2

(iii) 1, 1, 1, 1, 5, 8, 11
Difference between minimum and maximum value = 10
Mean = 4
Median = 1

(iv) 1, 1, 2, 2, 2, 9, 11
Difference between minimum and maximum value = 10
Mean = 4
Median = 2

(v) 1, 2, 3, 4, 5, 9, 11
Difference between minimum and maximum value = 10
Mean = 5
Median = 4

Teachers may wish to note the sets of data are not exhaustive. To come up with a set of data, it is recommended that the mean and median are decided, before working backwards.

Thinking Time (Page 507)

Some sets of data are shown as follows.

(i) 41, 56, 56, 58, 59, 60
Mean = 55
Mode = 56
Median = 57

(ii) 39, 56, 56, 57, 58, 59, 60
Mean = 55
Mode = 56
Median = 57

(ii) 35, 55, 56, 56, 58, 59, 60, 61
Mean = 55
Mode = 56
Median = 57

(iv) 36, 57, 57, 59, 60, 61
Mean = 55
Mode = 57
Median = 58

(v) 27, 58, 58, 59, 60, 61, 62
Mean = 55
Mode = 58
Median = 59

Teachers may wish to note the sets of data are not exhaustive. To come up with a set of data, it is recommended that the mean, mode and median are decided, before working backwards. Note that the number of data is not specified.

Thinking Time (Page 510)

$$\text{Mean monthly salary} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{12 \times 1500 + 5 \times 5000 + 2 \times 10\,000 + 4 \times 15\,000 + 1 \times 25\,000 + 1 \times 50\,000}{25}$$

$$= \frac{198\,000}{25}$$

$$= \$7920$$

The average monthly salary of the employees is $7920 refers to the mean monthly salary of the employees. The average monthly salary can mean the median monthly salary, which is $5000 or the modal monthly salary, which is $1500, as well.
Hence, Devi's statement does not give a good picture of how much the employees earn.
Number of employees who earn $1500 = 12

$$\text{Percentage of employees who earn \$1500} = \frac{12}{25} \times 100\%$$
$$= 48\% \approx 50\%$$

Lixin's statement that almost half of the employees earn $1500 is correct but it does not state the amount the other employees in the company earn.
Number of employees who earn at least $5000 = 13

$$\text{Percentage of employees who earn at least \$5000} = \frac{13}{25} \times 100\%$$
$$= 52\% > 50\%$$

The statement that more than 50% of the employees earn at least $5000 is correct. It also gives the best picture of how much money the employees earn in the company, since the statement allows us to infer the amount the rest of the employees earn.
Khairul's statement gives the best picture of how much the employees in the company earn.

Class Discussion (Comparison of Mean, Median and Mode)

1. (i) Mean = $\frac{15 + 17 + 13 + 18 + 20 + 19 + 15}{7}$

$= \frac{117}{7}$

$= 16\frac{5}{7}$

Total number of data = 7

Middle position $= \frac{7+1}{2}$

$= 4^{th}$ position

Rearranging the data in ascending order,

13, 15, 15, 17, 18, 19, 20

∴ Median = data in the 4^{th} position

= 17

Mode = 15

(ii) The mean and median will change while the mode will remain the same.

New mean $= \frac{117 + 55}{7 + 1}$

$= \frac{172}{8}$

$= 21\frac{1}{2}$

Total number of data = 7

Middle position $= \frac{8+1}{2}$

$= 4.5^{th}$ position

Rearranging the data in ascending order,

13, 15, 15, 17, 18, 19, 20, 55

∴ New median = mean of the data in the 4^{th} position and 5^{th} position

$= \frac{17 + 18}{2}$

= 17.5

New mode = 15

(iii) The mean is most affected by the addition of a large number.

It had the biggest difference of $21\frac{1}{2} - 16\frac{5}{7} = 4\frac{11}{14}$.

(iv) The mean will be most affected by extreme values.

The mode will remain unchanged by extreme values.

Hence, the median is the most appropriate measure to use.

2. (i) Mean = $\frac{6 + 7 + 8 + 8 + 7 + 9 + 5 + 6 + 6}{9}$

$= \frac{62}{9}$

$= 6\frac{8}{9}$

Total number of data = 9

Middle position $= \frac{9+1}{2}$

$= 5^{th}$ position

Rearranging the data in ascending order,

5, 6, 6, 6, 7, 7, 8, 8, 9

∴ Median = data in the 5^{th} position

= 7

Mode = 6

(ii) The mode best represents the sizes of shoes sold because it represents the size of the shoes most sold.

3. (i) Mean = $\frac{2 + 3 + 1 + 4 + 5 + 1 + 2 + 2 + 1 + 1}{10}$

$= \frac{22}{10}$

= 2.2

Total number of data = 10

Middle position $= \frac{10+1}{2}$

$= 5.5^{th}$ position

Rearranging the data in ascending order,

1, 1, 1, 1, 2, 2, 2, 3, 4, 5

∴ Median = mean of the data in the 5^{th} and the 6^{th} position

$= \frac{2+2}{2}$

= 2

Mode = 1

(ii) Even though the mean is not an integer, it still has a physical meaning.

i.e. 2.2 children per family is equivalent to <u>22</u> children in 10 families.

4. The mean is preferred when there are no extreme values in the set of data. Comparatively, the median is preferred when there are extreme values.

The mode is preferred when we want to know the most common value in a data set.

Practise Now 1

Mean score = $\frac{\text{Sum of scores}}{\text{Number of students}}$

$= \frac{79 + 58 + 73 + 66 + 50 + 89 + 91 + 58}{8}$

$= \frac{564}{8}$

= 70.5

Practise Now 2

$\frac{44 + 47 + y + 58 + 55}{5} = 52$

$44 + 47 + y + 58 + 55 = 260$

$204 + y = 260$

$\therefore y = 56$

Practise Now 3

1. **(i)** Since mean = $\frac{\text{sum of the 7 numbers}}{7}$,

then sum of the 7 numbers $= 7 \times \text{mean}$
$= 7 \times 11$
$= 77$

(ii) $3 + 17 + 20 + 4 + 15 + y + y = 77$
$59 + 2y = 77$
$2y = 18$
$y = 9$

2. Since mean = $\frac{\text{sum of the heights of 20 boys and 14 girls}}{34}$

then sum of the heights of 20 boys and 14 girls $= 34 \times \text{mean}$
$= 34 \times 161$
$= 5474$ cm

Since mean = $\frac{\text{sum of the heights of 14 girls}}{14}$

then sum of the heights of 14 girls $= 14 \times \text{mean}$
$= 14 \times 151$
$= 2114$ cm

Sum of the heights of 20 boys $= 5474 - 2114$
$= 3360$ cm

Mean height of the 20 boys $= \frac{3360}{20}$
$= 168$ cm

3. $\frac{16 + w + 17 + 9 + x + 2 + y + 7 + z}{9} = 11$

$16 + w + 17 + 9 + x + 2 + y + 7 + z = 99$
$51 + w + x + y + z = 99$
$w + x + y + z = 48$

Mean of w, x, y and z $= \frac{48}{4}$
$= 12$

Practise Now 4

(i) Total number of visitors $= 12 + 32 + 54 + 68 + 18 + 16$
$= 200$

(ii) Total amount of money spent by the visitors
$= 12 \times \$40 + 32 \times \$60 + 54 \times \$80 + 68 \times \$100 + 18 \times \$160 + 16 \times \$200$
$= \$19\,600$

(iii) Mean amount of money spent by the visitors $= \frac{\$19\,600}{200}$
$= \$98$

Practise Now 5

1. Mean number of siblings $= \frac{\Sigma fx}{\Sigma f}$

$= \frac{4 \times 0 + 5 \times 1 + 3 \times 2 + 2 \times 3 + 1 \times 4}{15}$

$= \frac{21}{15}$

$= 1.4$

2. Mean pH value of the solutions

$= \frac{\Sigma fx}{\Sigma f}$

$= \frac{1.4 + 1.4 + 1.9 + 2.3 + 2.6 + 2.7 + 2.7 + 2.7 + 2.8 + 3.3 + 3.6 + 3.6 + 3.6 + 3.9 + 4.1 + 4.2 + 4.2 + 4.5 + 4.5 + 4.9}{20}$

$= \frac{64.9}{20}$

$= 3.245$

3. Mean time taken by the group of students

$= \frac{\Sigma fx}{\Sigma f}$

$= \frac{5 \times 20 + 3 \times 30 + 10 \times 40 + 1 \times 50 + 1 \times 60}{5 + 3 + 10 + 1 + 1}$

$= \frac{700}{20}$

$= 35$ minutes

Practise Now 6

Age (x years)	Frequency (f)	Mid-value (x)	fx
$20 < x \leqslant 30$	12	25	300
$30 < x \leqslant 40$	10	35	350
$40 < x \leqslant 50$	20	45	900
$50 < x \leqslant 60$	15	55	825
$60 < x \leqslant 70$	18	65	1170
	$\Sigma f = 75$		$\Sigma fx = 3545$

Estimated mean age of the employees $= \frac{3545}{75}$
$= 47.3$ years (to 3 s.f.)

Practise Now 7

(a) Total number of data = 7

Middle position $= \frac{7 + 1}{2}$
$= 4^{\text{th}}$ position

Rearranging the data in ascending order, we have:
3, 9, 11, 15, 16, 18 and 20

$\therefore$ Median = data in the 4^{th} position
$= 15$

(b) Total number of data = 5

Middle position $= \frac{5 + 1}{2}$
$= 3^{\text{rd}}$ position

Rearranging the data in ascending order, we have:
11.2, 15.6, 17.3, 18.2 and 30.2

$\therefore$ Median = data in the 3^{rd} position
$= 17.3$

Practise Now 8

(a) Total number of data = 6

Middle position = $\frac{6+1}{2}$

= 3.5^{th} position

Rearranging the data in ascending order, we have:

12, 15, 15, 20, 25 and 32

$\therefore$ Median = mean of the data in the 3^{rd} and the 4^{th} position

= $\frac{15+20}{2}$

= 17.5

(b) Total number of data = 8

Middle position = $\frac{8+1}{2}$

= 4.5^{th} position

Rearranging the data in ascending order, we have:

6.7, 6.8, 7.3, 8, 8.8, 8.9, 8.9, 10

$\therefore$ Median = mean of the data in the 4^{th} and the 5^{th} position

= $\frac{8+8.8}{2}$

= 8.4

Practise Now 9

1. Total number of data = 15

Middle position = $\frac{15+1}{2}$

= 8^{th} position

$\therefore$ Median = data in the 8^{th} position

= 1

2. Total number of data = 16

Middle position = $\frac{16+1}{2}$

= 8.5^{th} position

$\therefore$ Median = mean of the data in the 8^{th} and the 9^{th} position

= $\frac{3+3}{2}$

= 3

Practise Now 10

Total number of data = 20

Middle position = $\frac{20+1}{2}$

= 10.5^{th} position

$\therefore$ Median distance = mean of the data in the 10^{th} and the 11^{th} position

= $\frac{4.4+4.7}{2}$

= 4.55 km

Practise Now 11

Total number of data = 28

Middle position = $\frac{28+1}{2}$

= 14.5^{th} position

$\therefore$ Median time = mean of the data in the 14^{th} and the 15^{th} position

= $\frac{7+7}{2}$

= 7 minutes

Practise Now 12

(i) Modal lengths = 60 cm, 110 cm

(ii) Modal length = 60 cm

Practise Now 13

(a) Mode = 0.4

(b) Modes = 32, 37

(c) Mode = 1

(d) Mode = \$3000

Practise Now 14

(a) $\frac{2\times 0 + x\times 1 + 3\times 2 + 4\times 3 + 1\times 4}{2+x+3+4+1} = 1.8$

$\frac{x+22}{x+10} = 1.8$

$x + 22 = 1.8(x+10)$

$x + 22 = 1.8x + 18$

$0.8x = 4$

$x = 5$

(b) We write the data as follows:

$0, 0, \underbrace{1, ..., 1}_{x}, 2, 2, 2, 3, 3, 3, 3, 4$

The greatest value of x occurs when the median is here. (arrow to the first 2)

The smallest value of x occurs when the median is here. (arrow to the second 2)

$\therefore 2 + x = 2 + 4 + 1$

$2 + x = 7$

$x = 5$

$\therefore$ Greatest value of $x = 5$

$\therefore 2 + x + 2 = 4 + 1$

$4 + x = 5$

$x = 1$

$\therefore$ Smallest value of $x = 1$

$\therefore$ Possible values of $x = 1, 2, 3, 4, 5$

(c) Greatest possible value of $x = 3$

Exercise 17A

1. Mean number of passengers

= $\frac{\text{Sum of number of passengers}}{\text{Number of coaches}}$

= $\frac{29+42+45+39+36+41+38+37+43+35+32+40}{12}$

= $\frac{457}{12}$

= 38.1 (to 3 s.f.)

2. Mean price of books

$= \frac{\text{Sum of prices of books}}{\text{Number of books}}$

$= \frac{19.90 + 24.45 + 34.65 + 26.50 + 44.05 + 38.95 + 56.40 + 48.75 + 29.30 + 35.65}{10}$

$= \frac{358.6}{10}$

= \$35.86

3. $\frac{7 + 15 + 12 + 5 + h + 13}{6} = 10$

$52 + h = 60$

$h = 8$

4. Since mean of masses of 5 boys = $\frac{\text{sum of the masses of 5 boys}}{5}$

then sum of the mass of 5 boys = 5 × mean

= 5 × 62

= 310 kg

Since mean of masses of 4 boys = $\frac{\text{sum of the masses of 4 boys}}{4}$

then sum of the mass of 4 boys = 4 × mean

= 4 × 64

= 256 kg

Mass of boy excluded = 310 – 256

= 54 kg

5. **(i)** Since mean of 8 numbers = $\frac{\text{sum of the 8 numbers}}{8}$,

then sum of the 8 numbers = 8 × mean

= 8 × 12

= 96

(ii) $6 + 8 + 5 + 10 + 28 + k + k + k = 96$

$57 + 3k = 96$

$3k = 39$

$k = 13$

6. **(i)** Total number of matches played = 6 + 8 + 5 + 6 + 2 + 2 + 1

= 30

(ii) Total number of goals scored

= 6 × 0 + 8 × 1 + 5 × 2 + 6 × 3 + 2 × 4 + 2 × 5 + 1 × 6

= 60

(iii) Mean number of goals scored per match $= \frac{60}{30}$

= 2

7. Mean number of days of absence

$= \frac{\Sigma fx}{\Sigma f}$

$= \frac{23 \times 0 + 4 \times 1 + 5 \times 2 + 2 \times 3 + 2 \times 4 + 1 \times 5 + 2 \times 6 + 1 \times 9}{23 + 4 + 5 + 2 + 2 + 1 + 2 + 1}$

$= \frac{54}{40}$

= 1.35 days

8. **(a)** Mean

$= \frac{\Sigma fx}{\Sigma f}$

$= \frac{1 \times 6 + 2 \times 7 + 1 \times 8 + 4 \times 9 + 3 \times 10 + 1 \times 11 + 1 \times 12}{1 + 2 + 1 + 4 + 3 + 1 + 1}$

$= \frac{117}{13}$

= 9

(b) Mean

$= \frac{\Sigma fx}{\Sigma f}$

$= \frac{7.2 + 7.3 + 7.5 + 7.5 + 8.2 + 8.7 + 8.8 + 8.8 + 8.9 + 8.9 + 8.9 + 9.1 + 9.3 + 9.7 + 9.7 + 10.2 + 10.7 + 10.8}{4 + 7 + 4 + 3}$

$= \frac{160.2}{18}$

= \$8.90

(c) Mean $= \frac{\Sigma fx}{\Sigma f}$

$= \frac{3 \times 3 + 5 \times 4 + 6 \times 5 + 4 \times 6 + 2 \times 7}{3 + 5 + 6 + 4 + 2}$

$= \frac{97}{20}$

= 4.85 years

9. **(i)** Since mean of 10 numbers = $\frac{\text{sum of 10 numbers}}{10}$,

then sum of 10 numbers = 10 × mean

= 10 × 14

= 140

Since mean of 3 numbers = $\frac{\text{sum of 3 numbers}}{3}$,

then sum of 3 numbers = 3 × mean

= 3 × 4

= 12

Sum of the remaining seven numbers = 140 – 12

= 128

(ii) $15 + 18 + 21 + 5 + m + 34 + 14 = 128$

$107 + m = 128$

$m = 21$

10. Since mean monthly wage of 12 workers = $\dfrac{\text{sum of monthly wages}}{12}$,

then sum of monthly wages = 12 × mean
= 12 × 1000
= \$12 000

Since mean monthly wage of 5 inexperienced workers

= $\dfrac{\text{sum of monthly wages}}{5}$,

then sum of monthly wages = 5 × mean
= 5 × 846
= \$4230

Sum of monthly wages of 7 experienced workers = 12 000 – 4230
= \$7770

Mean monthly wage of 7 experienced workers = $\dfrac{7770}{7}$
= \$1110

11. (i) Since mean height = $\dfrac{\text{sum of heights of 3 plants}}{3}$,

then sum of heights of 3 plants = 3 × mean
= 3 × 30
= 90 cm

Height of plant B = $\dfrac{3}{2+3+5} \times 90$
= $\dfrac{3}{10} \times 90$
= 27 cm

(ii) Since mean height = $\dfrac{\text{sum of heights of 4 plants}}{4}$

then sum of heights of 4 plants = 4 × mean
= 4 × 33
= 132 cm

Height of plant D = 132 – 90
= 42 cm

12. (a) Mean mark of students for English

$= \dfrac{\Sigma fx}{\Sigma f}$

$= \dfrac{0 \times 0 + 1 \times 1 + 6 \times 2 + 14 \times 3 + 4 \times 4 + 8 \times 5 + 2 \times 6 + 4 \times 7 + 0 \times 8 + 1 \times 9 + 0 \times 10}{40}$

$= \dfrac{160}{40}$

= 4 marks

Mean mark of students for Mathematics

$= \dfrac{\Sigma fx}{\Sigma f}$

$= \dfrac{0 \times 0 + 4 \times 1 + 1 \times 2 + 6 \times 3 + 5 \times 4 + 10 \times 5 + 3 \times 6 + 5 \times 7 + 3 \times 8 + 1 \times 9 + 2 \times 10}{40}$

$= \dfrac{200}{40}$

= 5 marks

(b) (i) Passing mark for English = 50% × 10
$= \dfrac{50}{100} \times 10$
= 5

Number of students who passed English = 8 + 2 + 4 + 1
= 15

Percentage of students who passed English = $\dfrac{15}{40} \times 100\%$
$= 37\frac{1}{2}\ \%$

(ii) Passing mark for Mathematics = 50% × 10
$= \dfrac{50}{100} \times 100$
= 5

Number of students who did not pass Mathematics
= 4 + 1 + 6 + 5
= 16

Percentage of students who did not pass Mathematics
$= \dfrac{16}{40} \times 100\%$
= 40%

13. (i)

Height (x cm)	Frequency (f)	Mid-value (x)	fx
$0 < x \leqslant 10$	4	5	20
$10 < x \leqslant 20$	6	15	90
$20 < x \leqslant 30$	14	25	350
$30 < x \leqslant 40$	6	35	210
$40 < x \leqslant 50$	10	45	450
	$\Sigma f = 40$		$\Sigma fx = 1120$

Estimate for the mean height of the plants = $\dfrac{1120}{40}$
= 28 cm

(ii) Number of plants not taller than 40 cm = 4 + 6 + 14 + 6
= 30

P(plant not taller than 40 cm) = $\dfrac{30}{40}$
$= \dfrac{3}{4}$

14. (i)

Time taken (t minutes)	**Frequency (f)**	**Mid-value (x)**	fx
$116 \leqslant t < 118$	1	117	117
$118 \leqslant t < 120$	6	119	714
$120 \leqslant t < 122$	23	121	2783
$122 \leqslant t < 124$	28	123	3444
$124 \leqslant t < 126$	27	125	3375
$126 \leqslant t < 128$	9	127	1143
$128 \leqslant t < 130$	5	129	645
$130 \leqslant t < 132$	1	131	131
	$\Sigma f = 100$		$\Sigma fx = 12\ 352$

Estimate for the mean travelling time of the lorries

$= \frac{12\ 352}{100}$

= 123.52 minutes

(ii) Number of lorries which took less than 124 minutes

= 1 + 6 + 23 + 28

= 58

Fraction of lorries which took less than 124 minutes $= \frac{58}{100}$

$= \frac{29}{50}$

15. (i)

Speed (x km/h)	**Frequency (f)**	**Mid-value (x)**	fx
$30 < x \leqslant 40$	16	35	560
$40 < x \leqslant 50$	25	45	1125
$50 < x \leqslant 60$	35	55	1925
$60 < x \leqslant 70$	14	65	910
$70 < x \leqslant 80$	10	75	750
	$\Sigma f = 100$		$\Sigma fx = 5270$

Estimate for the mean speed of the vehicles $= \frac{5270}{100}$

= 52.7 km/h

(ii) Required ratio = 16 : (14 + 10)

= 16 : 24

= 2 : 3

16. (i)

Mean distance (d million km)	**Frequency (f)**
$21.0 \leqslant d < 21.5$	7
$21.5 \leqslant d < 22.0$	0
$22.0 \leqslant d < 22.5$	1
$22.5 \leqslant d < 23.0$	1
$23.0 \leqslant d < 23.5$	8
$23.5 \leqslant d < 24.0$	3

(ii)

Mean distance (d million km)	**Frequency (f)**	**Mid-value (x)**	fx
$21.0 \leqslant d < 21.5$	7	21.25	148.75
$21.5 \leqslant d < 22.0$	0	21.75	0
$22.0 \leqslant d < 22.5$	1	22.25	22.25
$22.5 \leqslant d < 23.0$	1	22.75	22.75
$23.0 \leqslant d < 23.5$	8	23.25	186
$23.5 \leqslant d < 24.0$	3	23.75	71.25
	$\Sigma f = 20$		$\Sigma fx = 451$

Estimate for the mean of the mean distances of the moons from Jupiter

$= \frac{451}{20}$

= 22.55 million km

17. Since mean of x, y and $z = \frac{\text{Sum of } x, y, \text{ and } z}{3}$,

then sum of x, y and z = 3 × mean

= 3 × 6

= 18

Since mean of x, y, z, a and $b = \frac{\text{Sum of } x, y, z, a \text{ and } b}{5}$,

then sum of x, y, z, a and b = 5 × mean

= 5 × 8

= 40

Sum of a and b = 40 – 18

= 22

Mean of a and $b = \frac{22}{2}$

= 11

18. (i) Mean

$= \frac{\text{Sum of the lifespans of 30 light bulbs}}{30}$

$$= \frac{167 + 171 + 179 + 167 + 171 + 165 + 175 + 179 + 169 + 168 + 171 + 177 + 169 + 171 + 177 + 173 + 165 + 175 + 167 + 174 + 177 + 172 + 164 + 175 + 179 + 179 + 174 + 174 + 168 + 171}{30}$$

$= \frac{5163}{30}$

= 172.1 hours

(ii)

Lifespan (x hour)	**Frequency**
$164 \leqslant x < 167$	3
$167 \leqslant x < 170$	7
$170 \leqslant x < 173$	6
$173 \leqslant x < 176$	7
$176 \leqslant x < 179$	3
$179 \leqslant x < 182$	4

(iii)

Lifespan (x hour)	Frequency (f)	Mid-value (x)	fx
$164 \leqslant x < 167$	3	165.5	496.5
$167 \leqslant x < 170$	7	168.5	1179.5
$170 \leqslant x < 173$	6	171.5	1029
$173 \leqslant x < 176$	7	174.5	1221.5
$176 \leqslant x < 179$	3	177.5	532.5
$179 \leqslant x < 182$	4	180.5	722
	$\Sigma f = 30$		$\Sigma fx = 5181$

Estimate for the mean lifespan of the lightbulbs $= \dfrac{5181}{30}$

$= 172.7$ hours

(iv) The two values are different. The value in **(iii)** is an estimate of the actual value **(i)**.

Exercise 17B

1. **(a)** Total number of data = 7

Middle position $= \dfrac{7+1}{2}$

$= 4^{\text{th}}$ position

Rearranging the data in ascending order, we have:

1, 3, 5, 5, 5, 6, 6

$\therefore$ Median = data in the 4^{th} position

$= 5$

(b) Total number of data = 6

Middle position $= \dfrac{6+1}{2}$

$= 3.5^{\text{th}}$ position

Rearranging the data in ascending order, we have:

25, 28, 29, 30, 33, 37

$\therefore$ Median = mean of the data in the 3^{rd} and the 4^{th} position

$= \dfrac{29+30}{2}$

$= 29.5$

(c) Total number of data = 7

Middle position $= \dfrac{7+1}{2}$

$= 4^{\text{th}}$ position

Rearranging the data in ascending order, we have:

1.1, 1.2, 1.6, 2.8, 3.2, 4.1, 4.1

$\therefore$ Median = data in the 4^{th} position

$= 2.8$

(d) Total number of data = 8

Middle position $= \dfrac{8+1}{2}$

$= 4.5^{\text{th}}$ position

Rearranging the data in ascending order, we have:

4.7, 5.5, 8.4, 12, 13.5, 22.6, 31.3, 39.6

$\therefore$ Median = mean of the data in the 4^{th} and the 5^{th} position

$= \dfrac{12+13.5}{2}$

$= 12.75$

2. **(a)** Total number of data = 20

Middle position $= \dfrac{20+1}{2}$

$= 10.5^{\text{th}}$ position

$\therefore$ Median = mean of the data in the 10^{th} and the 11^{th} position

$= \dfrac{39+40}{2}$

$= 39.5$

(b) Total number of data = 21

Middle position $= \dfrac{21+1}{2}$

$= 11^{\text{th}}$ position

$\therefore$ Median = data in the 11^{th} position

$= 70$

(c) Total number of data = 17

Middle position $= \dfrac{17+1}{2}$

$= 9^{\text{th}}$ position

$\therefore$ Median = data in the 11^{th} position

$= 5.7$

(d) Total number of data = 42

Middle position $= \dfrac{42+1}{2}$

$= 21.5^{\text{th}}$ position

$\therefore$ Median = mean of the data in the 21^{th} and the 22^{th} position

$= \dfrac{40+45}{2}$

$= 42.5$

3. **(a)** Mode = 3

(b) Modes = 7.7, 9.3

4. **(a)** Mode = Red

(b) Modes = 78, 79

(c) Mode = 60

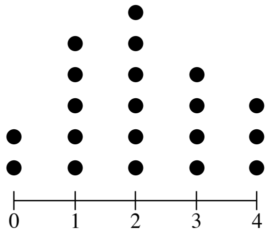
(d) Mode = 30

(e) Each value of x occurs only once. Hence, there is no mode.

5. **(i)** Modal temperature = 27 °C

(ii) Modal temperatures = 22 °C, 27 °C

6. **(a)**



(b) **(i)** Mean $= \dfrac{2 \times 0 + 5 \times 1 + 6 \times 2 + 4 \times 3 + 3 \times 4}{2 + 5 + 6 + 4 + 3}$

$= \dfrac{41}{20}$

$= 2.05$

(ii) Total number of data = 20

Middle position $= \dfrac{20 + 1}{2}$

$= 10.5^{\text{th}}$ position

$\therefore$ Median = mean of the data in the 10^{th} and the 11^{th} position

$= \dfrac{2 + 2}{2}$

$= 2$

(iii) Mode = 2

7. **(a)** The lengths of the pendulums measured by group A range from 49 cm to 85 cm.

The lengths of the pendulums measured by group B range from 53 cm to 83 cm. The length is clustered around 65 cm to 67 cm.

(b) I disagree with the statement. The modal length for group A is 53 cm, and is shorter than the modal lengths for group B, which are 66 cm and 73 cm.

8. Let the eighth number be x.

Total number of data = 8

Middle position $= \dfrac{8 + 1}{2}$

$= 4.5^{\text{th}}$ position

Rearranging the numbers in ascending order,

we can have x, 1, 2, 3, 4, 9, 12, 13 or 1, 2, 3, 4, 9, 12, 13, x or x is between any 2 numbers.

The median is the mean of the 4^{th} position and 5^{th} position.

Since $\dfrac{3 + 4}{2} = 3.5 \neq 4.5$ and $\dfrac{4 + 9}{2} = 6.5 \neq 4.5$, x must be in the 4^{th} position or 5^{th} position.

If x is in the 4^{th} position, $\dfrac{x + 4}{2} = 4.5$

$x + 4 = 9$

$x = 5$ (rejected, since $x \leqslant 4$)

If x is in the 5^{th} position, $\dfrac{4 + x}{2} = 4.5$

$4 + x = 9$

$x = 5$

The eighth number is 5.

9. **(a)** **(i)** Mean distance

$= \dfrac{23 + 24 + 26 + 29 + 30 + 31 + 32 + 32 + 32 + 34 + 34 + 35 + 38 + 42 + 42}{15}$

$= \dfrac{484}{15}$

$= 32.3$ km (to 3 s.f.)

(ii) Total number of data = 15

Middle position $= \dfrac{15 + 1}{2}$

$= 8^{\text{th}}$ position

$\therefore$ Median distance = data in the 8^{th} position

= 32 km

(iii) Modal distance = 32 km

(b) There are 3 prime numbers, i.e. 23, 29 and 31.

P(distance covered is a prime number) $= \dfrac{3}{15}$

$= \dfrac{1}{5}$

10. **(a)** **(i)** Mean allowance

$= \dfrac{4 \times 30 + 5 \times 31 + 9 \times 32 + 7 \times 33 + 4 \times 34 + 1 \times 35}{4 + 5 + 9 + 7 + 4 + 1}$

$= \dfrac{965}{30}$

= \$32.17 (to the nearest cent)

(ii) Total number of data = 30

Middle position $= \dfrac{30 + 1}{2}$

$= 15.5^{\text{th}}$ position

$\therefore$ Median allowance

= mean of the data in the 15^{th} and the 16^{th} position

$= \dfrac{32 + 32}{2}$

= \$32

(iii) Modal allowance = \$32

(b) Fraction of students who receive an allowance of at most \$32 a week

$= \dfrac{4 + 5 + 9}{30}$

$= \dfrac{18}{30}$

$= \dfrac{3}{5}$

11. **(a)** **(i)** $x + 2 + y + 6 + 14 = 40$

$x + y + 22 = 40$

$\therefore x + y = 18$ (shown)

(ii) Mean = 64

$\dfrac{x \times 2 + 2 \times 4 + y \times 6 + 6 \times 8 + 14 \times 10}{40} = 6.4$

$2x + 8 + 6y + 48 + 140 = 256$

$2x + 6y + 196 = 256$

$2x + 6y = 60$

$\therefore x + 3y = 30$ (shown)

(iii) $x + y = 18$ — (1)

$x + 3y = 30$ — (2)

(2) – (1):

$(x + 3y) - (x + y) = 30 - 18$

$x + 3y - x - y = 12$

$2y = 12$

$y = 6$

Substitute $y = 6$ into (1):

$x + 6 = 18$

$x = 12$

$\therefore x = 12, y = 6$

(b) (i) Total number of data = 40

Middle position = $\frac{40 + 1}{2}$

= 20.5^{th} position

$\therefore$ Median = mean of the data in the 20^{th} and the 21^{th} position

= $\frac{6 + 8}{2}$

= 7

(ii) Mode = 10

12. (a) We write the data as follows:

$\underbrace{0, ..., 0}_{5}, 1, 1, 2, \underbrace{3, ..., 3}_{x}$

Since the median is 2,

$\therefore 5 + 2 = x$

$x = 7$

(b) $\underbrace{0, ..., 0}_{5}, 1, 1, 2, \underbrace{3, ..., 3}_{x}$

The smallest value of x occurs when the median is here.

$\therefore 5 = 1 + 1 + x$

$5 = 2 + x$

$x = 3$

$\therefore$ Smallest value of $x = 3$

The greatest value of x occurs when the median is here.

$\therefore 5 + 1 = 1 + x$

$6 = 1 + x$

$x = 5$

$\therefore$ Greatest value of $x = 5$

$\therefore$ Possible values of $x = 3, 4, 5$

13. (i) Jun Wei's mean score = $\frac{45}{9}$

= 5

Raj's mean score = $\frac{42}{9}$

= 4.67 (to 3 s.f.)

(ii) Jun Wei scored better on most of the holes. The mean scores do not indicate this.

(iii) Total number of data = 9

Middle position = $\frac{9 + 1}{2}$

= 5^{th} position

Rearranging Jun Wei's scores in ascending order,

2, 2, 2, 3, 3, 4, 5, 7, 17

$\therefore$ Jun Wei's median score = data in the 5^{th} position

= 3

Rearranging Raj's scores in ascending order,

2, 3, 3, 4, 4, 6, 6, 6, 8

$\therefore$ Raj's median score = data in the 5^{th} position

= 4

(iv) Jun Wei's modal score = 2

Raj's modal score = 6

(v) The mode gives the best comparison. The mode shows the most common score of each player, thus demonstrates the ability of each player the best.

14. (i) The number of students who did less than or equal to 5 pull-ups, or more than or equal to 10 pull-ups are grouped together.

(ii) Total number of data = 21

Middle position = $\frac{21 + 1}{2}$

= 11^{th} position

$\therefore$ Median number by secondary 2A = data in the 11^{th} position

= 7

$\therefore$ Median number by secondary 2B = data in the 11^{th} position

= 7

(iii) Modal number by secondary 2A = 6

Modal number by secondary 2B = 8

(iv) The mode gives a better comparison. The mode shows the most common number of pull-ups by students in both classes, thus giving a better comparison.

15. (i) The number of SMS messages sent by students in June ranges from 65 to 95 messages. Most students sent between 75 to 80 messages in June.

The number of SMS messages sent by students in July ranges from 60 to 95 messages. Most students sent between 80 to 85 messages in July.

(ii) If a datum of 25 is added into June, the mean will be affected more than the median.

Most of the values range from 65 to 95, thus a value of 25 is an extreme value.

Extreme values affect the mean more than the median.

16. (a) $5 + 13 + 15 + x + 1 + y + 2 = 50$

$x + y + 36 = 50$

$x + y = 14 \quad — (1)$

Mean $= 2.18$

$$\frac{5 \times 0 + 13 \times 1 + 15 \times 2 + x \times 3 + 1 \times 4 + y \times 5 + 2 \times 6}{50} = 2.18$$

$$\frac{3x + 5y + 59}{50} = 2.18$$

$3x + 5y + 59 = 109$

$3x + 5y = 50 \quad — (2)$

$5 \times (1)$: $5x + 5y = 70 \quad — (3)$

(3) – (2):

$(5x + 5y) - (3x + 5y) = 70 - 50$

$5x + 5y - 3x - 5y = 20$

$2x = 20$

$x = 10$

Substitute $x = 10$ into (1):

$10 + y = 14$

$y = 4$

$\therefore x = 10, y = 4$

(b) (i) Total number of data = 50

Middle position $= \frac{50 + 1}{2}$

$= 25.5^{\text{th}}$ position

$\therefore$ Median = mean of the data in the 25^{th} and the 26^{th} position

$= \frac{2 + 2}{2}$

$= 2$

(ii) Mode = 2

(c) Number of years with at most p major hurricanes $= 36\% \times 50$

$= \frac{36}{100} \times 50$

$= 18$

Since $5 + 13 = 18$,

$\therefore p = 1$

17. (a) Mean $= 2.2$

$$\frac{4 \times 0 + 6 \times 1 + 3 \times 2 + x \times 3 + 3 \times 4 + 2 \times 5}{4 + 6 + 3 + x + 3 + 2} = 2.2$$

$$\frac{6 + 6 + 3x + 12 + 10}{x + 18} = 2.2$$

$3x + 34 = 2.2(x + 18)$

$3x + 34 = 2.2x + 39.6$

$0.8x = 5.6$

$x = 7$

(b) Total number of data $= 4 + 6 + 3 + 7 + 3 + 2$

$= 25$

Middle position $= \frac{25 + 1}{2}$

$= 13^{\text{th}}$ position

The median is the data in the 13^{th} position.

The greatest possible value of x occurs when

$4 + 6 + (3 - 1) = x + 3 + 2$

$12 = x + 5$

$\therefore x = 7$

(c) Since modal number = 3,

$\therefore$ Smallest possible value of $x = 7$

18. (i) Total number of students

$= x + 1 + x - 2 + x + 2 + x + x - 2 + x - 4 + x - 3$

$= 7x - 8$

$$\text{Mean} = \frac{(x + 1) \times 0 + (x - 2) \times 1 + (x + 2) \times 2 + x \times 3 + (x - 2) \times 4 + (x - 4) \times 5 + (x - 3) \times 6}{7x - 8}$$

$$= \frac{x - 2 + 2x + 4 + 3x + 4x - 8 + 5x - 20 + 6x - 18}{7x - 8}$$

$$= \frac{21x - 44}{7x - 8}$$

Mode = 2

$$\therefore \frac{21x - 44}{7x - 8} = 2$$

$21x - 44 = 2(7x - 8)$

$21x - 44 = 14x - 16$

$7x = 28$

$x = 4$

(ii)

Number of books	0	1	2	3	4	5	6
Number of students	5	2	6	4	2	0	1

Total number of data $= 7(4) - 8$

$= 20$

Middle position $= \frac{20 + 1}{2}$

$= 10.5^{\text{th}}$ position

$\therefore$ Median = mean of the data in the 10^{th} and the 11^{th} position

$= \frac{2 + 2}{2}$

$= 2$

Review Exercise 17

1. (a) Mean $= \frac{8 + 11 + 14 + 13 + 14 + 9 + 15}{7}$

$= \frac{84}{7}$

$= 12$

Total number of data = 7

Middle position $= \frac{7 + 1}{2}$

$= 4^{\text{th}}$ position

Rearrange the numbers in ascending order,

8, 9, 11, 13, 14, 14, 15

$\therefore$ Median = data in the 4^{th} position

$= 13$

Mode = 14

(b) Mean $= \dfrac{88 + 93 + 85 + 98 + 102 + 98}{6}$

$= \dfrac{564}{6}$

$= 94$

Total number of data = 6

Middle position $= \dfrac{6 + 1}{2}$

$= 3.5^{\text{th}}$ position

Rearranging the numbers in ascending order,

85, 88, 93, 98, 98, 102

$\therefore$ Median = mean of data in the 3^{rd} position and 4^{th} position

$= \dfrac{93 + 98}{2}$

$= 95.5$

Mode = 98

2. Since mean of 16 numbers $= \dfrac{\text{sum of the 16 numbers}}{16}$

then sum of the 16 numbers $= 16 \times$ mean

$= 16 \times 7$

$= 112$

Since mean of 6 numbers $= \dfrac{\text{sum of the 6 numbers}}{6}$,

then sum of the 6 numbers $= 6 \times$ mean

$= 6 \times 2$

$= 12$

Sum of the other set of 10 numbers $= 112 - 12$

$= 100$

Mean of the other set of 10 numbers, $x = \dfrac{100}{10}$

$= 10$

3. Since mode = 29 and $a < b$,

a or $b = 29$, as 29 will be the value that occurs most frequently.

Total number of data = 10

Middle position $= \dfrac{10 + 1}{2}$

$= 5.5^{\text{th}}$ position

Consider $a = 29$.

Rearranging the numbers in ascending order,

22, 24, 24, 25, 28, 29, 29, 29, 34, b

$\therefore$ Median = mean of the data in the 5^{th} position and 6^{th} position

$= \dfrac{28 + 29}{2}$

$= 28.5 \neq 27$

Hence, $b = 29$.

Rearranging the numbers in ascending order,

a, 22, 24, 24, 25, 28, 29, 29, 29, 34

By observation, $a = 26$, since median $= \dfrac{26 + 28}{2} = 27$

$\therefore a = 26, b = 29$

4. (a) (i) $h + 800 + 200 + k = 2000$

$h + k + 1000 = 2000$

$h + k = 1000$

(ii) $h + k = 1000$ — (1)

$9h = k$ — (2)

Substitute (2) into (1):

$h + 9h = 1000$

$10h = 1000$

$h = 100$

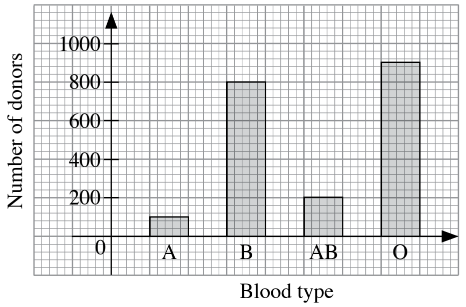
Substitute $h = 1000$ into (2):

$9(100) = k$

$k = 900$

$\therefore h = 100, k = 900$

(b) (i)



(ii) Mode = O

5. (i) Mean annual increment

$= \dfrac{\Sigma fx}{\Sigma f}$

$= \dfrac{2 \times 3 + 1 \times 4 + 6 \times 5 + 1 \times 6 + 2 \times 7 + 7 \times 8 + 1 \times 15}{20}$

$= \dfrac{131}{20}$

$= 6.55\%$

(ii) Total number of data = 20

Middle position $= \dfrac{20 + 1}{2}$

$= 10.5^{\text{th}}$ position

$\therefore$ Median annual increment

= mean of the data in the 10^{th} and 11^{th} position

$= \dfrac{6 + 7}{2}$

$= 6.5\%$

(iii) Modal annual increment = 8%

6. (a) (i) Mean current

$= \dfrac{52 + 68 + 69 + 69 + 70 + 70 + 70 + 71 + 72 + 72 + 72 + 72 + 73 + 74 + 74 + 76 + 76 + 77 + 77 + 77 + 84 + 84 + 85 + 90 + 93}{25}$

$= \dfrac{1867}{25}$

$= 74.68$ A

(ii) Total number of data = 25

Middle position = $\frac{25 + 1}{2}$

= 13^{th} position

$\therefore$ Median = data in the 13^{th} position

= 73 A

(iii) Mode = 72 A

(b) The current flowing through 25 electrical conductors ranges from 52 A to 93 A. The current is clustered around 68 A to 77 A. There is an extreme value of 52 A.

7. (a) (i) $21 + x + y = 18 + 17 = 100$

$x + y + 56 = 100$

$x + y = 44$ (shown)

(ii) Mean of the distribution

$= \frac{\Sigma fx}{\Sigma f}$

$= \frac{21 \times 1 + x \times 2 + y \times 3 + 18 \times 4 + 17 \times 5}{100}$

$= \frac{21 + 2x + 3y + 72 + 85}{100}$

$= \frac{2x + 3y + 178}{100}$

$\frac{2x + 3y + 178}{100} = 2.9$

$2x + 3y + 178 = 290$

$2x + 3y = 112$ (shown)

(iii) $x + y = 44$ — (1)

$2x + 3y = 112$ — (2)

$3 \times (1)$: $3x + 3y = 132$ — (3)

(3) – (2):

$(3x + 3y) - (2x + 3y) = 132 - 112$

$3x + 3y - 2x - 3y = 20$

$x = 20$

Substitute $x = 20$ into (1):

$20 + y = 44$

$y = 24$

$\therefore x = 20, y = 24$

(b) (i) Total number of data = 100

Middle position = $\frac{100 + 1}{2}$

= 50.5^{th} position

$\therefore$ Median

= mean of the data in the 50^{th} position and 51^{th} position

$= \frac{3 + 3}{2}$

= 3

(ii) Mode = 3

8. (a) Mean number of songs

$= \frac{\Sigma fx}{\Sigma f}$

$= \frac{5 \times 10 + 12 \times 15 + 4 \times 20 + m \times 25 + 5 \times 30}{5 + 12 + 4 + m + 5}$

$= \frac{25m + 460}{m + 26}$

$\frac{25m + 460}{m + 26} = 20.25$

$25m + 460 = 20.25(m + 26)$

$25m + 460 = 20.25m + 526.5$

$4.75m = 66.5$

$m = 14$

(b) We write the data as follows:

$\underbrace{10, ..., 10}_{5}, \underbrace{15, ..., 15}_{12}, \underbrace{20, ..., 20}_{4}, \underbrace{25, ..., 25}_{m}, \underbrace{30, ..., 30}_{5}$

Since median = 20,

the smallest possible value of m occurs when the median is here.

$5 + 12 = (4 - 1) + m + 5$

$17 = m + 8$

$m = 9$

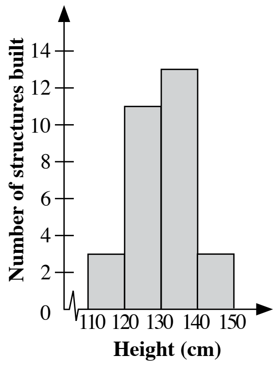
(c) Since mode = 15,

$\therefore$ greatest possible value of $m = 11$

9. (a)

Height (x cm)	Frequency
$110 \leqslant x < 120$	3
$120 \leqslant x < 130$	11
$130 \leqslant x < 140$	13
$140 \leqslant x < 150$	3

(b) (i)



(ii)

Height (x cm)	Frequency (f)	Mid-value (x)	fx
$110 \leqslant x < 120$	3	115	345
$120 \leqslant x < 130$	11	125	1375
$130 \leqslant x < 140$	13	135	1755
$140 \leqslant x < 150$	3	145	435
	$\Sigma f = 30$		$\Sigma fx = 3910$

Estimate for the mean height of the structures $= \dfrac{3910}{30}$

$= 130\dfrac{1}{3}$ cm

(iii) Percentage of structures shorter than 140 cm

$= \dfrac{3 + 11 + 13}{30} \times 100\%$

$= \dfrac{27}{30} \times 100\%$

$= 90\%$

10. (a) (i) Mean score of team Cheetah $= \dfrac{65 + 95 + 32 + 96 + 88}{5}$

$= \dfrac{376}{5}$

$= 75.2$

Mean score of team Jaguar $= \dfrac{50 + 90 + 65 + 87 + 87}{5}$

$= \dfrac{379}{5}$

$= 75.8$

Mean score of team Puma $= \dfrac{90 + 85 + 46 + 44 + 80}{5}$

$= \dfrac{345}{5}$

$= 69$

I would join team Jaguar as the team has the highest mean score.

(ii) Total number of data = 5

Middle position $= \dfrac{5 + 1}{2}$

$= 3^{\text{rd}}$ position

Rearranging the scores of team Cheetah,

32, 65, 88, 95, 96

Median score of team Cheetah = data in the 3^{rd} position

$= 88$

Rearranging the scores of team Jaguar,

50, 65, 87, 87, 90

Median score of team Jaguar = data in the 3^{rd} position

$= 87$

Rearranging the scores of team Puma,

44, 46, 80, 85, 90

Median score of team Puma = data in the 3^{rd} position

$= 80$

I would join team Cheetah as the team has the highest median score.

(b) I would report the median score as the median score of 87 is higher than the mean score of 75.8.

Challenge Yourself

1. Let the 4 numbers be a, b, c and d, where $a \leqslant b \leqslant c \leqslant d$.

Since mean $= \dfrac{\text{sum of the 4 numbers}}{4}$,

then sum of the 4 numbers $= 4 \times$ mean

$= 4 \times (x + y + 5)$

$= 4x + 4y + 20$

$\therefore a + b + c + d = 4x + 4y + 20$ — (1)

Total number of data = 4

Middle position $= \dfrac{4 + 1}{2}$

$= 2.5^{\text{th}}$ position

Arranging the numbers in ascending order,

a, b, c, d

Median = mean of the data in the 2^{nd} position and 3^{rd} position

$= \dfrac{b + c}{2}$

$\therefore \dfrac{b + c}{2} = x + y$ — (2)

Since mode $= x$,

x occurs twice. x cannot occur 3 times because median

$= \dfrac{x + x}{2} = x \neq x + y$.

Since the 4 numbers are whole numbers,

$\therefore a = b = x$

Substitute $b = x$ into (2):

$\dfrac{x + c}{2} = x + y$

$x + c = 2x + 2y$

$c = x + 2y$

Substitute $a = b = x$ and $c = x + 2y$ into (1):

$x + x + x + 2y + d = 4x + 4y + 20$

$d + 3x + 2y = 4x + 4y + 20$

$d = x + 2y + 20$

$\therefore$ The numbers are $x, x, x + 2y, x + 2y + 20$

2. $a + \dfrac{1}{2}b = 13 - \dfrac{1}{2}e$ — (1)

$c + \dfrac{1}{2}e + f = 8 - \dfrac{1}{2}b - d$ — (2)

(1) + (2):

$\left(a + \dfrac{1}{2}b\right) + \left(c + \dfrac{1}{2}e + f\right) = \left(13 - \dfrac{1}{2}e\right) + \left(8 - \dfrac{1}{2}b - d\right)$

$a + \dfrac{1}{2}b + c + \dfrac{1}{2}e + f = 21 - \dfrac{1}{2}b - d - \dfrac{1}{2}e$

$a + b + c + d + e + f = 21$

Mean of a, b, c, d, e and $f = \dfrac{a + b + c + d + e + f}{6}$

$= \dfrac{21}{6}$

$= 3.5$

3. Given: There are 3 (and only 3) boys with a height of 183 cm and one (and only one) boy with a height of 187 cm.

Since mean = $\frac{\text{sum of the heights of the 9 boys}}{9}$,

then sum of the heights of the 9 boys $= 9 \times \text{mean}$
$= 9 \times 183$
$= 1647$ cm

Since 3 boys have a height of 183 cm, and the tallest boy has a height of 187 cm,

Sum of the heights of the remaining boys $= 1647 - (3 \times 183) - 187$
$= 1647 - 549 - 187$
$= 911$ cm

Total number of data = 9

Middle position $= \frac{9 + 1}{2}$
$= 5^{th}$ position

Median = 183 cm and this data is in the 5^{th} position.

Since mode = 183 cm, it occurs 3 times. And the other heights can occur at most 2 times.

We let the heights of 4 boys be 186 cm, 186 cm, 185 cm and 182 cm.

Least possible height of the shortest boy
$= 911 - 186 - 186 - 185 - 182$
$= 172$ cm

Check:

Rearranging the heights of the 9 boys in ascending order:
172, 182, 183, 183, 183, 185, 186, 186, 187

Mean $= \frac{172 + 182 + 183 + 183 + 183 + 185 + 186 + 186 + 187}{9}$
$= \frac{1647}{9}$
$= 183$ cm

Median = data in the 5^{th} position
$= 183$ cm

Mode = 183 cm

Revision Exercise D1

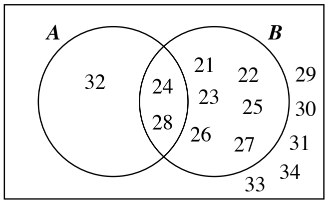
1. **(a)** $\xi = \{21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34\}$

$A = \{24, 28, 32\}$

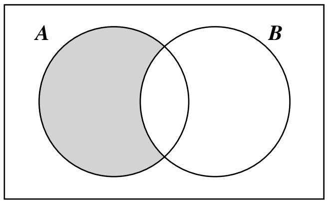
$B = \{21, 22, 23, 24, 25, 26, 27, 28\}$

$A' \cup B' = \{21, 22, 23, 25, 26, 27, 29, 30, 31, 32, 33, 34\}$

(b) ξ



(c) ξ



2. The sample space consists of the integers 10, 11, 12, ..., 99.

Total number of possible outcomes $= 99 - 9$

$= 90$

(i) There are 45 even numbers, i.e. 10, 12, 14, ..., 98.

P(number is an even number) $= \dfrac{45}{90}$

$= \dfrac{1}{2}$

(ii) There are 3 numbers less than 18, i.e. 11, 13 and 17.

P(number is a prime number less than 18) $= \dfrac{3}{90}$

$= \dfrac{1}{30}$

(iii) There are 9 multiples of 10, i.e. 10, 20, 30, 40, 50, 60, 70, 80 and 90.

P(number is a multiple of 10) $= \dfrac{9}{90}$

$= \dfrac{1}{10}$

(iv) There are 8 numbers divisible by both 3 and 4, i.e. 12, 24, 36, 48, 60, 72, 84, 96.

P(number is divisible by both 3 and 4) $= \dfrac{8}{90}$

$= \dfrac{4}{45}$

3. Total number of possible outcomes $= 52 - 13$

$= 39$

(i) There is no ten of hearts in the remaining pack of cards.

P(drawing the ten of hearts) $= \dfrac{0}{39}$

$= 0$

(ii) There are 13 red cards in the remaining pack of cards, i.e. 13 diamonds.

P(drawing a red card) $= \dfrac{13}{39}$

$= \dfrac{1}{3}$

(iii) There are 6 cards that are either a king or an ace.

P(drawing either a king or an ace) $= \dfrac{6}{39}$

$= \dfrac{2}{13}$

(iv) There are 3 cards that are a two, i.e. the two of diamonds, the two of clubs and the two of spades.

P(drawing a card which is a two) $= \dfrac{3}{39}$

$= \dfrac{1}{13}$

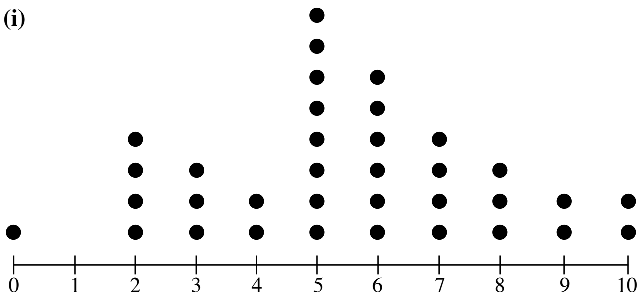
P(drawing a card which is not a two)

$= 1 -$ P(drawing a card which is a two)

$= 1 - \dfrac{1}{13}$

$= \dfrac{12}{13}$

4. **(i)**



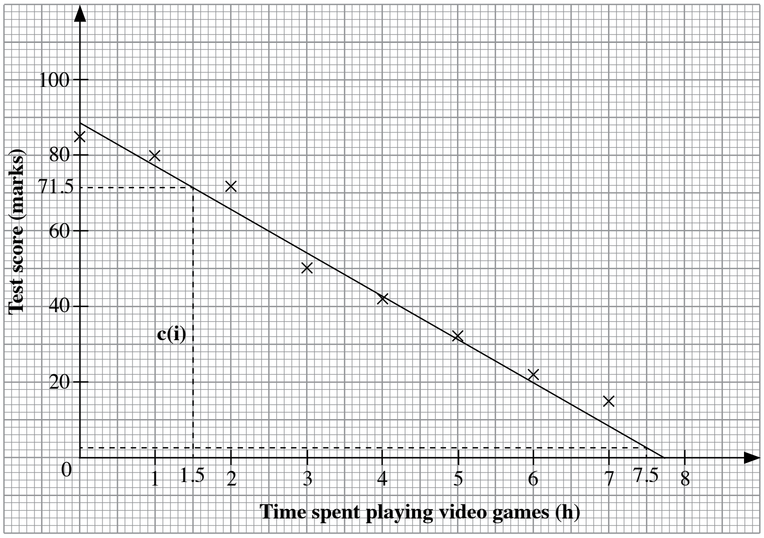
(ii) The most common number of times is 5 times.

(iii) Percentage of students who did not fall ill last year

$= \dfrac{1}{35} \times 100\%$

$= 2\dfrac{6}{7}\%$

5.



(c) From graph,

(i) When $x = 1.5$, $y = 71.5$

$\therefore$ It is estimated that a student who spends 1.5 hours playing video games daily will score 71.5 marks.

(ii) When $x = 7.5$, $y = 3$

$\therefore$ It is estimated that a student who spends 7.5 hours playing video games daily will score 3 marks.

(d) Gradient $= \dfrac{\text{rise}}{\text{run}}$

$= \dfrac{71.5 - 3}{1.5 - 7.5}$

$= -11.4$ (to 3 s.f.)

y-intercept = 89

$\therefore$ The equation of the best fit line is $y = -11.4x + 89$

(e) A strong, negative correlation exists between playing video games and test scores.

6. **(i)** Total number of Secondary Two students

$= 40 + 30 + 50 + 35 + 15 + 5$

$= 175$

(ii) Required ratio = 30 : 50

= 3 : 5

(iii) P(student consumes at least 3 servings of rice)

$= \dfrac{35 + 15 + 5}{175}$

$= \dfrac{55}{175}$

$= \dfrac{11}{35}$

7. **(i)** Mean

$= \dfrac{\Sigma fx}{\Sigma f}$

$= \dfrac{6 \times 10 + 10 \times 11 + 20 \times 12 + 22 \times 13 + 14 \times 14 + 6 \times 15 + 2 \times 16}{6 + 10 + 20 + 22 + 14 + 6 + 2}$

$= \dfrac{1014}{80}$

$= 12.675$

(ii) Total number of data = 80

Middle position $= \dfrac{80 + 1}{2}$

$= 40.5^{\text{th}}$ position

$\therefore$ Median = mean of the data in the 40^{th} position and 41^{th} position

$= \dfrac{13 + 13}{2}$

$= 13$

(iii) Mode = 13

8. **(i)**

Lifespan (x days)	**Frequency (f)**	**Mid-value (x)**	***fx***
$0 < x \leqslant 10$	6	5	30
$10 < x \leqslant 20$	11	15	165
$20 < x \leqslant 30$	22	25	550
$30 < x \leqslant 40$	29	35	1015
$40 < x \leqslant 50$	32	45	1440
	$\Sigma f = 100$		$\Sigma fx = 3200$

Estimate for the mean lifespan of the bees $= \dfrac{3200}{100}$

= 32 days

(ii) Fraction of bees whose lifespans are at least 40 days $= \dfrac{32}{100}$

$= \dfrac{8}{25}$

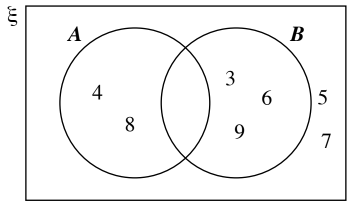
Fraction of bees whose lifespans are at most 40 days

$= 1 - \dfrac{8}{25}$

$= \dfrac{17}{25}$

Revision Exercise D2

1. ξ = {3, 4, 5, 6, 7, 8, 9}
   A = {4, 8}
   B = {3, 6, 9}
   (i)



   (ii) n($A \cap B$) = 0
   (iii) $A \cup B$ = {3, 4, 6, 8, 9}

2. Total number of possible outcomes = 28
   (i) Number of boys who have visited the park = 28 – 16 – 6
   = 6

   $$\text{P(student is a boy who has visited the park)} = \frac{6}{28} = \frac{3}{14}$$

   (ii) Number of students who have not visisted the park
   = 28 – 2 – 6
   = 20

   $$\text{P(student has not visited the park)} = \frac{20}{28} = \frac{5}{7}$$

3. Total number of boys and girls after the sports fair
   = 48 + 2 + x + 2x
   = 50 + 3x
   Total number of boys after the sports fair = 48 + x

   $$\text{P(boy selected at random)} = \frac{2}{5}$$
   $$\frac{48 + x}{50 + 3x} = \frac{2}{5}$$
   $$5(48 + x) = 2(50 + 3x)$$
   $$240 + 5x = 100 + 6x$$
   $$x = 140$$

   Number of girls who joined the team = 2 × 140
   = 280

4. **(i)**

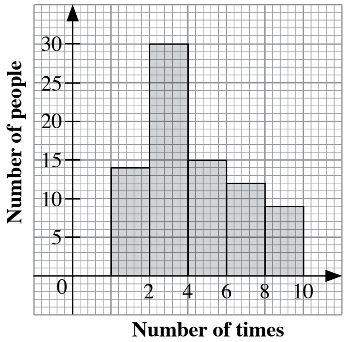
Stem	**Leaf**
10	3 4
10	5 6 7 7 7 8 8 8 8 9 9 9
11	1 2 2 2 2 3 4
11	5 6 8 8 9
12	2 3
12	6
13	
13	
14	0

   Key : 10 | 3 means 10.3 seconds

   (ii) Number of students who took less than 11.5 seconds
   = 2 + 12 + 7
   = 21
   Percentage of students who scored an A for the run

   $$= \frac{21}{30} \times 100\%$$
   = 70%

5. **(i)**



   (ii) Total number of people = 14 + 30 + 15 + 12 + 9
   = 80

   $$\text{P(withdrew at least } p \text{ times last week)} = \frac{33}{40}$$

   Number of people who withdrew at least p times last week

   $$= \frac{33}{40} \times 80$$
   = 66
   Since 30 + 15 + 12 + 9 = 66,
   ∴ p = 2

6. Since mean of a, b, c and $d = \dfrac{\text{sum of } a,\ b,\ c \text{ and } d}{4}$
   then sum of a, b, c and d = 4 × mean
   = 4 × 8
   = 32

   Since mean of a, b, c, d, e and $f = \dfrac{\text{sum of } a,\ b,\ c,\ d,\ e \text{ and } f}{6}$
   then sum of a, b, c, d, e and f = 6 × mean
   = 6 × 7.6
   = 45.6

   Sum of e and f = 45.6 – 32
   = 13.6

   $$\text{Mean of } e \text{ and } f = \frac{13.6}{2}$$
   = 6.8

7. (a) $3 + 4 + x + 7 + y + 5 + 4 = 36$

$$x + y + 23 = 36$$

$$x + y = 13 \quad \text{— (1)}$$

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$6 = \frac{3 \times 3 + 4 \times 4 + x \times 5 + 7 \times 6 + y \times 7 + 5 \times 8 + 4 \times 9}{36}$$

$$6 = \frac{5x + 7y + 143}{36}$$

$$5x + 7y + 143 = 216$$

$$5x + 7y = 73 \quad \text{— (2)}$$

From (1), $y = 13 - x$ — (3)

Substitute (3) into (2):

$$5x + 7(13 - x) = 73$$

$$5x + 91 - 7x = 73$$

$$2x = 18$$

$$x = 9$$

Substitute $x = 9$ into (3):

$$y = 13 - 9$$

$$= 4$$

$\therefore x = 9, y = 4$

(b) (i) Total number of data = 36

$$\text{Middle position} = \frac{36 + 1}{2}$$

$$= 18.5^{\text{th}} \text{ position}$$

$\therefore$ Median

$= \text{mean of the data in the } 18^{\text{th}} \text{ position and } 19^{\text{th}} \text{ position}$

$$= \frac{6 + 6}{2}$$

$$= 6$$

(ii) Mode = 5

Problems in Real-World Contexts

1. Kate's mean time $= \dfrac{\text{2 minutes 21 seconds + 1 minute 50 seconds + 2 minutes 31 seconds}}{3}$

$= \dfrac{\text{6 minutes 42 seconds}}{3}$

= 2 minutes 14 seconds

Lixin's mean time $= \dfrac{\text{1 minute 58 seconds + 2 minutes 11 seconds + 2 minutes 20 seconds}}{3}$

$= \dfrac{\text{6 minutes 29 seconds}}{3}$

= 2 minutes 10 seconds (to the nearest second)

Nora's mean time $= \dfrac{\text{1 minute 59 seconds + 2 minutes 1 second + 2 minutes}}{3}$

$= \dfrac{\text{6 minutes}}{3}$

= 2 minutes

Based on the mean timings, Nora is the fastest swimmer. Moreover, her timings are consistent because they differ from her mean timing of 2 : 00 by at most one second. Hence, Nora should be selected to participate in the swimming competition.

The median is not used as the three students had only 3 attempts and the number of data is too low to consider the median as a basis for selection.

Similarly, the mode is not used as the number of data is too low and no times were repeated.

For weaker students, more scaffolding questions can be given, for example:

(i) *For the 1st attempt, who is the fastest swimmer? Explain how you got your answer. (Fastest swimmer – shortest time)*

(ii) *What criteria would you use to select the participant for the swimming competition?*

(a) *Fastest overall, i.e. Kate has the shortest time overall (1 : 50)*

(b) *Fastest based on mean timing, i.e. Nora has the shortest mean time (2 : 00)*

Teachers may wish to ask students who should be selected to participate in the swimming competition if the fastest timing of each student is considered and the potential drawback for the choice.

The following conclusions may be made:

(a) *Nora's timing is the most consistent among the three students.*

(b) *Nora's timing may not improve much.*

(c) *Kate's timing may improve with training as her best time is much better than Nora's.*

2. (i) At the side of the pyramid with the entrance,

number of rhombus-shaped glass panels $= 603 - 3 \times 153$

$= 603 - 459$

$= 144$

number of triangular glass panels $= 70 - 3 \times 18$

$= 16$

(ii) Volume of the pyramid $= \frac{1}{3} \times$ base area $\times$ height

$= \frac{1}{3} \times 35 \times 35 \times 20.6$

$= 8411\frac{2}{3}\text{ m}^3$

(iii) Using Pythagoras' Theorem,

Slant height, $l = \sqrt{\left(\frac{35}{2}\right)^2 + 20.6^2}$

$= 27.03$ m (to 4 s.f.)

Total area of glass panels on each side of the pyramid without the entrance

$= \frac{1}{2} \times 35 \times 27.03$

$= 473\text{ m}^2$ (to 3 s.f.)

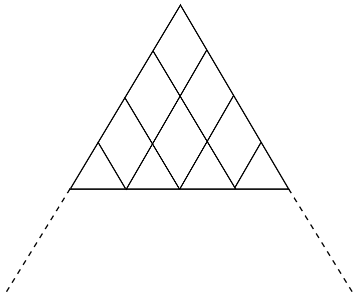
Teachers may expand the question and ask students to consider how the rhombus-shaped glass panels and triangular panels are arranged at each side of the pyramid without the entrance.

Students should note

(i) there is 1 rhombus at row 1

(ii) there are 2 rhombuses at row 2 and so on

(iii) there are n rhombuses at row n, so row n represents the bottommost row where the base is.



By varying the number of rhombuses/rows, using number patterns, teachers can ask students to find the number of rhombuses/rows/triangular panels required (e.g. when number of rhombuses = 153, number of rows = 17 and number of triangular panels = 18)

3. (a) In 2010,

percentage of reported cases where victims were cheated

$= \frac{175}{346} \times 100\%$

$= 50.6\%$ (to 3 s.f.)

In 2011,

percentage of reported cases where victims were cheated

$= \frac{183}{298} \times 100\%$

$= 61.4\%$ (to 3 s.f.)

In 2012,

percentage of reported cases where victims were cheated

$= \frac{181}{327} \times 100\%$

$= 55.4\%$ (to 3 s.f.)

(b) In 2010,

mean amount of money cheated $= \frac{3\,800\,000}{175}$

$= \$22\,000$ (to the nearest thousand)

In 2011,

mean amount of money cheated $= \frac{6\,400\,000}{183}$

$= \$35\,000$ (to the nearest thousand)

In 2012,

mean amount of money cheated $= \frac{7\,400\,000}{181}$

$= \$41\,000$ (to the nearest thousand)

Teachers may wish to note that for each of the given years, the mean amount of money cheated per case is an estimate because the total amount of money cheated per case is correct to 2 significant figures. Thus unless we want each of the answers to be corrected to 1 significant figure (which we prefer not to because we would like to obtain $35 000 for the mean amount of money cheated per case for the year 2011, we have to write it as an estimate to the nearest thousand, knowing it may not be accurate to the nearest thousand. Notice also that the question requires students to leave their answers 'to the nearest thousand', not 'correct to the nearest thousand'.

(c) Yes, the lucky draw scams should be a concern for the police. The total amount of money cheated has been increasing and the mean amount of money cheated has been increasing for the past 3 years.

Beside the total amount of money cheated and the mean amount of money cheated, teachers may prompt students to look at the other data given and computed

(i) Number of reported cases

(ii) Number of cases where victims were successfully cheated

(iii) Percentage of cases where victims were successfully cheated and decide if these should also be a cause for concern for the police.

4. The average walking speed of a human is approximately 5 km/h.

In 5 minutes, Vishal will walk $\frac{5}{60} \times 5 = \frac{5}{12}$ km

$= 0.417$ km (to 3 s.f.)

$= 417$ m

Distance of route which Vishal takes to walk from Clementi MRT Station to his block ≈ 650 m

Since 417 m < 650 m, Vishal's claim is not true.

Teachers may instruct students to obtain an estimate of the distance of the route Vishal walks and the average walking speed of a human by searching relevant sources, e.g. maps, the Internet etc.

Teachers may want to ask the students to do the following.

(i) Get students to find out their average walking speed. This can be done at the school running track.

(ii) Using their walking speed, each student can compute the time taken to cover the distance in the given route.

(iii) If none of the students' timing is close to 5 minutes, then we can refute Vishal's claim.

Teachers can change the scenario to one situated near their school. Then, the students can verify the claim by walking along the route and recording their timings. They can then compute the timings of the class statistically and make relevant conclusions.

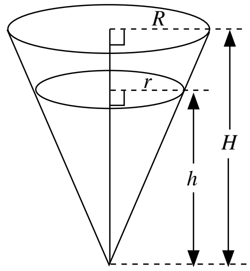
5. (i) Ethan's method is obviously wrong. Since the cross section of the lower half of the glass is smaller than the cross section of its upper half, the volume of the wine will be less than half the volume of the glass.

(ii) *Teachers may conduct a poll to find out students' guesses. It is likely that most students will think that either Jun Wei's or Michael's method is correct.*

(iii) Let the height and the radius of the glass be H and R respectively. Let the height and the radius of the cocktail be h and r respectively.

Volume of glass $= \frac{1}{3}\pi R^2 H$

Volume of cocktail $= \frac{1}{3}\pi r^2 h$



Using similar triangles,

$$\frac{r}{R} = \frac{h}{H}$$

$$\therefore \frac{r^2}{R^2} = \frac{h^2}{H^2} \quad \text{— (1)}$$

$$\frac{\text{Volume of cocktail}}{\text{Volume of glass}} = \frac{1}{2}$$

$$\frac{\frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi R^2 H} = \frac{1}{2}$$

$$\frac{r^2 h}{R^2 H} = \frac{1}{2} \quad \text{— (2)}$$

Substitute (1) into (2):

$$\frac{h^2 h}{H^2 H} = \frac{1}{2}$$

$$\frac{h^3}{H^3} = \frac{1}{2}$$

$$\left(\frac{h}{H}\right)^3 = \frac{1}{2}$$

$$\frac{h}{H} = \sqrt[3]{\frac{1}{2}}$$

$$= \frac{1}{\sqrt[3]{2}}$$

$$\therefore h = \frac{1}{\sqrt[3]{2}} H$$

$$= 0.794H \text{ (to 3 s.f.)}$$

∴ Rui Feng's method of filling the glass to four-fifths of its depth is the closest to getting half a glass of cocktail.

*As mentioned in **(ii)**, it is likely that most students will think that either Jun Wei's or Michael's method is correct. After students have found the answer for **(iii)**, teachers may wish to guide them to see that they have underestimated the volume of the upper part of the glass, i.e. $\frac{1}{5}$ of the volume of its upper part ≈ $\frac{4}{5}$ of the volume of its lower part.*

6. **(a)** If the speed just before the brakes are applied doubles, the braking distance does not always double.
From the table, when the speed of the car just before the brakes are applied increased from 20 km/h to 40 km/h, the braking distance increases from 2 m to 8 m.
When the speed of the car just before the brakes are applied increased from 50 km/h to 100 km/h, the braking distance increases from 12 m to 45 m.

(b) **(i)** From the linear trendline,
Braking distance of car ≈ 16 m
Using the equation of the linear trendline,
Braking distance of car $= 0.3694 \times 45$
$= 16.6$ m (to 3 s.f.)
From the quadratic trendline,
Braking distance of car ≈ 10 m
Using the equation of the quadratic trendline,
Braking distance of car $= 0.0042 \times 45^2 + 0.0385 \times 45$
$= 8.505 + 1.7325$
$= 10.2$ m (to 3 s.f.)

(ii) From the linear trendline,
Braking distance of car ≈ 2 m
Using the equation of the linear trendline,
Braking distance of car $= 0.3694 \times 5$
$= 1.85$ m (to 3 s.f.)
From the quadratic trendline,
Braking distance of car ≈ 0.5 m
Using the equation of the quadratic trendline,
Braking distance of car $\approx 0.0042 \times 5^2 + 0.0385 \times 5$
$= 0.105 + 0.1925$
$= 0.298$ m (to 3 s.f.)

(c) The quadratic trendline provides a better model of the braking distance of the car as the quadratic curve is a better fit to the points than the linear trendline.
Moreover, from **(b)**, we observe that it is unlikely that the braking distance and the speed of the car just before the brakes are applied are in direct proportion because if the speed of the car just before the brakes are applied doubles, the braking distance does not always double.

(d)

Speed (km/h)	0	10	20	30	40	50
Braking distance (m)	0	1	2	5	8	12
Distance travelled (m)	0	5.56	11.1	16.7	22.2	27.8

Speed (km/h)	60	70	80	90	100
Braking distance (m)	17	24	31	38	45
Distance travelled (m)	33.3	38.9	44.4	50	55.6

Regardless of the speed the car it is travelling, the braking distance of the car is always less than the distance travelled in 2 seconds. Hence, the car will always be able to come to a stop in time if the driver follows the two-second rule.

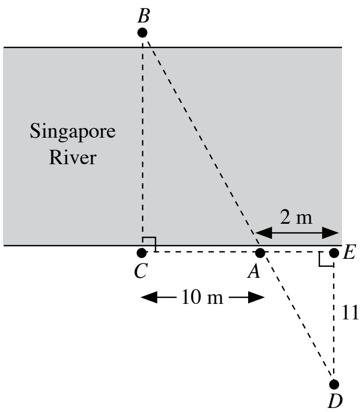
Teachers may also wish to note that the braking time is not used to determine whether the car is able to come to a stop in time, as the speed of the car is not constant after the brakes are applied.

As the braking distance of the car is much less than its distance travelled in 2 seconds, it is tempting to think that the two-second rule can be shortened. However, it is not possible for the driver to apply the brakes immediately when he sees that the vehicle in front has started to slow down or stop, i.e. we need to take into account his reaction time before he applies the brakes.

Teachers should note that the reaction time of the driver is not considered in this question. The reaction time is determined by the alertness of the driver, and will affect and determine if he can keep a safe distance from the vehicle in front.

7. **(a)** Some examples of assumptions that need to be made are as follows:
- The width of the river may not be the same for this stretch of the river (the bridge is longer than the line *BC* in the figure in **(b)**).
- The far bank at this part of the river should preferably have an object to act as a point of reference since it is too troublesome, or sometimes not feasible, to travel across the river by boat or by another bridge at other parts of the river, to the far bank to erect a pole as a point of reference. Otherwise, a person has no choice but to travel across the river to the far bank to erect a pole as a point of reference.

(b) The figure shows a drawing of the river. *B* represents a tree near the edge of the river on the far bank (see the photo in the problem).
The points *C*, *A* and *E* represent vertical poles on the railing on the near bank near the edge of the river. *CAE* is parallel to the river. If there is no railing on the near bank, poles can be erected at *C*, *A* and *E*. *C* is directly opposite *B*.
We walk inland from *E* perpendicular to the railing *CAE* until we see that the pole at *A* coincides with the tree at *B*. This will be the position of the point *D* where a vertical pole will be erected, i.e. *DAB* is a straight line.



(not to scale)

Measure the lengths of CA, AE and DE,

Suppose $CA = 10$ m, $AE = 2$ m and $DE = 11.2$ m.

Since $\triangle ABC$ is similar to $\triangle ADE$,

then $\frac{BC}{DE} = \frac{AC}{AE}$

i.e. $\frac{BC}{11.2} = \frac{10}{2}$

$\therefore BC = \frac{10}{2} \times 11.2$

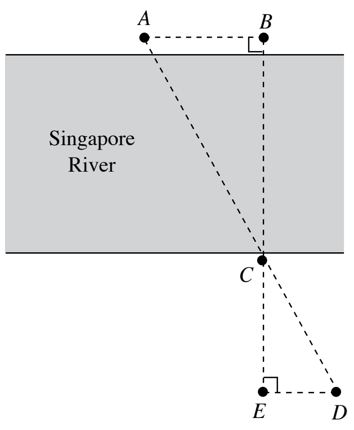
$= 56$ m

$\therefore$ Width of this stretch of Singapore River ≈ 56 m

(c) The mathematical solution may not be the same as the solution for the real-world problem because there is a need to take into account the first assumption made in **(a)**, and the distance of the tree at B and the railing CAE from the edge of the river. The latter is not stated as one of the assumption in **(a)** because we have to take this into account, i.e. we cannot say that these distances are negligible.

(d) Some examples of alternative methods of solution which may not be feasible are as follows:

- Using congruent triangles instead of the similar triangles as shown in **(b)** because there may not be enough space on the near bank such that $DE = BC$.
- Using similar triangles as shown in the following figure because a person has to travel across the river to the far bank to measure AB (the need for two points of reference on the far bank, instead of one point of reference as stated in one of the assumptions in **(a)**, is less an issue since if a person has to travel across to the far bank, he can easily erect two poles to use as points of reference.)



(not to scale)

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